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UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama

Sidang Akademik 2004/2005

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**ZCA 110/4 - KALKULUS DAN ALJABAR LINEAR**

Masa: 3 jam

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Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab kesemua ENAM soalan. Kesemuanya wajib dijawab dalam Bahasa Malaysia.

Diberi bersama kertas soalan ini ialah *A Brief Table of Integrals* (5 muka surat).

1.

(a) Tulis sistem persamaan linear berikut,

$$-x_1 + x_2 + 2x_3 = 5$$

$$2x_1 - x_2 + 5x_3 = 3$$

$$2x_1 + 5x_2 - 2x_4 = 0$$

$$3x_2 - x_3 + x_4 = 1.$$

dengan menggunakan nyataan matriks. Apakah matriks koefisien  $A$  sistem persamaan ini? Cari kofaktor  $\alpha_{34}$  dan  $\alpha_{43}$  bagi matriks  $A$  dan dengan menggunakan keputusan ini cari determinan bagi matriks koefisien  $A$ .

(b) Cari penyelesaian sistem ini dengan menggunakan kaedah Cramer.

(16 Markah)

2.

Cari titik-titik dan persamaan-persamaan bagi tangen pada lengkung  $y = 2x^3 - 3x^2 - 12x + 20$  di mana tangennya adalah;

(a) bersudut tepat dengan garis  $y = 1 - \frac{x}{24}$ ,

(b) selari dengan garis  $y = \sqrt{2} - 12x$ .

(16 Markah)

3.

(a) Nilaikan kamiran-kamiran berikut:

(i)  $\int 2^{\tan x} \sec^2 x \, dx$

(ii)  $\int_1^3 \frac{(\ln(v+1))^2}{v+1} \, dv$

(iii)  $\int_{-2}^2 \frac{3 \, dt}{4 + 3t^2}$

(iv)  $\int \frac{dt}{(t+1)\sqrt{t^2+2t-8}}$

(b) Cari luas kawasan di antara graf,  $f(x) = 1 - \frac{x^2}{4}$ ,  $-2 \leq x \leq 3$ , dan paksi- $x$ .

(16 Markah)

4.

Diberi terbitan pertama  $y'(x) = x^4 - 2x^2$ , suatu fungsi  $y = f(x)$ .

(a) Jika fungsi  $y(x)$  ini melalui pusat koordinat  $(0, 0)$ , cari persamaan fungsi ini.

(b) Cari titik-titik minimum, maksimum, dan titik perubahan kecekungan fungsi ini.

(c) Cari selang-selang di mana fungsi ini bertambah dan selang-selang di mana ia berkurang.

(d) Cari selang-selang di mana fungsi ini bercekung ke atas dan selang-selang di mana ia bercekung ke bawah.

(e) Lukis fungsi  $y(x)$  ini atas selang  $[-2, 2]$ , di kertas graf yang dibekalkan. Tunjukkan titik-titik minimum, maksimum, dan titik perubahan kecekungannya atas

graf ini.

(16 Markah)

5.

Nilaikan kamiran-kamiran berikut:

(a)  $\int \frac{\tan x \, dx}{\tan x + \sec x}$

(b)  $\int \frac{x^3 + x^2}{x^2 + x - 2} \, dx$

(c)  $\int \frac{9 \, dv}{81 - v^4}$

(d)  $\int \frac{z + 1}{z^2(z^2 + 4)} \, dz$

(e)  $\int x^5 \sin x \, dx$

(f)  $\int_1^\infty \frac{\ln y}{y^3} \, dy.$

(18 Markah)

6.

(a) Bagi siri,

$$\sum_{n=1}^{\infty} \frac{(x + 4)^n}{n3^n},$$

cari (i) jejari siri dan (ii) selang menumpunya. Untuk nilai apa  $x$  siri itu menumpu (iii) mutlak dan (iv) menumpu bersyarat?

(b) Cari perkembangan Siri Fourier bagi fungsi,

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 2, & 0 < x < \pi. \end{cases}$$

(c) Cari perkembangan Siri Kosinus Fourier bagi fungsi,

$$f(x) = \begin{cases} 1, & 0 < x < 1/2 \\ 0, & 1/2 < x < 1. \end{cases}$$

(18 Markah)

## A Brief Table of Integrals

1.  $\int u dv = uv - \int v du$
2.  $\int a^u du = \frac{a^u}{\ln a} + C, \quad a \neq 1, \quad a > 0$
3.  $\int \cos u du = \sin u + C$
4.  $\int \sin u du = -\cos u + C$
5.  $\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$
6.  $\int (ax + b)^{-1} dx = \frac{1}{a} \ln |ax + b| + C$
7.  $\int x(ax + b)^n dx = \frac{(ax + b)^{n+1}}{a^2} \left[ \frac{ax + b}{n+2} - \frac{b}{n+1} \right] + C, \quad n \neq -1, -2$
8.  $\int x(ax + b)^{-1} dx = \frac{x}{a} - \frac{b}{a^2} \ln |ax + b| + C$
9.  $\int x(ax + b)^{-2} dx = \frac{1}{a^2} \left[ \ln |ax + b| + \frac{b}{ax + b} \right] + C$
10.  $\int \frac{dx}{x(ax + b)} = \frac{1}{b} \ln \left| \frac{x}{ax + b} \right| + C$
11.  $\int (\sqrt{ax + b})^n dx = \frac{2(\sqrt{ax + b})^{n+2}}{a(n+2)} + C, \quad n \neq -2$
12.  $\int \frac{\sqrt{ax + b}}{x} dx = 2\sqrt{ax + b} + b \int \frac{dx}{x\sqrt{ax + b}}$
13. (a)  $\int \frac{dx}{x\sqrt{ax - b}} = \frac{2}{\sqrt{b}} \tan^{-1} \sqrt{\frac{ax - b}{b}} + C$   
(b)  $\int \frac{dx}{x\sqrt{ax + b}} = \frac{1}{\sqrt{b}} \ln \left| \frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}} \right| + C$
14.  $\int \frac{\sqrt{ax + b}}{x^2} dx = -\frac{\sqrt{ax + b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax + b}} + C$
15.  $\int \frac{dx}{x^2\sqrt{ax + b}} = -\frac{\sqrt{ax + b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax + b}} + C$
16.  $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
17.  $\int \frac{dx}{(a^2 + x^2)^2} = \frac{x}{2a^2(a^2 + x^2)} + \frac{1}{2a^3} \tan^{-1} \frac{x}{a} + C$
18.  $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C$
19.  $\int \frac{dx}{(a^2 - x^2)^2} = \frac{x}{2a^2(a^2 - x^2)} + \frac{1}{4a^3} \ln \left| \frac{x+a}{x-a} \right| + C$
20.  $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a} + C = \ln(x + \sqrt{a^2 + x^2}) + C$
21.  $\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + C$
22.  $\int x^2 \sqrt{a^2 + x^2} dx = \frac{x}{8} (a^2 + 2x^2) \sqrt{a^2 + x^2} - \frac{a^4}{8} \ln(x + \sqrt{a^2 + x^2}) + C$
23.  $\int \frac{\sqrt{a^2 + x^2}}{x} dx = \sqrt{a^2 + x^2} - a \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right| + C$
24.  $\int \frac{\sqrt{a^2 + x^2}}{x^2} dx = \ln(x + \sqrt{a^2 + x^2}) - \frac{\sqrt{a^2 + x^2}}{x} + C$
25.  $\int \frac{x^2}{\sqrt{a^2 + x^2}} dx = -\frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + \frac{x\sqrt{a^2 + x^2}}{2} + C$
26.  $\int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right| + C$
27.  $\int \frac{dx}{x^2\sqrt{a^2 + x^2}} = -\frac{\sqrt{a^2 + x^2}}{a^2 x} + C$

## T-2 A Brief Table of Integrals

- $$28. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$
- $$29. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$
- $$30. \int x^2 \sqrt{a^2 - x^2} dx = \frac{a^4}{8} \sin^{-1} \frac{x}{a} - \frac{1}{8} x \sqrt{a^2 - x^2} (a^2 - 2x^2) + C$$
- $$31. \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$
- $$32. \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\sin^{-1} \frac{x}{a} - \frac{\sqrt{a^2 - x^2}}{x} + C$$
- $$33. \int \frac{x^2}{\sqrt{a^2 - x^2}} dx = \frac{a^2}{2} \sin^{-1} \frac{x}{a} - \frac{1}{2} x \sqrt{a^2 - x^2} + C$$
- $$34. \int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$
- $$35. \int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$$
- $$36. \int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \frac{x}{a} + C = \ln |x + \sqrt{x^2 - a^2}| + C$$
- $$37. \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln |x + \sqrt{x^2 - a^2}| + C$$
- $$38. \int (\sqrt{x^2 - a^2})^n dx = \frac{x(\sqrt{x^2 - a^2})^n}{n+1} - \frac{na^2}{n+1} \int (\sqrt{x^2 - a^2})^{n-2} dx, \quad n \neq -1$$
- $$39. \int \frac{dx}{(\sqrt{x^2 - a^2})^n} = \frac{x(\sqrt{x^2 - a^2})^{2-n}}{(2-n)a^2} - \frac{n-3}{(n-2)a^2} \int \frac{dx}{(\sqrt{x^2 - a^2})^{n-2}}, \quad n \neq 2$$
- $$40. \int x(\sqrt{x^2 - a^2})^n dx = \frac{(\sqrt{x^2 - a^2})^{n+2}}{n+2} + C, \quad n \neq -2$$
- $$41. \int x^2 \sqrt{x^2 - a^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{x^2 - a^2} - \frac{a^4}{8} \ln |x + \sqrt{x^2 - a^2}| + C$$
- $$42. \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \sec^{-1} \left| \frac{x}{a} \right| + C$$
- $$43. \int \frac{\sqrt{x^2 - a^2}}{x^2} dx = \ln |x + \sqrt{x^2 - a^2}| - \frac{\sqrt{x^2 - a^2}}{x} + C$$
- $$44. \int \frac{x^2}{\sqrt{x^2 - a^2}} dx = \frac{a^2}{2} \ln |x + \sqrt{x^2 - a^2}| + \frac{x}{2} \sqrt{x^2 - a^2} + C$$
- $$45. \int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C = \frac{1}{a} \cos^{-1} \left| \frac{a}{x} \right| + C$$
- $$46. \int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C$$
- $$47. \int \frac{dx}{\sqrt{2ax - x^2}} = \sin^{-1} \left( \frac{x-a}{a} \right) + C$$
- $$48. \int \sqrt{2ax - x^2} dx = \frac{x-a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x-a}{a} \right) + C$$
- $$49. \int (\sqrt{2ax - x^2})^n dx = \frac{(x-a)(\sqrt{2ax - x^2})^n}{n+1} + \frac{na^2}{n+1} \int (\sqrt{2ax - x^2})^{n-2} dx$$
- $$50. \int \frac{dx}{(\sqrt{2ax - x^2})^n} = \frac{(x-a)(\sqrt{2ax - x^2})^{2-n}}{(n-2)a^2} + \frac{n-3}{(n-2)a^2} \int \frac{dx}{(\sqrt{2ax - x^2})^{n-2}}$$
- $$51. \int x \sqrt{2ax - x^2} dx = \frac{(x+a)(2x-3a)\sqrt{2ax - x^2}}{6} + \frac{a^3}{2} \sin^{-1} \left( \frac{x-a}{a} \right) + C$$
- $$52. \int \frac{\sqrt{2ax - x^2}}{x} dx = \sqrt{2ax - x^2} + a \sin^{-1} \left( \frac{x-a}{a} \right) + C$$
- $$53. \int \frac{\sqrt{2ax - x^2}}{x^2} dx = -2 \sqrt{\frac{2a-x}{x}} - \sin^{-1} \left( \frac{x-a}{a} \right) + C$$
- $$54. \int \frac{x dx}{\sqrt{2ax - x^2}} = a \sin^{-1} \left( \frac{x-a}{a} \right) - \sqrt{2ax - x^2} + C$$
- $$55. \int \frac{dx}{x \sqrt{2ax - x^2}} = -\frac{1}{a} \sqrt{\frac{2a-x}{x}} + C$$
- $$56. \int \sin ax dx = -\frac{1}{a} \cos ax + C$$
- $$57. \int \cos ax dx = \frac{1}{a} \sin ax + C$$
- $$58. \int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$
- $$59. \int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

60.  $\int \sin^n ax \, dx = -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n} \int \sin^{n-2} ax \, dx$
61.  $\int \cos^n ax \, dx = \frac{\cos^{n-1} ax \sin ax}{na} + \frac{n-1}{n} \int \cos^{n-2} ax \, dx$
62. (a)  $\int \sin ax \cos bx \, dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + C, \quad a^2 \neq b^2$   
 (b)  $\int \sin ax \sin bx \, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$   
 (c)  $\int \cos ax \cos bx \, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$
63.  $\int \sin ax \cos ax \, dx = -\frac{\cos 2ax}{4a} + C$
64.  $\int \sin^n ax \cos ax \, dx = \frac{\sin^{n-1} ax}{(n+1)a} + C, \quad n \neq -1$
65.  $\int \frac{\cos ax}{\sin ax} \, dx = \frac{1}{a} \ln |\sin ax| + C$
66.  $\int \cos^n ax \sin ax \, dx = -\frac{\cos^{n-1} ax}{(n+1)a} + C, \quad n \neq -1$
67.  $\int \frac{\sin ax}{\cos ax} \, dx = -\frac{1}{a} \ln |\cos ax| + C$
68.  $\int \sin^n ax \cos^m ax \, dx = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(m+n)} + \frac{n-1}{m+n} \int \sin^{n-2} ax \cos^m ax \, dx, \quad n \neq -m \quad (\text{reduces } \sin^n ax)$
69.  $\int \sin^n ax \cos^m ax \, dx = \frac{\sin^{n+1} ax \cos^{m-1} ax}{a(m+n)} + \frac{m-1}{m+n} \int \sin^n ax \cos^{m-2} ax \, dx, \quad m \neq -n \quad (\text{reduces } \cos^m ax)$
70.  $\int \frac{dx}{b+c \sin ax} = \frac{-2}{a\sqrt{b^2-c^2}} \tan^{-1} \left[ \sqrt{\frac{b-c}{b+c}} \tan \left( \frac{\pi}{4} - \frac{ax}{2} \right) \right] + C, \quad b^2 > c^2$
71.  $\int \frac{dx}{b+c \sin ax} = \frac{-1}{a\sqrt{c^2-b^2}} \ln \left| \frac{c+b \sin ax + \sqrt{c^2-b^2} \cos ax}{b+c \sin ax} \right| + C, \quad b^2 < c^2$
72.  $\int \frac{dx}{1+\sin ax} = -\frac{1}{a} \tan \left( \frac{\pi}{4} - \frac{ax}{2} \right) + C$
73.  $\int \frac{dx}{1-\sin ax} = \frac{1}{a} \tan \left( \frac{\pi}{4} + \frac{ax}{2} \right) + C$
74.  $\int \frac{dx}{b+c \cos ax} = \frac{2}{a\sqrt{b^2-c^2}} \tan^{-1} \left[ \sqrt{\frac{b-c}{b+c}} \tan \frac{ax}{2} \right] + C, \quad b^2 > c^2$
75.  $\int \frac{dx}{b+c \cos ax} = \frac{1}{a\sqrt{c^2-b^2}} \ln \left| \frac{c+b \cos ax + \sqrt{c^2-b^2} \sin ax}{b+c \cos ax} \right| + C, \quad b^2 < c^2$
76.  $\int \frac{dx}{1+\cos ax} = \frac{1}{a} \tan \frac{ax}{2} + C$
77.  $\int \frac{dx}{1-\cos ax} = -\frac{1}{a} \cot \frac{ax}{2} + C$
78.  $\int x \sin ax \, dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax + C$
79.  $\int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax + C$
80.  $\int x^n \sin ax \, dx = -\frac{x^n}{a} \cos ax + \frac{n}{a} \int x^{n-1} \cos ax \, dx$
81.  $\int x^n \cos ax \, dx = \frac{x^n}{a} \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \, dx$
82.  $\int \tan ax \, dx = \frac{1}{a} \ln |\sec ax| + C$
83.  $\int \cot ax \, dx = \frac{1}{a} \ln |\sin ax| + C$
84.  $\int \tan^2 ax \, dx = \frac{1}{a} \tan ax - x + C$
85.  $\int \cot^2 ax \, dx = -\frac{1}{a} \cot ax - x + C$
86.  $\int \tan^n ax \, dx = \frac{\tan^{n-1} ax}{a(n-1)} - \int \tan^{n-2} ax \, dx, \quad n \neq 1$
87.  $\int \cot^n ax \, dx = -\frac{\cot^{n-1} ax}{a(n-1)} - \int \cot^{n-2} ax \, dx, \quad n \neq 1$
88.  $\int \sec ax \, dx = \frac{1}{a} \ln |\sec ax + \tan ax| + C$
89.  $\int \csc ax \, dx = -\frac{1}{a} \ln |\csc ax + \cot ax| + C$
90.  $\int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$
91.  $\int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$
92.  $\int \sec^n ax \, dx = \frac{\sec^{n-2} ax \tan ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax \, dx, \quad n \neq 1$

## T-4 A Brief Table of Integrals

93.  $\int \csc^n ax \, dx = -\frac{\csc^{n-2} ax \cot ax}{a(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2} ax \, dx, \quad n \neq 1$
94.  $\int \sec^n ax \tan ax \, dx = \frac{\sec^n ax}{na} + C, \quad n \neq 0$
95.  $\int \csc^n ax \cot ax \, dx = -\frac{\csc^n ax}{na} + C, \quad n \neq 0$
96.  $\int \sin^{-1} ax \, dx = x \sin^{-1} ax + \frac{1}{a} \sqrt{1-a^2x^2} + C$
97.  $\int \cos^{-1} ax \, dx = x \cos^{-1} ax - \frac{1}{a} \sqrt{1-a^2x^2} + C$
98.  $\int \tan^{-1} ax \, dx = x \tan^{-1} ax - \frac{1}{2a} \ln(1+a^2x^2) + C$
99.  $\int x^n \sin^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \sin^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} dx}{\sqrt{1-a^2x^2}}, \quad n \neq -1$
100.  $\int x^n \cos^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \cos^{-1} ax + \frac{a}{n+1} \int \frac{x^{n+1} dx}{\sqrt{1-a^2x^2}}, \quad n \neq -1$
- \* 101.  $\int x^n \tan^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \tan^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} dx}{\sqrt{1+a^2x^2}}, \quad n \neq -1$
102.  $\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$
103.  $\int b^{ax} \, dx = \frac{1}{a} \frac{b^{ax}}{\ln b} + C, \quad b > 0, \quad b \neq 1$
104.  $\int x e^{ax} \, dx = \frac{e^{ax}}{a^2} (ax - 1) + C$
105.  $\int x^n e^{ax} \, dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx$
106.  $\int x^n b^{ax} \, dx = \frac{x^n b^{ax}}{a \ln b} - \frac{n}{a \ln b} \int x^{n-1} b^{ax} \, dx, \quad b > 0, \quad b \neq 1$
107.  $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$
108.  $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$
109.  $\int \ln ax \, dx = x \ln ax - x + C$
110.  $\int x^n (\ln ax)^m \, dx = \frac{x^{n+1} (\ln ax)^m}{n+1} - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} \, dx, \quad n \neq -1$
111.  $\int x^{-1} (\ln ax)^m \, dx = \frac{(\ln ax)^{m+1}}{m+1} + C, \quad m \neq -1$
112.  $\int \frac{dx}{x \ln ax} = \ln |\ln ax| + C$
113.  $\int \sinh ax \, dx = \frac{1}{a} \cosh ax + C$
114.  $\int \cosh ax \, dx = \frac{1}{a} \sinh ax + C$
115.  $\int \sinh^2 ax \, dx = \frac{\sinh 2ax}{4a} - \frac{x}{2} + C$
116.  $\int \cosh^2 ax \, dx = \frac{\sinh 2ax}{4a} + \frac{x}{2} + C$
117.  $\int \sinh^n ax \, dx = \frac{\sinh^{n-1} ax \cosh ax}{na} - \frac{n-1}{n} \int \sinh^{n-2} ax \, dx, \quad n \neq 0$
118.  $\int \cosh^n ax \, dx = \frac{\cosh^{n-1} ax \sinh ax}{na} + \frac{n-1}{n} \int \cosh^{n-2} ax \, dx, \quad n \neq 0$
119.  $\int x \sinh ax \, dx = \frac{x}{a} \cosh ax - \frac{1}{a^2} \sinh ax + C$
120.  $\int x \cosh ax \, dx = \frac{x}{a} \sinh ax - \frac{1}{a^2} \cosh ax + C$
121.  $\int x^n \sinh ax \, dx = \frac{x^n}{a} \cosh ax - \frac{n}{a} \int x^{n-1} \cosh ax \, dx$
122.  $\int x^n \cosh ax \, dx = \frac{x^n}{a} \sinh ax - \frac{n}{a} \int x^{n-1} \sinh ax \, dx$
123.  $\int \tanh ax \, dx = \frac{1}{a} \ln |\cosh ax| + C$
124.  $\int \coth ax \, dx = \frac{1}{a} \ln |\sinh ax| + C$
125.  $\int \tanh^2 ax \, dx = x - \frac{1}{a} \tanh ax + C$
126.  $\int \coth^2 ax \, dx = x - \frac{1}{a} \coth ax + C$
127.  $\int \tanh^n ax \, dx = -\frac{\tanh^{n-1} ax}{(n-1)a} + \int \tanh^{n-2} ax \, dx, \quad n \neq 1$
128.  $\int \coth^n ax \, dx = -\frac{\coth^{n-1} ax}{(n-1)a} + \int \coth^{n-2} ax \, dx, \quad n \neq 1$
129.  $\int \operatorname{sech} ax \, dx = \frac{1}{a} \sin^{-1} (\tanh ax) + C$
130.  $\int \operatorname{csch} ax \, dx = \frac{1}{a} \ln \left| \tanh \frac{ax}{2} \right| + C$

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$$131. \int \operatorname{sech}^2 ax \, dx = \frac{1}{a} \tanh ax + C$$

$$133. \int \operatorname{sech}^n ax \, dx = \frac{\operatorname{sech}^{n-2} ax \tanh ax}{(n-1)a} + \frac{n-2}{n-1} \int \operatorname{sech}^{n-2} ax \, dx, \quad n \neq 1$$

$$134. \int \operatorname{csch}^n ax \, dx = -\frac{\operatorname{csch}^{n-2} ax \coth ax}{(n-1)a} - \frac{n-2}{n-1} \int \operatorname{csch}^{n-2} ax \, dx, \quad n \neq 1$$

$$135. \int \operatorname{sech}^n ax \tanh ax \, dx = -\frac{\operatorname{sech}^n ax}{na} + C, \quad n \neq 0$$

$$137. \int e^{ax} \sinh bx \, dx = \frac{e^{ax}}{2} \left[ \frac{e^{bx}}{a+b} - \frac{e^{-bx}}{a-b} \right] + C, \quad a^2 \neq b^2$$

$$139. \int_0^{\infty} x^{n-1} e^{-x} \, dx = \Gamma(n) = (n-1)!, \quad n > 0$$

$$141. \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx = \begin{cases} \frac{1 \cdot 3 \cdot 5 \cdots (n-1)}{2 \cdot 4 \cdot 6 \cdots n} \cdot \frac{\pi}{2}, & \text{if } n \text{ is an even integer } \geq 2 \\ \frac{2 \cdot 4 \cdot 6 \cdots (n-1)}{3 \cdot 5 \cdot 7 \cdots n}, & \text{if } n \text{ is an odd integer } \geq 3 \end{cases}$$

$$132. \int \operatorname{csch}^2 ax \, dx = -\frac{1}{a} \coth ax + C$$

$$136. \int \operatorname{csch}^n ax \coth ax \, dx = -\frac{\operatorname{csch}^n ax}{na} + C, \quad n \neq 0$$

$$138. \int e^{ax} \cosh bx \, dx = \frac{e^{ax}}{2} \left[ \frac{e^{bx}}{a+b} + \frac{e^{-bx}}{a-b} \right] + C, \quad a^2 \neq b^2$$

$$140. \int_0^{\infty} e^{-ax^2} \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}, \quad a > 0$$