
UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2008/2009

November 2008

MAT 251 – Introduction to Operations Research
[Pengantar Penyelidikan Operasi]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of ELEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEBELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all eight** [8] questions.

Arahan: Jawab **semua lapan** [8] soalan.]

1. During the USM convocation, the number of guards needed daily for the whole week are shown according to the table below:

Duration	Number of Guards
12.00 – 4.00 a.m.	18
4.00 – 8.00 a.m.	25
8.00 – 12.00 noon	20
12.00 – 4.00 p.m.	30
4.00 – 8.00 p.m.	25
8.00 – 12.00 p.m.	22

Every guard must work the 8 hour shift continuously. Formulate this problem as a linear programming problem.

[10 marks]

2. Given the following linear programming,

$$\begin{aligned} \text{Maximize } Z &= 25x_1 + 40x_2 && \text{[Profit]} \\ \text{Subject to} & && \\ & 2x_1 + 3x_2 &\leq 200 & \text{Resource 1} \\ & -2x_1 + 6x_2 &\leq 54 & \text{Resource 2} \\ & 6x_1 - 5x_2 &\geq 108 & \text{Resource 3} \\ & x_1 &\leq 80 & \text{Resource 4} \end{aligned}$$

$$\text{where } x_1 \geq 0, x_2 \geq 0$$

- (i) Solve the problem graphically.
- (ii) Which are binding and the non-binding constraints?
- (iii) State the status of each resource whether they are scarce or surplus?
- (iv) What is the leftover amount for each of the surplus resources?
- (v) Get the dual value of resource 1.
- (vi) Find the range for resource 1 that will not affect the optimum solution found in (ii).
- (vii) If the profit per unit of x_1 changes by δ , find the range for δ that will not affect the optimum solution found in (ii).
- (viii) What is the profit range for x_1 that will make this factory produces x_2 only?

[20 marks]

1. Sempena peristiwa konvokesyen USM, bilangan pengawal yang diperlukan setiap hari sepanjang minggu adalah seperti berikut :

Tempoh	Bilangan Pengawal
12.00 – 4.00 pagi	18
4.00 – 8.00 pagi	25
8.00 – 12.00 tgh	20
12.00 – 4.00 petang	30
4.00 – 8.00 petang	25
8.00 – 12.00 malam	22

Setiap pengawal mesti bekerja satu syif selama 8 jam secara berterusan.
Rumuskan masalah sebagai suatu model pengaturcaraan linear.

[10 markah]

2. Diberikan suatu masalah pengaturcaraan linear seperti berikut,

$$\text{Maksimumkan } Z = 25x_1 + 40x_2 \quad [\text{Keuntungan}]$$

$$\text{Terhadap } 2x_1 + 3x_2 \leq 200 \quad \text{Sumber 1}$$

$$-2x_1 + 6x_2 \leq 54 \quad \text{Sumber 2}$$

$$6x_1 - 5x_2 \geq 108 \quad \text{Sumber 3}$$

$$x_1 \leq 80 \quad \text{Sumber 4}$$

$$\text{dan } x_1 \geq 0, x_2 \geq 0$$

- (i) Selesaikan masalah ini dengan graf.
- (ii) Kekangan manakah yang terikat dan yang tidak terikat?
- (iii) Nyatakan status semua sumber sama ada berkurangan atau berlebihan.
- (iv) Berapakah amaun baki bagi setiap sumber?
- (v) Dapatkan nilai dual bagi sumber 1.
- (vi) Dapatkan julat bagi sumber 1 dengan tidak menjejaskan penyelesaian optimum di (ii).
- (vii) Sekiranya keuntungan seunit x_1 berubah sebanyak δ , dapatkan julat bagi δ yang tidak akan menjejaskan penyelesaian optimum di (ii).
- (viii) Apakah julat keuntungan seunit x_1 yang akan membuatkan kilang ini menghasilkan x_2 sahaja?

[20 markah]

3. Consider the following problem,

$$\text{Minimize } Z = 20x_1 + 35x_2 + 30x_3$$

$$\begin{aligned} \text{Subject to } \quad 3x_1 + 5x_2 + x_3 &\geq 20 \\ 2x_1 \quad \quad + 4x_3 &= 15 \end{aligned}$$

$$\text{where } x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Using the two phase method, do two iterations towards solving it.

[10 marks]

4. A problem to determine the level of production of 3 products (x_1 , x_2 and x_3) using 4 resources (the constraints are in the form of \leq and the slack variables are respectively s_1 , s_2 , s_3 , and s_4) results in the following tableau. Z represents profit (in RM).

Basic	x_1	x_2	x_3	s_1	s_2	s_3	s_4	Solution
Z	0	0	5	0	3	0	0	91
x_1	1	0	-3	4	-2	0	0	5
x_2	0	1	4	0	-3	0	0	7
s_3	0	0	0	-1	0	1	0	2
s_4	0	0	2	3	-1	0	1	4

- (i) Describe the status of each resources (binding, non-binding, scarce or surplus)?
- (ii) What is the remainder of each of the resources above?
- (iii) How much can resource 1 change without affecting the solution above?
- (iv) How much will the profit reduces if 1 unit of product 3 is produced?
- (v) How much will change in per unit profit of product 2 before production of product 3 starts?
- (vi) How much will change in resource 2 that makes the above solution infeasible?
- (vii) Get an alternative basic optimal solution. Give the general expression for all of the alternative optimal solutions.
- (viii) If an additional constraint, $3x_1 + 2x_2 + 10x_3 \leq 20$ is imposed, will the original solution remains optimal?

[15 marks]

3. Pertimbangkan masalah berikut,

$$\text{Minimumkan } Z = 20x_1 + 35x_2 + 30x_3$$

$$\begin{aligned} \text{Terhadap } 3x_1 + 5x_2 + x_3 &\geq 20 \\ 2x_1 + x_3 &= 15 \end{aligned}$$

$$\text{dengan } x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Dengan menggunakan kaedah dua fasa, lakukan dua lelaran ke arah menyelesaikan masalah ini.

[10 markah]

4. Suatu masalah untuk menentukan aras pengeluaran 3 jenis produk (x_1 , x_2 dan x_3) yang menggunakan 4 sumber (dengan kekangan asal semuanya berbentuk \leq dan pembolehubah lalainya ialah s_1 , s_2 , s_3 , dan s_4) menghasilkan tablo maksimum berikut. Z ialah keuntungan (di dalam RM).

Asas	x_1	x_2	x_3	s_1	s_2	s_3	s_4	Penyelesaian
Z	0	0	5	0	3	0	0	91
x_1	1	0	-3	4	-2	0	0	5
x_2	0	1	4	0	-3	0	0	7
s_3	0	0	0	-1	0	1	0	2
s_4	0	0	2	3	-1	0	1	4

- (i) Nyatakan status setiap sumber (terikat, tak terikat, berkurangan atau berlebihan)?
- (ii) Berapakah baki setiap sumber bagi penyelesaian di atas?
- (iii) Berapakah amaun sumber 1 boleh berubah tanpa menjejaskan penyelesaian di atas?
- (iv) Berapakah jumlah keuntungan akan menyusut jika sebanyak 1 unit produk 3 dihasilkan?
- (v) Berapakah keuntungan seunit produk 2 perlu berubah untuk mula menghasilkan produk 3?
- (vi) Berapakah amaun sumber 2 perlu berubah untuk menjadikan penyelesaian di atas tak tersaur?
- (vii) Dapatkan satu penyelesaian alternatif yang asas. Berikan ungkapan umum untuk semua penyelesaian optimum alternatif.
- (viii) Sekiranya suatu kekangan tambahan $3x_1 + 2x_2 + 10x_3 \leq 20$ dikenakan, adakah penyelesaian asal masih optimum?

[15 markah]

5. Given the data for a transportation problem in the table below. The cost of transportation (in thousand RM) are shown at the centre of the table, while the supply and demand (in Million units) limits are also shown in the following table.

Source	Destination				Supply
	A	B	C	D	
1	10	11	7	15	20
2	5	15	10	12	25
3	11	8	7	21	25
Demand	15	12	18	20	

- Solve the problem by using the transportation method so that the total cost of transportation can be minimized. What is the minimal total cost?
- Find the range for C_{11} , which is the cost of transportation from Source 1 to Destination A so that the optimal solution found in (a) remains.
- Find the range for C_{13} , which is the cost of transportation from Source 1 to Destination C so that the optimal solution found in (a) remains.
- Get the new solution if the supply from Source 3 and demand at Destination B is increased by 2.
- Get the new solution if the supply from Source 2 and demand at Destination C is increased by 3.

[15 marks]

6. A company has five workers. On a particular day, six tasks are scheduled to be completed. A cost is estimated for each worker-job combination and is shown in the table.

Worker	Job					
	1	2	3	4	5	6
A	7	5	3	6	4	6
B	4	3	7	5	3	3
C	8	5	8	9	7	6
D	5	6	4	8	5	7
E	9	4	5	3	6	8

Set up the assignment tableau and solve for each of the following situations:

- Find an assignment that completes as many tasks as possible at minimum cost when each worker can do only one job. List out your solution. Which job is left undone?
- If worker C cannot be assigned to job 6, **just show** the new initial assignment tableau towards solving it. [Do not do any iteration].

[10 marks]

5. Diberikan data bagi suatu masalah pengangkutan di dalam jadual di bawah. Kos pengangkutan (dalam ribu RM) ditunjukkan dibahagian tengah jadual, manakala had bekalan dan permintaan (juta unit) ditunjukkan di dalam jadual berikut.

Punca	Destinasi				Bekalan
	A	B	C	D	
1	10	11	7	15	20
2	6	15	10	12	25
3	11	8	7	21	25
Permintaan	15	12	18	20	

- (a) Selesaikan masalah ini menggunakan kaedah pengangkutan supaya jumlah kos dapat diminimumkan. Berapakah jumlah kos yang minimum?
 (b) Dapatkan julat bagi C_{11} , iaitu kos pengangkutan dari Punca 1 ke Destinasi A supaya penyelesaian optimum yang didapati di (a) kekal.
 (c) Dapatkan julat bagi C_{13} , iaitu kos pengangkutan dari Punca 1 ke Destinasi C supaya penyelesaian optimum yang didapati di (a) kekal.
 (d) Dapatkan penyelesaian baru jika bekalan di Punca 3 dan permintaan di Destinasi B meningkat sebanyak 2.
 (e) Dapatkan penyelesaian baru jika bekalan di Punca 2 dan permintaan di Destinasi C meningkat sebanyak 3.

[15 markah]

6. Sebuah syarikat mempunyai lima orang pekerja. Pada suatu hari, enam tugas perlu dilaksanakan. Kos bagi setiap pekerja-tugas telah dianggarkan dan ia ditunjukkan di dalam jadual.

Pekerja	Tugasan					
	1	2	3	4	5	6
A	7	5	3	6	4	6
B	4	3	7	5	3	3
C	8	5	8	9	7	6
D	5	6	4	8	5	7
E	9	4	5	3	6	8

Binakan tablo umpukan dan selesaikan bagi setiap keadaan berikut:

- (a) Dapatkan suatu umpukan untuk menyiapkan sebanyak tugas yang mungkin dengan kos yang minimum jika seorang pekerja hanya boleh melakukan satu tugas sahaja. Senaraikan jawapan anda. Tugas yang mana tidak dapat dilaksanakan?
 (b) Jika pekerja C tidak boleh melaksanakan tugas 6, **tunjukkan** tablo umpukan permulaan ke arah menyelesaikannya. [Jangan lakukan sebarang lelaran].

[10 markah]

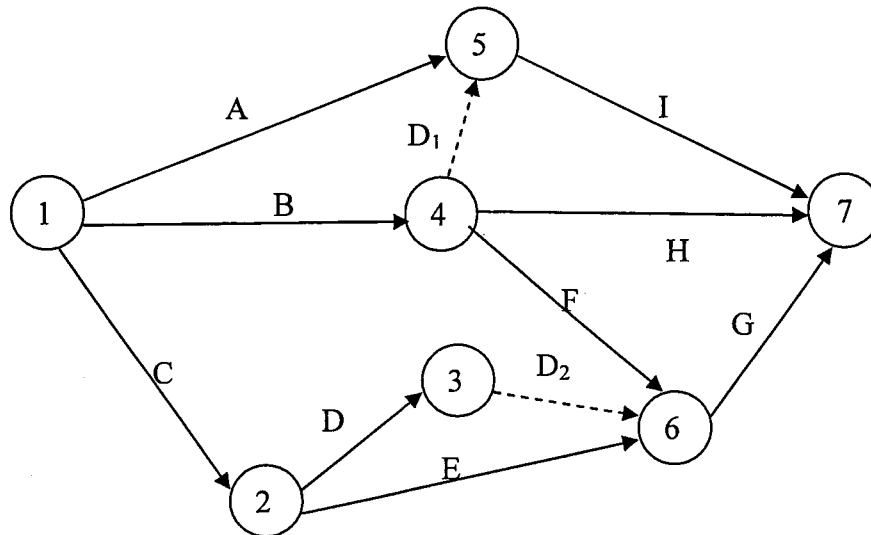
7. Consider the list of activities and its predecessors that are involved in doing a project.

Activity	Precedence Activities	Duration (days)
A	-	5
B	-	4
C	-	7
D	A,C	6
E	B,C	8
F	D, E	7
G	E	5

- (a) Draw a project network diagram.
- (b) Determine the critical path of the project. Give the minimum time and list the critical activities.

[10 marks]

8. The network diagram below represents a project.



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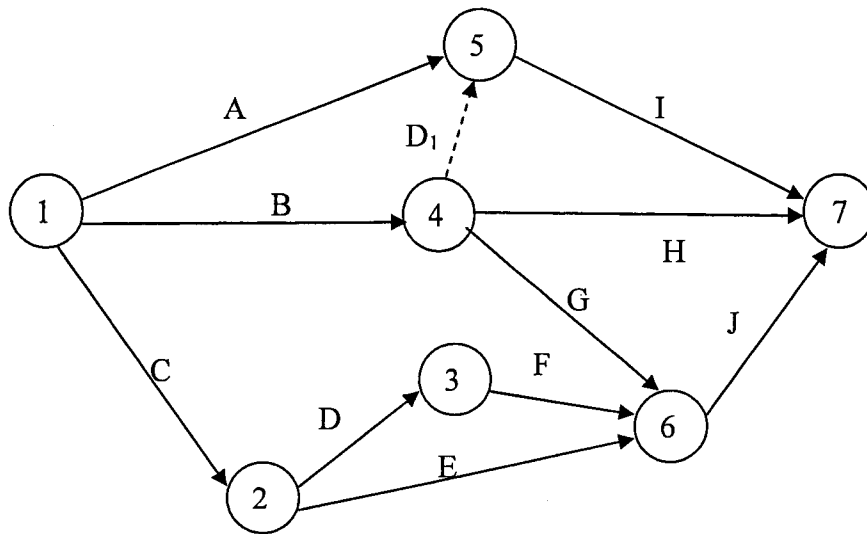
7. Pertimbangkan senarai kegiatan dan kegiatan pendahulu untuk membuat suatu projek.

Kegiatan	Kegiatan Pendahulu	Jangkamasa (hari)
A	-	5
B	-	4
C	-	7
D	A,C	6
E	B,C	8
F	D, E	7
G	E	5

- (a) Lakarkan gambarajah aliran projek ini.
- (b) Tentukan lintasan genting projek ini. Berikan masa terpendek dan senaraikan kegiatan-kegiatan yang genting.

[10 markah]

8. Gambarajah rangkaian berikut mewakili suatu projek:



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The normal and crash durations, and the normal and crash costs are given as follows:

Activity	Duration (days)		Cost (RM)	
	Normal	Crash	Normal	Crash
A	5	4	60	80
B	4	3	70	100
C	3	2	150	150
D	5	3	180	300
E	6	4	200	400
F	4	3	180	220
G	3	2	120	200
H	5	4	100	200
I	6	4	400	600
J	7	5	250	300

The indirect cost per day is RM80. The contract agreement states that there is a penalty cost of RM110 per day if it later than 20 days and there is a bonus of RM150 per day if it is earlier than 20 days.

- (i) Determine all possible critical paths.
- (ii) List all of the critical activities.
- (iii) Get the minimum total cost for the project.
- (iv) Do just **two** iterations to crash the project. Show the total cost for each iterations.

[10 marks]

Jangkamasa biasa dan nahas, serta kos biasa dan nahas bagi setiap kegiatan diberikan seperti berikut:

<i>Kegiatan</i>	<i>Jangkamasa (hari)</i>		<i>Kos Langsung (RM)</i>	
	<i>Biasa</i>	<i>Nahas</i>	<i>Biasa</i>	<i>Nahas</i>
<i>A</i>	5	4	60	80
<i>B</i>	4	3	70	100
<i>C</i>	3	2	150	150
<i>D</i>	5	3	180	300
<i>E</i>	6	4	200	400
<i>F</i>	4	3	180	220
<i>G</i>	3	2	120	200
<i>H</i>	5	4	100	200
<i>I</i>	6	4	400	600
<i>J</i>	7	5	250	300

Kos tak langsung projek ialah RM80 sehari. Ternyata di dalam perjanjian kontrak bahawa denda sebanyak RM110 sehari dikenakan jika projek ini lewat daripada 20 hari dan ditawarkan bonus sebanyak RM150 sehari jika ia awal daripada 20 hari.

- (i) *Tentukan semua lintasan genting yang mungkin.*
- (ii) *Senaraikan kesemua kegiatan genting.*
- (iii) *Dapatkan jumlah kos minimum projek ini.*
- (iv) *Lakukan hanya dua lelaran pemampatan bagi projek ini. Berikan jumlah kos projek bagi setiap lelaran.*

[10 markah]