UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2003/2004

Februari/Mac 2004

JIM 312 – Teori Kebarangkalian

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH SATU** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab SEMUA soalan.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

...2/-

1. (a) Suatu sistem kommunikasi mempunyai 4 antena secaman yang disusun secara linear. Sistem ini dikatakan berfungsi jika tiada kerosakan berlaku pada sebarang 2 antena yang bersebelahan. Andaikan 1 mewakili antena yang elok dan 0 mewakili antena yang rosak, dan memang terdapat 2 antena yang rosak di dalam sistem ini.

   (i) Senaraikan kesemua susunan antena di dalam sistem ini.

   (ii) Berdasarkan (i) senaraikan susunan-susunan antena yang membolehkan sistem ini berfungsi.

   (iii) Berapakah kebarangkalian sistem ini berfungsi?

(50 markah)

(b) Fungsi jisim kebarangkalian bagi pembolehubah rawak X diberikan oleh $p(x) = c\lambda^x/x!$, $x = 0, 1, 2, \ldots, \lambda > 0$. Cari nilai c.

(20 markah)

(c) Dua biji dadu adil dilemparkan. Andaikan X ialah nilai paling besar yang dicerap daripada 2 dadu tersebut dan Y ialah hasiltambah kedua-dua nilai yang dicerap. Dapatkan fungsi jisim kebarangkalian tercantum p(x, y).

(30 markah)

...3/-

2. (a) Empat buah bas membawa sejumlah 148 orang pelajar. Bas-bas tersebut masing-masing membawa 40, 33, 25 dan 50 orang pelajar. Seorang pelajar dipilih secara rawak. Andaikan X mewakili bilangan pelajar di dalam bas yang mengandungi pelajar terpilih ini. Seorang daripada 4 orang pemandu bas turut dipilih secara rawak. Andaikan Y mewakili bilangan pelajar di dalam bas yang dipandu beliau.

   (i) Bandingkan E[X] dan E[Y].

   (ii) Bandingkan Var[X] dan Var[Y].

(50 markah)

(b) Diberikan $M_X(t) = \exp\{t(2t+1)\}$.

   (i) Cari E[X] dan Var [Y].

   (ii) Camkan taburan X.

(20 markah)

(c) X, Y dan Z adalah pembolehubah-pembolehubah rawak tak bersandar dan tertabur secara secaman seragam (0, 1). Hitungkan $P(X \geq YZ)$.

(30 markah)

3. (a) Fungsi jisim kebarangkalian tercantum bagi X dan Y diberikan oleh jadual berikut

p(xy)		Y = 1	Y = 2
X	1	1/8	1/4
	2	1/8	1/2

(i) Dapatkan fungsi jisim kebarangkalian bersyarat X diberikan Y = 1, 2.

(ii) Tentukan sama ada X dan Y bersandar ataupun sebaliknya.

(50 markah)

(b) Diberikan $S^2 = \sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2$.

(i) Tunjukkan S^2 juga boleh ditulis sebagai $\sum_{i=1}^{n}(X_i - \mu)^2 - n(\overline{X} - \mu)^2$, yang mana $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

(ii) Katakan $X_1, X_2, \ldots, X_n$ adalah sampel rawak daripada taburan $N(\mu, \sigma^2)$. Sekiranya $\overline{X}$ dan $S^2/n-1$ tak bersandar, tunjukkan $S^2/\sigma^2 \sim \chi^2_{n-1}$.

(50 markah)

4. (a) X_1 dan X_2 adalah pembolehubah rawak tak bersandar yang bertabur secara secaman N(0, 1). Andaikan $Y_1 = X_1 + X_2$ dan $Y_2 = X_1 - X_2$.

(i) Dapatkan fungsi ketumpatan tercantum (Y_1, Y_2).

(ii) Seterusnya dengan menggunakan (i) tentukan taburan Y_1 dan Y_2.

(50 markah)

(b) X_1 dan X_2 adalah sampel rawak. Katakan $Y = X_1 + X_2$. Dapatkan taburan Y sekiranya X_1 dan X_2 disampel daripada taburan

(i) Binomial (n, p).

(ii) Poisson (λ).

(20 markah)

...5/-

(c) S_1^2 dan S_2^2 mewakili varians-varians sampel yang masing-masingnya dicerap daripada sampel bersaiz $n_1 = 25$ dan $n_2 = 31$. Kedua-dua sampel ini terdiri daripada cerapan-cerapan bertaburan normal yang mempunyai varians $\sigma_1^2 = 10$ dan $\sigma_2^2 = 15$. Cari $P\left(S_1^2 / S_2^2 > 1.26 \right)$.

(30 markah)

5. (a) Jika 2 pasangan suami isteri duduk sebaris, berapakah kebarangkalian seorang suami itu tidak duduk bersebelahan dengan isterinya?

(25 markah)

(b) X adalah pembolehubah rawak gamma (n, 1). Berapa besarkah n supaya

$$P\left(\left| \frac{X}{n} - 1 \right| > 0.01 \right) < 0.01?$$

(25 markah)

(c) $Z \sim N(0,\ 1)$. Dapatkan Cov $(Z,\ Z^2)$.

(25 markah)

(d) (X, Y) tertabur secara normal bivariat $N\left(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho\right)$. Tunjukkan apabila $\rho = 0$, X dan Y tak bersandar.

(25 markah)

...6/-

Rumus-Rumus

Modul 1

Pelajaran 1

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2. $P(A) = P(A \cap \bar{B}) + P(A \cap B)$

3. $P(\bar{A}) = 1 - P(A)$

4. ${}^{n}P_{r} = \frac{n!}{(n-r)!}$

5. $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

6. $N = \frac{n!}{n_1!\, n_2! \ldots n_k!}$

Pelajaran 2

1. $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

2. $P(A \cap B) = P(A)P(B)$

3. $P(A) = P(A \mid B)\, P(B) + P(A \mid \bar{B})\, P(\bar{B})$

4. $P(B_i \mid A) = \frac{P(A \cap B_i)}{\sum_{j=1}^{n} P(A \mid B_j)\, P(B_j)}$

Pelajaran 3

1. $P(a \leq X \leq b) = \int_{a}^{b} f(x)\, dx$

2. $P(a < X < b) = \sum_{a<x<b} p(x)$

3. $F(t) = P(X \leq t)$

4. $P(a < X \leq b) = F(b) - F(a)$

5. $\frac{d}{dt} F(t) = f(t)$

6. $F_Y(t) = F_X(g^{-1}(t))$

7. $F_Y(t) = 1 - F_X(g^{-1}(t))$

8. $f_Y(t) = f_X(g^{-1}(t)) \mid J \mid$

9. $J = \frac{dg^{-1}(t)}{dt}$

10. $f_Y(t) = \sum_{i=1}^{k} f_X (g_i^{-1}(t)) \mid J_i \mid)$

11. $J_i = \frac{d}{dt} g_i^{-1}(t)$

12. $P_Y(y) = \sum_{x \in A} P_X(x)$

Modul 2

Pelajaran 1

1. $E(X) = \sum_{x \in \text{Julat } X} xp(x)$

2. $1 + x + x^2 + \ldots + x^n + \ldots = \frac{1}{1-x}, \ |x| < 1$

3. $1 + 2x + \ldots + n x^{n-1} + \ldots = \frac{1}{(1-x)^2}, \ |x| < 1$

4. $E(X) = \int_{-\infty}^{\infty} x f(x) \, dx$

5. $E(X) = \int_{0}^{\infty} [1 - f(x)] \, dx - \int_{-\infty}^{0} F(x) \, dx$

6. $E[G(X)] = \sum_{x \in \text{Julat } X} G(x) \, p(x)$

7. $E[G(X)] = \int_{-\infty}^{\infty} G(x)\, f(x)\, dx$

8. $E[c] = c$

9. $E[cX] = c\, E[X]$

10. $E[X + c] = E[X] + c$

11. $\mathrm{Var}(X) = E[X - E[X]]^2$

12. $\mathrm{Var}(X) = E[X^2] - \mu_X^2$

13. $\mathrm{Var}(X) = \sum_{x \in \text{Julat } X} x^2 p(x) - \mu_X^2$

14. $\mathrm{Var}(X) = \int_{-\infty}^{\infty} x^2 f(x)\, dx - \mu_X^2$

15. $\mathrm{Var}(a) = 0$

16. $\mathrm{Var}(aX + b) = a^2\, \mathrm{Var}(X)$

17. $F_X(t_k) = k,\ 0 < k < 1$

Pelajaran 2

1. $m_k = E[X^k]$

2. $m_k = \sum_{x \in \text{Julat } X} x^k\, p(x)$

3. $m_k = \int_{-\infty}^{\infty} x^k f(x)\, dx$

4. $\mu_k = E[(X - \mu_X)^k$

5. $\gamma_1 = \mu_3 / \sigma_X^3$

6. $\gamma_2 = \dfrac{\mu_4}{\sigma_X^4} - 3.$

7. $\mu_{[k]} = E[X(X-1)(X-2) \ldots (X-k+1)]$

8. $m(t) = E[e^{tX}]$

9. $m(t) = \sum_{x \in \text{Julat } X} e^{tx} \, p(x)$

10. $m(t) = \int_{-\infty}^{\infty} e^{tx} \, f(x) \, dx$

11. $m_Y(t) = E[e^{tg(X)}]$

12. $m_Y(t) = \sum_{x \in \text{Julat } X} e^{tg(x)} \, p(x)$

13. $m_Y(t) = \int_{-\infty}^{\infty} e^{tg(x)} \, f(x) \, dx$

14. $m_Y(t) = e^{bt} \, m_X(at)$

15. $m^{(i)}(0) = m_i$

16. $k(t) = \ell n \, m(t)$

17. $\psi(t) = E[t^X]$

18. $f(t) = \sum_{i=0}^{\infty} \frac{f^{(i)}(a)}{i!} (t-a)^i$

19. $\psi^{(i)}(0) = i! \, p(i)$

20. $P(|X| \geq a) < \frac{1}{a^2} E[X^2]$

21. $P(|X-\mu| \geq a\sigma) \leq \frac{1}{a^2}$

22. $P(|X-\mu| < a\sigma) \geq 1 - \frac{1}{a^2}$

23. $P(X \geq a) \leq \frac{E[X]}{a}$

24. $E[X^n] = \int_{0}^{\infty} n x^{n-1} (1-F(x)) \, dx$

Pelajaran 3

1. (i) $p(x) = \begin{cases} q, & x = 0 \\ p, & x = 1 \\ 0, & \text{ditempat lain} \end{cases}$ $X \sim$ Bernoulli (p)

   (ii) $E[X] = p$

   (iii) $Var(X) = pq$

   (iv) $m(t) = q + pe^t$

2. (i) $p(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & x = 0, 1, 2, \ldots, n \\ 0, & \text{di tempat lain} \end{cases}$ $X \sim$ Binomial (n, p)

   (ii) $E[X] = np$

   (iii) $Var(X) = npq$

   (iv) $m(t) = (q + pe^t)^n$

3. (i) $p(x) = \begin{cases} \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}, & x = 0, 1, 2, \ldots, n \\ 0, & \text{di tempat lain} \end{cases}$ $X \sim$ hipergeometri (N, k, n)

   (ii) $E[X] = \dfrac{nK}{N}$

   (iii) $Var(X) = \dfrac{nK(N-K)(N-n)}{N^2(N-1)}$

4. $(a + b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$

5. (i) $p(x) = \begin{cases} q^{x-1}p, & x = 1, 2, 3, \ldots \\ 0, & \text{di tempat lain} \end{cases}$ $\qquad X \sim$ geometri (p)

   (ii) $E[X] = 1/p$

   (iii) $Var(X) = q/p^2$

   (iv) $m(t) = \dfrac{pe^t}{1 - qe^t}$

6. (i) $p(x) = \begin{cases} \dbinom{x-1}{r-1} p^r q^{x-r}, & x = r, r+1, r+2 \\ & r = 2, 3, 4, \ldots \\ 0, & \text{di tempat lain} \end{cases}$ $\qquad X \sim$ negatif binomial (r, p)

   (ii) $E[X] = r/p$

   (iii) $Var(X) = rq/p^2$

   (iv) $m(t) = \left[\dfrac{pe^t}{1 - qe^t}\right]^r$

7. (i) $p(x) = \begin{cases} e^{-\lambda}\dfrac{\lambda^x}{x!}, & x = 0, 1, 2, \ldots \\ 0, & \text{di tempat lain} \end{cases}$ $\qquad X \sim$ Poisson (λ)

   (ii) $E[X] = \lambda$

   (iii) $Var(X) = \lambda$

   (iv) $m(t) = e^{\lambda(e^t - 1)}$

8. $\underset{x \to 0}{\text{had}} (1 + x)^{1/x} = e$

9. $\underset{x \to \infty}{\text{had}} \left(1 + \dfrac{1}{x}\right)^x = e$

10. $\underset{x \to 0}{\text{had}} (1 + ax)^{1/x} = e^a$

Pelajaran 4

1. (i) $f(x) = \begin{cases} \dfrac{1}{b-a}, & a<x<b \\ 0, & \text{di tempat lain} \end{cases}$ X ~ seragam (a, b)

   (ii) $E[X] = \dfrac{a+b}{2}$

   (iii) $Var(X) = \dfrac{(b-a)^2}{12}$

   (iv) $m(t) = \dfrac{e^{bt} - e^{at}}{t(b-a)}$

2. (i) $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, -\infty < x < \infty$ X ~ $N(\mu, \sigma^2)$

   (ii) $E[X] = \mu$

   (iii) $Var(X) = \sigma^2$

   (iv) $m(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

3. $\underset{n\to\infty}{\text{had}}\, P\left[a \le \dfrac{S_n - np}{\sqrt{npq}} \le b\right] \to P(Z \ge a) - P(Z > b)$

4. $\underset{\lambda\to\infty}{\text{had}}\, P\left[a \le \dfrac{X-\lambda}{\sqrt{\lambda}} < b\right] \to P(Z > a) - P(Z \ge b)$

5. (i) $f(x) = \begin{cases} \lambda e^{-\lambda}, & x \ge 0 \\ 0, & \text{di tempat lain} \end{cases}$ X ~ eksponen (λ)

   (ii) $E[X] = 1/\lambda$

   (iii) $Var(X) = 1/\lambda^2$

   (iv) $m(t) = \dfrac{\lambda}{\lambda - t}$

6. $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx$

7. $\Gamma(n) = (n-1)\,\Gamma(n-1)$

8. $\Gamma(n) = (n-1)!$

9. (i) $f(x) = \begin{cases} \dfrac{\lambda^n x^{n-1}}{\Gamma(n)} e^{-\lambda x}, & x > 0 \\ 0, & \text{di tempat lain} \end{cases}$ $X \sim \text{Gamma}(n, \lambda)$

(ii) $E[X] = n/\lambda$

(iii) $\text{Var}(X) = n/\lambda^2$

(iv) $m(t) = \left(\dfrac{\lambda}{\lambda - t}\right)^n$

10. (i) $f(x) = \begin{cases} \dfrac{1}{2^{\upsilon/2}\Gamma\left(\dfrac{\upsilon}{2}\right)} x^{\upsilon/2-1} e^{-x/2}, & x > 0 \\ 0, & \text{di tempat lain} \end{cases}$ $X \sim \chi^2_\upsilon$

(ii) $E[X] = \upsilon$

(iii) $\text{Var}(X) = 2\upsilon$

(iv) $m(t) = \left(\dfrac{1}{1-2t}\right)^{\upsilon/2}$

11. $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\, dt$

12. $B(x, y) = \int_0^\infty \dfrac{t^{x-1}}{(1+t)^{x+y}}\, dt$

13. $B(x, y) = \dfrac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$

14. (i) $f(x) = \begin{cases} \dfrac{1}{B(a,b)} x^{a-1}(1-x)^{b-1} & , 0 < x < 1 \\ 0 \ , & \text{di tempat lain} \end{cases}$ $\quad X \sim \text{Beta}(a, b)$

(ii) $F_X(p) = \sum_{x=a}^{n} \binom{n}{x} p^x (1-p)^{n-x}$

(iii) $E[X] = \dfrac{a}{a+b}$

(iv) $\text{Var}(X) = \dfrac{ab}{(a+b+1)(a+b)^2}$

Modul 3

Pelajaran 1

1. $P(X \le x, Y \le y) = \sum_{t_1 \le x} \sum_{t_2 \le y} p(t_1, t_2)$

2. $P(X \le x, Y \le y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(t_1, t_2)\, dt_1\, dt_2$

3. $F(x, y) = P(X \le x, Y \le y)$

4. $f(x, y) = \dfrac{\partial^2 F(x, y)}{dxdy}$

Pelajaran 2

1. $p(x) = \sum_{y} p(x, y)$

2. $p(y) = \sum_{x} p(x, y)$

3. $f(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$

4. $f(y) = \int_{-\infty}^{\infty} f(x, y)\, dx$

5. $F(x) = F(x, \infty)$

6. $F(y) = F(\infty, y)$

7. $f(x) = \dfrac{\partial F(x, \infty)}{\partial x}$

8. $f(y) = \dfrac{\partial F(\infty, y)}{\partial y}$

9. $p(x \mid y) = \dfrac{p(x, y)}{p(y)}$

10. $f(x \mid y) = \dfrac{f(x, y)}{f(y)}$

11. $p(x, y) = p(x)\, p(y)$

12. $f(x, y) = f(x)\, f(y)$

Pelajaran 3

1. $E[g(X, Y)] = \sum_x \sum_y g(x, y)\, p(x, y)$

2. $E[g(X, Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f(x,y) dx\, dy$

3. $E[g_1(X, Y) + g_2(X, Y)] = E[g_1(X, Y)] + E[g_2(X, Y)]$

4. $E[h_1(X)\, h_2(Y)] = E[h_1(X)]\, E[h_2(Y)]$

5. (i) $\text{Cov}(X, Y) = E[X - \mu_X)(Y - \mu_Y)]$

   (ii) $\text{Cov}(X, Y) = E[XY] - \mu_X \mu_Y$

6. $\text{Cov}(aX, bY) = ab\, \text{Cov}(X, Y)$

7. $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\, \text{Cov}(X, Y)$

8. $\text{Var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \text{Var}(X_i) + 2\sum_{i<j}\sum \text{Cov}(X, Y)$

9. $\rho(X, Y) = \dfrac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$

10. $E[g(X, Y) \mid Y = y] = \sum_{x} g(x, y)\, p(x \mid y)$

11. $E[g(X, Y) \mid Y = y] = \int_{-\infty}^{\infty} g(x,y) f(x \mid y)\, dx$

12. $E[E[X \mid Y = y]] = E[X]$

13. $E[E[Y \mid X = x]] = E[Y]$

14. $E[E[g(X) \mid Y = y]] = E[g(X)]$

15. $E[E[g(Y) \mid X = x]] = E[g(Y)]$

16. $\text{Var}(X \mid Y = y) = E[X^2 \mid Y = y] - (E[X \mid Y = y)^2$

17. $m(t_1, t_2) = E\left[e^{t_1 X_1 + t_2 X_2}\right]$

18. $m(t_1, t_2, \ldots, t_n) = E\left[e^{\sum_{i=1}^{n} t_i X_i}\right]$

19. $m(t_1) = \lim_{t_2 \to 0} m(t_1, t_2)$

20. $m(t_1, t_2, \ldots, t_n) = m(t_1)\, m(t_2) \ldots m(t_n)$

Pelajaran 4

1. (i) $p(x_1, x_2, \ldots, x_k) = \dfrac{n!}{x_1!\, x_2! \ldots x_k!}\, p_1^{x_1}\, p_2^{x_2} \ldots p_k^{x_k}$

   (ii) $p(x_i) = \binom{n}{x_i} p_i^{x_i} (1 - p_i)^{n - x_i}$

   (iii) $p(x_i, x_j) = \dfrac{n!}{x_i!\, x_j!\, (n - x_i - x_j)!}\, p_i^{x_i}\, p_j^{x_j} (1 - p_i - p_j)^{n - x_i - x_j}$

   (iv) $E[X_i X_j] = n(n - 1)\, p_i p_j$

   (v) $\text{Cov}(X_i, X_j) = -n p_i p_j$

2. (i) $$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_X}{\sigma_X}\right)^2 - 2\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right) + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2\right]\right\},$$

$$-\infty < x < \infty, -\infty < y < \infty$$

(ii) $$f(x|y) = \frac{1}{\sigma_x\sqrt{2\pi(1-\rho^2)}} \exp\left\{-\frac{1}{2(1-\rho^2)\sigma_X^2}\left[x-\mu_X-\rho\frac{\sigma_X}{\sigma_Y}(y-\mu_Y)\right]^2\right\}$$

$$-\infty < x < \infty$$

(iii) $$m(t_1,t_2) = \exp\left[t_1\mu_X + t_2\mu_Y + \frac{1}{2}\left(t_1^2\sigma_X^2 + 2\rho t_1 t_2\sigma_X\sigma_Y + t_2^2\sigma_Y^2\right)\right]$$

(iv) $E[XY] = \mu_X\mu_Y + \rho\,\sigma_X\sigma_Y$

(v) $\mathrm{Cov}(X, Y) = \rho\,\sigma_X\sigma_Y$

Modul 4

Pelajaran 1

1. $M_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k$

2. $E[M_k] = m_k$

3. $\mathrm{Var}(M_k) = \frac{1}{n}\,[m_{2k} - m_k)^2]$

4. $E[\bar{X}] = \mu$

5. $\mathrm{Var}(\bar{X}) = \frac{1}{n}\,\sigma^2$

6. $S^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (X_i - \bar{X})^2$

7. $E[S^2] = \sigma^2$

8. $\text{Var}(S^2) = \frac{1}{n}\left(\mu_4 - \frac{(n-3)}{(n-1)}\sigma^4\right)$

9. $\sum_{i=1}^{n}(X_i - \mu)^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2 + n(\bar{X} - \mu)^2$

10. $\bar{X} - \mu = \frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)$

Pelajaran 2

1. $p(u, v) = p_{X,Y}\left(g_1^{-1}(u, v),\ g_2^{-1}(u, v)\right)$

2. $f(u, v) = f_{X,Y}\left(g_1^{-1}(u, v), g_2^{-1}(u, v)\right) \mid J \mid$

3. $J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$

4. $f(u,v) = \sum_{i=1}^{m} |J_i| f_{X,Y}\left(g_i^{-1}(u,v), h_i^{-1}(u,v)\right)$

5. $J_i = \begin{vmatrix} \frac{\partial g_i^{-1}(u,v)}{\partial u} & \frac{\partial g_i^{-1}(u,v)}{\partial v} \\ \frac{\partial h_i^{-1}(u,v)}{\partial u} & \frac{\partial h_i^{-1}(u,v)}{\partial v} \end{vmatrix}$

6. $m_{u,v}(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{t_1 g(x,y) + t_2 h(x,y)} f(x,y)\,dx\,dy$

7. $m_u(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{tg(x,y)} f(x,y)\,dx\,dy$

8. (i) $f_{u=X+Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(x, u-x)\, dx$

(ii) $f_{u=X+Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(u-y, y)\, dy$

9. (i) $f_{u=X-Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(x, x-u)\, dx$

(ii) $f_{u=X-Y}(u) = \int_{-\infty}^{\infty} f_{X,Y}(u+y, y)\, dy$

10. (i) $f_{u=XY}(u) = \int_{-\infty}^{\infty} \frac{1}{|x|} f_{X,Y}(x, u/x)\, dx$

(ii) $f_{u=XY}(u) = \int_{-\infty}^{\infty} \frac{1}{|y|} f_{X,Y}(u/y, y)\, dy$

11. $f_{u=X/Y}(u) = \int_{-\infty}^{\infty} |y|\, f_{X,Y}(uy, y)\, dy$

Pelajaran 3

1. (i) $f(x) = \frac{\Gamma[(n+1)/2]}{\Gamma(n/2)\sqrt{\pi n}} \left(1 + \frac{x^2}{n}\right)^{-(n+1)/2}, -\infty < x < \infty$ $\quad X \sim t_n$

(ii) $T = \frac{Z}{\sqrt{V/n}}$

(iii) $E[X] = 0$

(iv) $Var\,[X] = \frac{n}{n-2}$

2. (i) $$f(x) = \begin{cases} \dfrac{\Gamma[m+n)/2}{\Gamma(m/2)\Gamma(n/2)} \left(\dfrac{m}{n}\right)^{m/2} \dfrac{x^{(m-2)/2}}{[1+(m/n)x]^{(m+n)/2}}, x>0 \\ 0 \quad , \text{di tempat lain} \end{cases}$$ $X \sim F_{m,n}$

(ii) $F = \dfrac{U/m}{V/m}$

(iii) $E[X] = \dfrac{n}{n-2}$

(iv) $Var(X) = \dfrac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$

- oooOooo -

Senarai Rumus Tambahan

1. $\sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = e^{\lambda}$

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