UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2007/2008

June 2008
Jun 2008

EMH 331/4 – Finite Element Method in Mechanical Engineering
Kaedah Unsur Terhingga Dalam Kejuruteraan Mekanik

Duration : 3 hours
Masa : 3 jam

INSTRUCTIONS TO CANDIDATE:
ARAHAN KEPADA CALON :

Please check that this paper contains **EIGHT (8)** printed pages, **TWO (2)** page appendix and **SIX (6)** questions before you begin the examination.
*Sila pastikan bahawa kertas soalan ini mengandungi **LAPAN (8)** mukasurat bercetak, **DUA (2)** mukasurat lampiran dan **ENAM (6)** soalan sebelum anda memulakan peperiksaan.*

Answer **FIVE (5)** questions.
*Jawab **LIMA (5)** soalan.*

Answer all questions in **ENGLISH** OR **BAHASA MALAYSIA** OR a combination of both.
*Calon boleh menjawab semua soalan dalam **BAHASA MALAYSIA** ATAU **BAHASA INGGERIS** ATAU kombinasi kedua-duanya.*

Each question must begin from a new page.
Setiap soalan mestilah dimulakan pada mukasurat yang baru.

Appendix/*Lampiran*:

1. The given selected formula [2 pages/*mukasurat*]

Q1. [a] Derive the stiffness matrix, k for a linear elastic bar element by using:

(i) Direct stiffness method

(ii) Galerkin's method where the governing equation is given by

$$\frac{d}{dx}\left(AE\frac{du}{dx}\right)=0.$$

A is cross-sectional area, E is Young's modulus and u is the displacement function.

Terbitkan matriks kekakuan, k untuk elemen bar linear elastik dengan menggunakan:

(i) Kaedah kekakuan terus

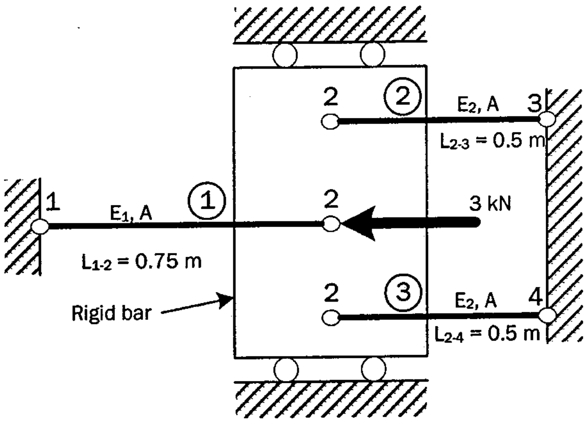
(ii) Kaedah Galerkin di mana persamaan diberi oleh $\frac{d}{dx}\left(AE\frac{du}{dx}\right)=0$

A ialah luas keratan rentas, E ialah modulus Young dan u ialah fungsi anjakan.

(50 marks/*markah*)

[b] For the bar assemblage shown in Figure Q1[b], using finite element method determine the nodal displacements, the forces in each element and the nodal reactions.

Untuk aturan bar yang ditunjukkan dalam Rajah S1[b], dengan menggunakan kaedah unsur terhingga tentukan anjakan nod, daya-daya dalam setiap elemen dan daya tindakbalas nod.



$E_1 = 70\ GPa$

$E_2 = 200\ GPa$

$A = 0.2\ m^2$

Figure Q1[b]
Rajah S1[b]

(50 marks/*markah*)

Q2. [a] List and briefly describe the general steps in the finite element method.

Senaraikan dan terangkan dengan ringkas langkah-langkah umum dalam kaedah unsur terhingga.

(20 marks/*markah*)

[b] For the assemblage shown in Figure Q2[b], determine the nodal displacements, the forces in each element and the nodal reactions.

Untuk aturan yang ditunjukkan dalam Rajah S2[b], tentukan anjakan nod, daya-daya dalam setiap elemen dan daya tindakbalas nod.

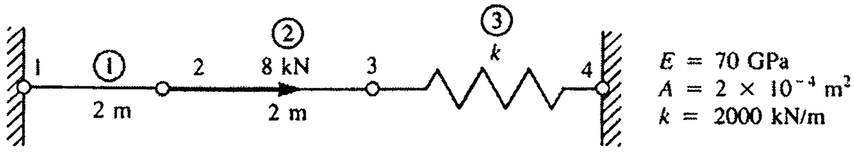


Figure Q2[b]
Rajah S2[b]

(30 marks/*markah*)

[c] Evaluate the stiffness matrix for the triangular elemens shown in Figure Q2[c]. The coordinates are given in units of millimeters. Assume plane stress conditions. Let Young's modulus $E = 210$ GPa, Poisson ratio, $\nu = 0.25$, and thickness $t = 5$ mm.

Kirakan matriks kekakuan untuk elemen segitiga seperti yang ditunjukkan dalam Rajah S2[c]. Koordinat yang diberikan adalah dalam unit millimeter. Andaikan keadaan tegasan satah. Ambil modulus Young $E = 210$ GPa, nisbah Poisson, $\nu = 0.25$, dan ketebalan $t = 5$ mm.

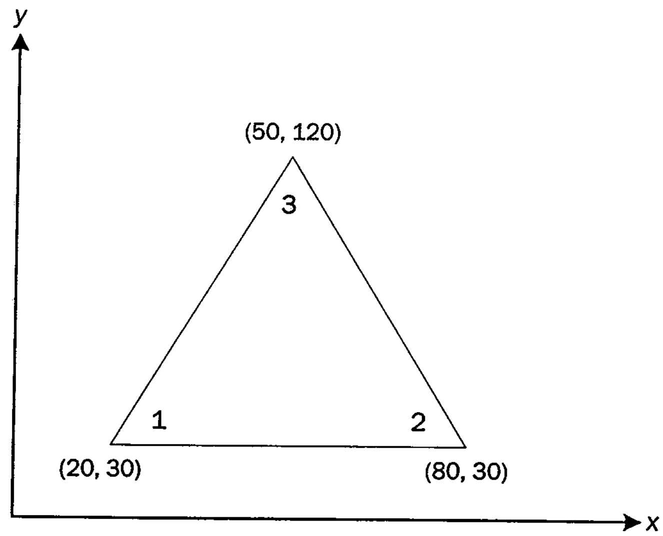


Figure Q2[c]
Rajah S2[c]

(50 marks/*markah*)

Q3. [a] **The fin shown in Figure Q3[a] is insulated on the perimeter. The left end has a constant temperature of 70 ^{0}C. A positive heat flux of q = 1500 W/m^2 acts on the right end. Let thermal conductivity, k_x = 20 W/(m. ^{0}C) and cross-sectional area A = 0.4 m^2. Determine the temperatures at L/2 and L from the left end.**

Satu fin seperti yang ditunjukkan dalam Rajah S3[a] diinsulasi pada ukur lilitnya. Hujung kiri fin tersebut dikenakan suhu tetap 70 ^{0}C. Aliran haba positif q = 1500 W/m^2 bertindak pada hujung kanan. Ambil konduksi terma, k_x = 20 W/(m. ^{0}C) dan luas keratan rentas A = 0.4 m^2. Tentukan suhu pada L/2 dan L dari hujung kiri.

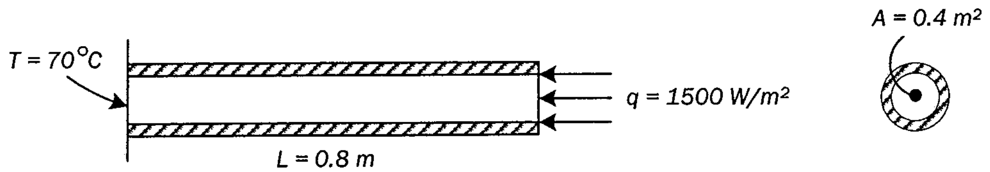


Figure Q3[a]
Rajah S3[a]

(40 marks/*markah*)

[b] **For the triangular element shown in Figure Q3[b], determine the stiffness matrix k and force matrix f. The conductivities are $k_x = k_y$ = 7 W/(cm. °C) and the convection coefficient is h = 5 W/(cm^2.°C). Convection occurs across the node 1-2 surface. The heat source Q^* is the line source of Q^*= 25 W/cm as located in the figure. The free stream temperature is T_∞ = 20°C. The coordinates are expressed in units of cm. Take the thickness of the element to be 1 cm.**

Untuk elemen segitiga seperti yang ditunjukkan dalam Rajah S3[b], tentukan matriks kekakuan k dan matriks daya f. Kekonduksian ialah $k_x = k_y$ = 7 W/(cm. °C) dan pemalar perolakan ialah h = 5 W/(cm^2.°C). Perolakan berlaku pada permukaan nod 1-2. Sumber haba Q ialah sumber garisan Q^* = 25 W/cm dilokasi yang ditunjukkan dalam rajah. Suhu sekeliling ialah T_∞ = 20°C. Koordinat ditunjukkan dalam unit cm. Ambil ketebalan elemen 1 cm.*

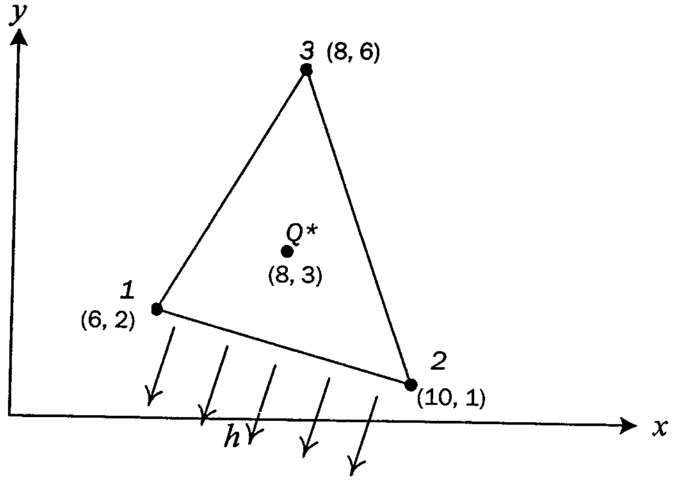


Figure Q3[b]
Rajah S3[b]

(60 marks/*markah*)

Q4. [a] **For the smooth pipe of variable cross section shown in Figure Q4[a], determine the nodal fluid heads or potential at the junctions, the velocities in each section of pipe and the volumetric flow rate. The nodal fluid heads or potential at the left end is $p_1 = 25$ m²/s and that at the right end is $p_4 = 1$ m²/s.**

Untuk satu paip seragam yang mempunyai berbagai luas keratan rentas seperti yang ditunjukkan dalam Rajah S4[a], tentukan kepala aliran nod atau potensi pada simpang, halaju pada setiap bahagian paip dan kadar aliran isipadu. Potensi pada hujung kiri ialah $p_1 = 25\ m^2/s$ dan potensi pada hujung kanan ialah $p_4 = 1\ m^2/s$.

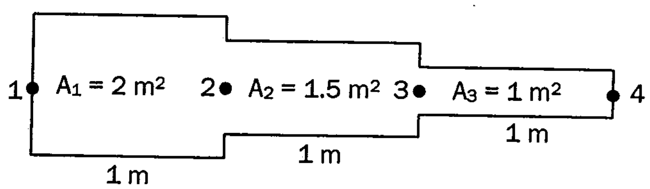


Figure Q4[a]
Rajah S4[a]

(40 marks/*markah*)

[b] **For the plane truss shown in Figure Q4[b], determine the displacement at node 2 and the axial stress in each bar. Bar 1 is subjected to a temperature rise of 25°C. Let Young's modulus E = 210 GPa, coefficient of thermal expansion α = 12 x 10^{-6} (mm/mm)/°C and cross sectional area 1 x 10^{-2} m^2 for bar element 1, and cross sectional area 1 x 10^{-2} m^2 for bar element 2. Both bar elements have length of 0.6 m.**

Untuk kekuda satah yang ditunjukkan dalam Rajah S4[b], tentukan anjakan pada nod 2 dan tegasan paksi dalam setiap bar. Bar elemen 1 dikenakan suhu meningkat 25°C. Ambil modulus Young E = 210 GPa, pekali pengembangan terma α = 12 x 10^{-6} (mm/mm)/°C dan luas keratan rentas 1 x 10^{-2} m^2 untuk bar elemen 1, dan luas keratan rentas 1 x 10^{-2} m^2 untuk bar elemen 2. Panjang kedua-dua elemen tersebut ialah 0.6 m.

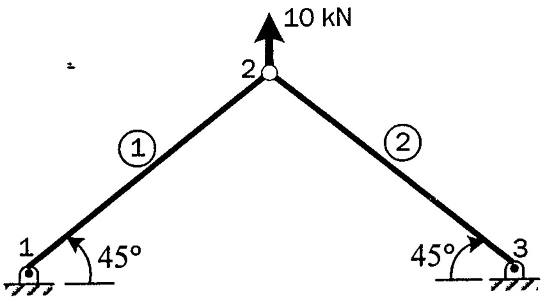


Figure Q4[b]
Rajah S4[b]

(60 marks/*markah*)

Q5. [a] **Use the principle of minimum potential energy to find the equilibrium of displacement of spring system as shown in Figure Q5[a]. Briefly discuss your findings.**

Gunakan kaedah prinsip minima tenaga potensi untuk menentukan perseimbangan anjakan pada pegas yang ditunjukkan dalam Rajah S5[a]. Terangkan dengan ringkas penemuan yang dicapai.

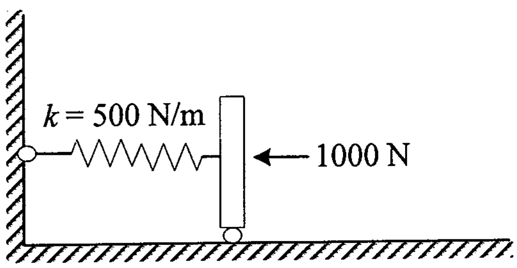


Figure Q5[a]
Rajah S5[a]

(30 marks/*markah*)

[b] **For the plane trusses shown in Figure Q5[b], determine the horizontal and vertical displacements of node 1 and the stresses in each element. All elements have $E = 210$ GPa and cross sectional area, $A = 6.0 \times 10^{-4} m^2$.**

Untuk kekuda satah yang ditunjukkan dalam Rajah S5[b], tentukan anjakan menegak dan melintang pada nod 1 dan tegasan di setiap elemen. Semua elemen mempunyai $E = 210$ GPa dan luas keratan rentas $A = 6.0 \times 10^{-4} m^2$.

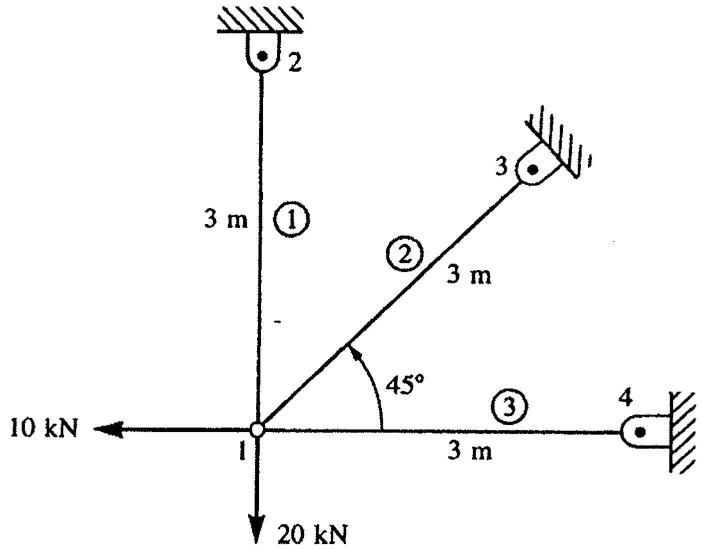


Figure Q5[b]
Rajah S5[b]

(70 marks/*markah*)

Q6. **[a]** **For the beam shown in Figure Q6[a], with the node numbers 1-2-3, determine**

(i) **The displacements**
(ii) **The slopes at the node**
(iii) **The forces in each element**
(iv) **The reactions**

Untuk rasuk yang ditunjukkan dalam Rajah S6[a] dengan nombor-nombor nod 1-2-3, tentukan

(i) *Anjakan*
(ii) *Kecerunan pada nod*
(iii) *Daya pada setiap element*
(iv) *Daya tindakbalas*

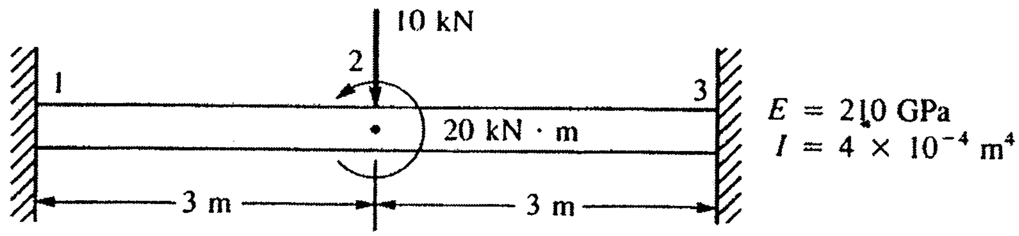


Figure Q6[a]
Rajah S6[a]

(50 marks/*markah*)

[b] **For the plane truss shown in Figure Q6[b], determine the displacements at node 1 and the axial stresses in each bar. Bar 1 is subjected to a temperature rise of 40°C. Let *E =210 GPa*, coefficient of thermal expansion $\alpha = 12 \times 10^{-6}$ *(mm/mm)/°C*, and cross sectional area $A = 1 \times 10^{-2}$ for both bar elements.**

Untuk kekuda satah yang ditunjukkan dalam Rajah S6[b], tentukan anjakan pada nod 1 dan tegasan paksi dalam setiap bar. Bar elemen 1 dikenakan suhu meningkat 40°C. Ambil modulus Young E = 210 GPa, pekali pengembangan terma $\alpha = 12 \times 10^{-6}$ *(mm/mm)/°C dan luas keratan rentas* $A = 1 \times 10^{-2}$ m^2 *untuk kedua dua bar elemen.*

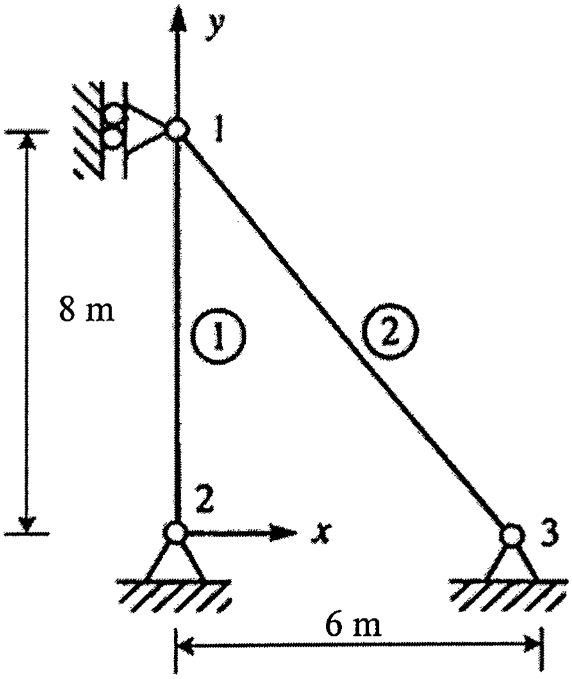


Figure Q6[b]
Rajah S6[b]

(50 marks/*markah*)

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The given selected formula

Bar and Truss:

$$\begin{Bmatrix} \hat{d}_x \\ \hat{d}_y \end{Bmatrix} = \begin{bmatrix} C & S \\ -S & C \end{bmatrix} \begin{Bmatrix} d_x \\ d_y \end{Bmatrix} \qquad \underline{\hat{d}} = \underline{T}\underline{d} \qquad \underline{T} = \begin{bmatrix} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ 0 & 0 & C & S \\ 0 & 0 & -S & C \end{bmatrix}$$

$$\underline{k} = \frac{AE}{L} \begin{bmatrix} C^2 & CS & -C^2 & -CS \\ & S^2 & -CS & -S^2 \\ & & C^2 & CS \\ Symmetry & & & S^2 \end{bmatrix} \qquad \underline{\sigma} = \underline{C}'\underline{d} \qquad \underline{C}' = \frac{E}{L}\begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} C & S & 0 & 0 \\ 0 & 0 & C & S \end{bmatrix}$$

Beam and Frame:

$$\underline{\hat{k}} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Plane Stress

$$\{\sigma\} = [D]\{\varepsilon\}$$

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$$[k] = tA[B]^T[D][B]$$

Plane Strain

$$\{\sigma\} = [D]\{\varepsilon\}$$

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$$

Triangular element strain

$$\{\varepsilon\} = \frac{1}{2A} \begin{bmatrix} \beta_i & 0 & \beta_j & 0 & \beta_m & 0 \\ 0 & \gamma_i & 0 & \gamma_j & 0 & \gamma_m \\ \gamma_i & \beta_i & \gamma_j & \beta_j & \gamma_m & \beta_m \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_m \\ v_m \end{Bmatrix}$$

$$\{\varepsilon\} = \begin{bmatrix} \underline{B}_i & \underline{B}_j & \underline{B}_m \end{bmatrix} \begin{Bmatrix} \underline{d}_i \\ \underline{d}_j \\ \underline{d}_m \end{Bmatrix}$$

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$$[B_i]=\frac{1}{2A}\begin{bmatrix}\beta_i & 0\\ 0 & \gamma_i\\ \gamma_i & \beta_i\end{bmatrix} \qquad [B_j]=\frac{1}{2A}\begin{bmatrix}\beta_j & 0\\ 0 & \gamma_j\\ \gamma_j & \beta_j\end{bmatrix} \qquad [B_m]=\frac{1}{2A}\begin{bmatrix}\beta_m & 0\\ 0 & \gamma_m\\ \gamma_m & \beta_m\end{bmatrix}$$

Characteristics of triangular element

$$2A=\begin{bmatrix}1 & x_i & y_i\\ 1 & x_j & y_j\\ 1 & x_m & \gamma_m\end{bmatrix} \qquad 2A=x_i(y_j-y_m)+x_j(y_m-y_i)+x_m(y_i-y_j)$$

$$\alpha_i=x_jy_m-y_jx_m \qquad \alpha_j=y_ix_m-x_iy_m \qquad \alpha_m=x_iy_j-y_ix_j$$

$$\beta_i=y_j-y_m \qquad \beta_j=y_m-y_i \qquad \beta_m=y_i-y_j$$

$$\gamma_i=x_m-x_j \qquad \gamma_j=x_i-x_m \qquad \gamma_m=x_j-x_i$$

x displacement:

$$u(x,y)=\frac{1}{2A}\{(\alpha_i+\beta_ix+\gamma_iy)u_i+(\alpha_j+\beta_jx+\gamma_jy)u_j+(\alpha_m+\beta_mx+\gamma_my)u_m\}$$

y displacement:

$$v(x,y)=\frac{1}{2A}\{(\alpha_i+\beta_ix+\gamma_iy)v_i+(\alpha_j+\beta_jx+\gamma_jy)v_j+(\alpha_m+\beta_mx+\gamma_my)v_m\}$$

$$N_i=\frac{1}{2A}(\alpha_i+\beta_ix+\gamma_iy)$$

$$N_j=\frac{1}{2A}(\alpha_j+\beta_jx+\gamma_jy)$$

$$N_m=\frac{1}{2A}(\alpha_m+\beta_mx+\gamma_my)$$