UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2004/2005

October 2004

ZCT 211E/2 - Vector Analysis
[Analisis Vektor]

Duration : 2 hours
[Masa : 2 jam]

Please check that this examination paper consists of **FOUR** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **EMPAT** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer all **FIVE** (5) questions. Students are allowed to answer all questions in Bahasa Malaysia or in English.

*[**Arahan:** Jawab kesemua **LIMA** (5) soalan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]*

1. (a) What vector must be added to the two vectors $\hat{i}-2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$, so that the resultant may be a unit vector along the x-axis?

[(a) Apakah vektor yang perlu ditambah kepada dua vektor $\hat{i}-2\hat{j}+2\hat{k}$ dan $2\hat{i}+\hat{j}-\hat{k}$ supaya paduannya adalah satu vektor unit sepanjang paksi-x?

(20/100)

(b) If $\vec{A}=4\hat{i}+6\hat{j}+2\hat{k}$ and $\vec{B}=\hat{i}+6\hat{j}+k$, find the magnitudes and direction cosines of $(\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$.

[(b) Jika $\vec{A}=4\hat{i}+6\hat{j}+2\hat{k}$ dan $\vec{B}=\hat{i}+6\hat{j}+k$, cari magnitud dan kosinus arah bagi $(\vec{A}+\vec{B})$ dan $(\vec{A}-\vec{B})$.

(40/100)

(c) A car is driven eastward for a distance of 5 kilometers, then northward for 3 kilometers and then in a direction 30° east of north for 10 kilometers. Draw the vector diagram and determine the total displacment of the car from its starting point.

[(c) Sebuah kereta dipandu ke timur untuk jarak 5 kilometer, kemudian ke utara untuk jarak 3 kilometer, dan seterusnya di dalam arah 30° timur daripada utara untuk jarak 10 kilometer. Lukiskan gambarajah vektor dan tentukan jumlah sesaran kereta tersebut daripada titik permulaannya.

(40/100)

2. (a) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, show that

[(a) Jika $\vec{a}$ dan $\vec{b}$ adalah vektor unit dan θ adalah sudut diantaranya, tunjukkan

$$\sin\frac{\theta}{2}=\frac{1}{2}\left|\vec{a}-\vec{b}\right| .$$

(20/100)

(b) Show that $\vec{a}$ is perpendicular to $\vec{b}$ if $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-b\right|$.

[(b) Tunjukkan bahawa $\vec{a}$ tegak lurus kepada $\vec{b}$ jika $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-b\right|$.

(20/100)

(c) If $\vec{A}=4\hat{i}-5\hat{j}+3\hat{k}$, $\vec{B}=2\hat{i}-10\hat{j}-7\hat{k}$ and $\vec{C}=5\hat{i}+7\hat{j}-4\hat{k}$ deduce the values of

[(c) Jika $\vec{A}=4\hat{i}-5\hat{j}+3\hat{k}$, $\vec{B}=2\hat{i}-10\hat{j}-7\hat{k}$ dan $\vec{C}=5\hat{i}+7\hat{j}-4\hat{k}$ dapatkan nilai bagi

(i) $(\vec{A}\times\vec{B}).\vec{C}$ and *[dan]*

(ii) $\vec{A}\times(\vec{B}\times\vec{C})$.

(60/100)

3. (a) Show that $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=0$.

[(a) Tunjukkan bahawa $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=0$.*]*

(15/100)

(b) Find the volume of a paralleopiped whose three coterminus edges are described by the vectors $\hat{i}+2\hat{j}$, $4\hat{j}$ and $\hat{j}+3\hat{k}$.

[(b) Cari isipadu bagi paralelepiped di mana tiga sisi koterminus adalah diberi oleh vektor $\hat{i}+2\hat{j}$, $4\hat{j}$ *dan* $\hat{j}+3\hat{k}$.*]*

(15/100)

(c) Differentiate $\frac{\vec{r}\times\vec{a}}{\vec{r}.\vec{a}}$, $\vec{a}$ being constant vector and $\vec{r}$ is the position vector.

[(c) Bezakan $\frac{\vec{r}\times\vec{a}}{\vec{r}.\vec{a}}$, $\vec{a}$ *adalah vektor malar dan* $\vec{r}$ *ialah vektor kedudukan.]*

(20/100)

(d) Find $\frac{\partial s}{\partial y}, \frac{\partial^2 s}{\partial x^2}, \frac{\partial^2 s}{\partial y^2}, \frac{\partial^2 s}{\partial x\partial y}$ for the following functions:

[(d) Cari $\frac{\partial s}{\partial y}, \frac{\partial^2 s}{\partial x^2}, \frac{\partial^2 s}{\partial y^2}, \frac{\partial^2 s}{\partial x\partial y}$ *bagi fungsi-fungsi berikut]*

(i) $\vec{s}=x\cos y\,\hat{i}+x\sin y\,\hat{j}+ae^{my}\hat{k}$, where *m* is an integer *[di sini m adalah integer]*

(ii) $\vec{s}=\frac{1}{2}\vec{a}(x+y)\hat{i}+\frac{1}{2}\vec{b}(x-y)\hat{j}+\frac{1}{2}xy\hat{k}$.

(50/100)

4. (a) Prove $\nabla r^n = nr^{n-2}\vec{r}$, where *n* is an integer.

[(a) Buktikan $\nabla r^n = nr^{n-2}\vec{r}$*, di sini n adalah integer]*

(25/100)

(b) Show that $\nabla.\nabla\phi=\nabla^2\phi$ where $\nabla^2\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

[(b) Tunjukkan bahawa $\nabla.\nabla\phi=\nabla^2\phi$ *di sini* $\nabla^2\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.*]*

(25/100)

(c) Find curl of the following functions:
[(c) Cari keikalan fungsi-fungsi berikut:]

(i) $\vec{f} = \dfrac{x\hat{i} + y\hat{j}}{x + y}$

(ii) $\vec{g} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$

(50/100)

5. (a) (i) Write the Maxwell's equations governing electromagnetic fields in a dielectric medium.
[(a) (i) Tuliskan persamaan Maxwell bagi medan elektromagnet dalam medium dielektrik.]

(ii) What was the Maxwell's contribution toward Ampere's law?
[(ii) Apakah sumbangan Maxwell terhadap Hukum Ampere?]

(25/100)

(b) (i) Write the Stoke's theorem in space.
[(i) Tuliskan teorem Stokes dalam ruang.]

(ii) If $\oint \vec{E}.d\vec{r} = -\dfrac{1}{c}\dfrac{\partial}{\partial t}\iint_s \vec{H}.d\vec{s}$,

[(ii) Jika $\oint \vec{E}.d\vec{r} = -\dfrac{1}{c}\dfrac{\partial}{\partial t}\iint_s \vec{H}.d\vec{s}$,

where s is any surface bounded by curve c,
[di sini sebarang permukaan s disempadani oleh lengkung c,]

show that $\nabla \times \vec{E} = -\dfrac{1}{c}\dfrac{\partial \vec{H}}{\partial t}$

[tunjukkan bahawa $\nabla \times \vec{E} = -\dfrac{1}{c}\dfrac{\partial \vec{H}}{\partial t}$*]*

(50/100)

(c) Given that $\vec{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$ when t = 2 and $\vec{r}(t) = 4\hat{i} - 2\hat{j} + 3\hat{k}$ when t = 3.
[(c) Di beri $\vec{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$ *bila* t = 2 dan $\vec{r}(t) = 4\hat{i} - 2\hat{j} + 3\hat{k}$ *bila* t = 3.

Show that $\int_2^3 \vec{r}.\dfrac{d\vec{r}}{dt}dt = 10$.

Tunjukkan $\int_2^3 \vec{r}.\dfrac{d\vec{r}}{dt}dt = 10$.

(25/100)