UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2008/2009

November 2008

MAT 514 – Mathematical Modelling
[Pemodelan Matematik]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **SEVEN** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instructions: Answer **all four** [4] questions.

[Arahan: *Jawab **semua empat** [4] soalan.]*

1. (a) Taking all necessary assumptions derive the unlimited population growth model

$$\frac{dP}{dt} = kP$$

with the growth-rate k and hence obtain the Malthusian model of population growth.

(b) The following table provides the population in US; US census figures, in million, of people (shown in columns: Year, t and Actual population)

Year	t	Actual	Year	t	Actual
1790	0	3.9	1900	110	75.0
1800	10	5.3	1910	120	91.0
1810	20	7.2	1920	130	105.0
1820	30	9.6	1930	140	122.0
1830	40	12.0	1940	150	131.0
1840	50	17.0	1950	160	151.0
1850	60	23.0	1960	170	179.0
1860	70	31.0	1970	180	203.0
1870	80	38.0	1980	190	226.0
1880	90	50.0	1990	200	249.0
1890	100	62.0	2000	210	281.0

(i) Formulate the population model and solve.

(ii) Determine the values of growth-rate k.

(iii) Make predictions about the population in the years 1920 and 2000.

(iv) Compare your solution to the actual data. Do you believe your prediction?

[100 marks]

1. (a) *Dengan membuat andaian yang bersesuaian, terbitkan model pertumbuhan populasi tanpa had*

$$\frac{dP}{dt} = kP$$

dengan kadar pertumbuhan k dan seterusnya peroleh model pertumbuhan populasi Malthusian

(b) *Jadual berikut menunjukan populasi di US; angka banci US, dalam jutaan manusia (ditunjukan dalam lajur : Tahun t dan populasi sebenar)*

Tahun	*t*	*Sebenar*	*Tahun*	*t*	*Sebenar*
1790	0	3.9	1900	110	75.0
1800	10	5.3	1910	120	91.0
1810	20	7.2	1920	130	105.0
1820	30	9.6	1930	140	122.0
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(i) *Rumuskan model populasi dan selesaikan.*

(ii) *Tentukan nilai kadar pertumbuhan k.*

(iii) *Ramalkan populasi pada tahun* 1920 *dan* 2000.

(iv) *Bandingkan penyelesaian anda dengan data sebenar. Adakah anda percaya ramalan anda?*

[100 markah]

2. (a) Derive an expression

$$mx'' + kx = 0$$

for the simple harmonic motion. Also find the amplitude, the frequency and the period of the motion.

(b) A block weighing 8-*lb* is attached to the end of a spring, causing the spring to stretch 6 inch beyond its natural length. The block is then pulled down 3 inch and released. Assume that there are no damping forces or external applied forces.

(i) Formulate the mathematical model that describes the motion of the block.

(ii) Determine the motion of the block.

(iii) Consider the spring mass system in the above example assuming that damping is present and the damping coefficient is given by $\gamma = 1$ *lb*-sec/ft. Determine the motion of the block.

[100 marks]

3. (a) A carpenter makes tables and bookcases. He is trying to determine how many of each type of furniture he should make each week. The carpenter wishes to determine a weekly production schedule for tables and bookcases that maximizes his profits. The carpenter realizes a net unit profit of RM 25 per table and RM 30 per bookcase. He has up to 690 board feet of lumber to devote weekly to the project and up to 120 hr of labor. He can use lumber and labor productively elsewhere if they are not used in the production of tables and bookcases. He estimates that it requires 20 board feet of lumber and 5 hr of labor to complete a table and 30 board feet of lumber and 4 hr of labor for a bookcase.

Formulate a mathematical model and solve for how many of each piece of furniture he should make each week to maximize its profit.

(b) Write notes on Stress, Strain and Newton's law of viscosity. Stating clearly all basic assumptions, derive the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

for incompressible flow.

[100 marks]

2. (a) *Terbitkan ungkapan*

$$mx'' + kx = 0$$

untuk ayunan okum d mudah. Dapatkan okum de, kekerapan dan kalaan pergerakan.

(b) *Suatu bungkah berat* 8-*lb diikat pada hujung spring, menyebabkan spring meregangkan* 6 *inci melebihi kepanjangan biasanya. Bungkah kemudian ditarik* 3 *inci ke bawah dan dilepaskan. Andaikan tiada daya kelembapan atau daya luaran dikenakan.*

(i) *Rumuskan model matematik yang menghuraikan pergerakan bungkah.*

(ii) *Tentukan pergerakan bungkah*

(iii) *Pertimbangkan okum spring-jisim diatas dengan andaian terdapat kelembapan dan pekali kelembapan diberi oleh* $\gamma = 1$ *lb-saat/kaki. Tentukan pergerakan bungkah.*

[100 markah]

3. (a) *Seorang tukang kayu membuat meja dan rak buku. Beliau ingin mengetahui jadual penghasilan mingguan untuk meja dan rak buku yang memaksimumkan keuntungan. Tukang kayu memperoleh keuntungan bersih unit sebanyak RM* 25 *per meja dan RM* 30 *per rak buku. Beliau mempunyai* 690 *kaki kayu untuk projek beliau dan* 120 *jam waktu buruh. Beliau boleh gunakan kayu dan buruh ditempat lain secara produktif jika kayu dan buruh tidak digunakan untuk menghasikan meja dan rak buku. Beliau menganggarkan* 20 *kaki kayu dan* 5 *jam waktu buruh diperlukan untuk menghasilkan suatu meja dan* 30 *kaki kayu dan* 4 *jam waktu buruh untuk menghasilkan rak buku*

Rumuskan model matematik dan selesaikan untuk setiap jenis yang beliau perlu buat setiap minggu untuk memaksimumkan keuntungan.

(b) *Tulis nota berkaitan tegasan, keterikan dan okum kelikatan Newton. Sebutkan semua andaian dan terbitkan persamaan kelanjaran*

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

untuk aliran tak mampat.

[100 markah]

4. (a) Explain the concept of shallow water theory. Stating clearly all basic assumptions of shallow water theory, establish the vertically integrated shallow water model.

(b) Explaining all the basic assumptions derive the following fundamental one-dimensional diffusion transport equation

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} + r_g - r_d$$

for a substance well mixed with a fluid. Hence write down the three-dimensional form of the equation.

A tank with a fixed volume of 200 L has water flowing through an inlet and an outlet at a constant rate of 5 L/min. The water in the tank is continuously mixed by a mechanical device. The concentration of dissolved oxygen (DO) in the tank is initially zero, and the inlet flow is at the saturation concentration C_s of DO.

(i) Formulate the mathematical models for the following cases:
The tank is open to the atmosphere so that oxygen can transfer across the air-water interface; and the tank is completely sealed with oxygen entering the system only through the inlet.

(ii) Solve the models.

[100 marks]

4. (a) *Terangkan konsep teori air cetek. Sebutkan semua andaian teori air cetek dan bangunkan model air cetek terkamirkan secara mencacang.*

(b) *Dengan menerangkan semua andaian terbitkan persamaan asas pengangkatan resapan satu dimensi*

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} + r_g - r_d$$

untuk suatu bahan yang bercampur dengan baik bersama suatu bendalir. Seterusnya tulis bentuk tiga-dimensi persamaan ini.

Suatu tangki dengan isipadu tetap 200 *L mempunyai air yang mengalir masuk melalui input dan mengalir keluar melalui output pada kadar malar* 5 *L/min. Air dalam tangki dicampur secara berterusan oleh suatu alat mekanikal. Kepekatan oksigen terlarut (DO) dalam tangki pada awalnya sifar dan aliran pada input ialah pada kepekatan ketepuan saturation* C_s *DO.*

(i) *Rumuskan model-model matematik untuk kes-kes berikut:*
Tangki dibuka kepada atmosfera sedemikian oksigen boleh mengalir melalui antaramuka udara-air; Tangki ditutup sepenuhnya dengan oksigen boleh masuk hanya melalui input.

(ii) *Selesaikan model-model ini.*

[100 markah]

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