UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2008/2009

November 2008

MAT 203 – Vector Calculus
[Kalkulus Vektor]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all three** [3] questions.

*[**Arahan:** Jawab **semua tiga** [3] soalan.]*

1. Write down only your answer in the exam answer book. Marks will be awarded for correct answer only

(a) If $\underset{\sim}{u} = < u_1, u_2 .., u_{1004} >$

$\underset{\sim}{v} = < v_1, v_2 .., v_{1004} >$

where $u_i = 2i, v_i = 2i - 1$ for $i = 1, 2, .., 1004$

Evaluate $\underset{\sim}{u} \cdot \underset{\sim}{u} - \underset{\sim}{v} \cdot \underset{\sim}{v}$

(b) Compute the directional derivative of

$f(x, y, z) = xe^y + \ln(xz)$ at $(1, 0, 1)$ in the direction of $\underset{\sim}{u} = < 1, 2, 2 >$

(c) Let $R = \{(x, y) : x^2 + y^2 \leq 4\pi^2\}$. Compute

$$\iint_R \sin(x^2 + y^2) dA$$

(d) Compute $\iint_R xy^2 dA$ where $R = \{(x, y) : 1 \leq xy \leq 4 \text{ and } 1 \leq xy^2 \leq 4\}$

Hint: use change of variables $u = xy, v = xy^2$

(e) Let $\underset{\sim}{F}(x, y) = < y, -x >$ and C is the part of an ellipse $4x^2 + y^2 = 4$ joining the point $(0, 2)$ to $(1, 0)$ in the clockwise direction. Compute

$$\int_c \underset{\sim}{F} \cdot d\underset{\sim}{r}$$

(f) Compute the maximum value of $z = xy$ with the constraint $9x^2 + 4y^2 = 36$

(g) Find the distance of the point $(2, -3, 4)$ from the plane $2x - y + 3z = 0$.

(h) Let $z = f(x, y)$ be a surface with $(0, 2)$ lying on the level curve $f(x, y) = 5$. Also we know that $f_x(0, 2) = -3$ and $f_y(0, 2) = 4$. Compute the maximum rate of increase at $(0, 2)$.

(i) Compute magnitude of an orthogonal projection $\underset{\sim}{u} = < 1, 3, 4, 6 >$ onto $\underset{\sim}{v} = < 1, 2, 0, -2 >$

(j) Determine the value of a such that

$\underset{\sim}{r}(t) = < t, at^2, -1 >$

Just touches the plane $2x + y + z + = 0$

[40 marks]

1. *Tulis hanya jawapan anda sahaja dalam buku jawapan. Markah akan diberi kepada jawapan yang betul sahaja.*

(a) *Jika* $\underset{\sim}{u} = < u_1, u_2 .., u_{1004} >$

$\underset{\sim}{v} = < v_1, v_2 .., v_{1004} >$

dengan $u_i = 2i, v_i = 2i - 1$ *untuk* $i = 1, 2, .., 1004$

Nilaikan $\underset{\sim}{u} \cdot \underset{\sim}{u} - \underset{\sim}{v} \cdot \underset{\sim}{v}$

(b) *Kira terbitan berarah untuk*

$f(x, y, z) = xe^y + \ln(xz)$ *di* $(1,0,1)$ *pada arah* $\underset{\sim}{u} =< 1,2,2 >$

(c) *Andai* $R = \{(x, y) : x^2 + y^2 \leq 4\pi^2\}$. *Kira*

$$\iint_R \sin(x^2 + y^2) dA$$

(d) *Kira* $\iint_R xy^2 dA$ *dengan* $R = \{(x, y) : 1 \leq xy \leq 4 \text{ dan } 1 \leq xy^2 \leq 4\}$

Petunjuk: guna penukaran pembolehubah $u = xy, v = xy^2$

(e) *Andai* $\underset{\sim}{F}(x, y) =< y, -x >$ dan *C sebahagian elips*

$4x^2 + y^2 = 4$ *menghubungkan titik* $(0,2)$ ke $(1,0)$ *pada arah jam. Kirakan*

$$\int_c \underset{\sim}{F} \cdot d\underset{\sim}{r}$$

(f) *Kira nilai maksimum* $z = xy$ dengan kekangan $9x^2 + 4y^2 = 36$.

(g) *Cari jarak titik* $(2, -3, 4)$ *dari satah* $2x - y + 3z = 0$.

(h) *Andai* $z = f(x, y)$ *sebagai suatu permukaan dengan* $(0,2)$ *terletak pada lengkung aras* $f(x, y) = 5$. *Juga kita tahu* $f_x(0,2) = -3$ *dan* $f_y(0,2) = 4$. *Kira kadar perubahan maksimum pada* $(0,2)$.

(i) *Kira magnitud unjuran serenjang* $\underset{\sim}{u} =< 1,3,4,6 >$ terhadap $\underset{\sim}{v} =< 1,2,0,-2 >$

(j) *Tentukan nilai a supaya lengkung*

$\underset{\sim}{r}(t) =< t, at^2, -1 >$

hanya menyentuh satah $2x + y + z + = 0$

[40 markah]

2. (a) Use Green's theorem to find the line integral of $\underset{\sim}{F} = 3y\,\underset{\sim}{i} + xy\,\underset{\sim}{j}$ around the unit circle oriented counterclockwise.

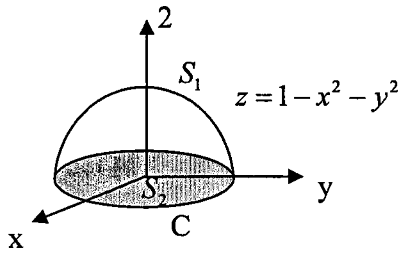
(b) (i) Find $f(x,y)$ such that $\underset{\sim}{F}(x,y) = < y\cos x, \sin x >$ is the gradient of f.

(ii) Use the Fundamental Theorem of Line Integrals to evaluate $\int_C \underset{\sim}{F}\cdot d\underset{\sim}{r}$ where C is the parabola $y = 2x^2$ from $(0,0)$ to $(1,2)$.

(c) Consider the vector field $\underset{\sim}{F}$, with oriented surfaces S_1 and S_2 and a curve C.

$$\underset{\sim}{F}(x,y,z) = < z, -xy, 2z - x >$$

S_1 is part of paraboloid $z = 1 - x^2 - y^2$ above the xy-plane,
S_2 is the part on the xy plane enclosed by the paraboloid $z = 1 - x^2 - y^2$, and
C is the common boundary of S_1 and S_2



(i) Find the curl and divergence of $\underset{\sim}{F}$.
(ii) Evaluate the line integral $\oint_C \underset{\sim}{F}\cdot d\underset{\sim}{r}$
(iii) Evaluate the surface integral $\iint_{S_1} \text{curl}\, \underset{\sim}{F}\cdot d\underset{\sim}{s}$
(iv) Using divergence theorem

Show that $\iint_{S_1} \underset{\sim}{F}\cdot ds = \iint_{S_2} \underset{\sim}{F}\cdot ds$

[30 marks]

2. (a) *Guna teorem Green untuk mencari kamiran garis untuk* $\underset{\sim}{F} = 3y\,\underset{\sim}{i} + xy\,\underset{\sim}{j}$ *mengelingi bulatan satu unit pada arah lawan jam.*

(b) (i) *Cari* $f(x,y)$ *supaya* $\underset{\sim}{F}(x,y) = < y\cos x, \sin x >$ *adalah gradient f.*

(ii) *Guna Teorem Asas Pengkamiran garis untuk menilai* $\int_C \underset{\sim}{F} \cdot d\underset{\sim}{r}$

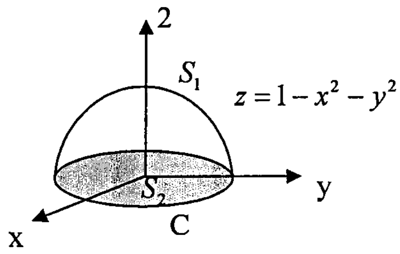
dengan C adalah suatu parabola $y = 2x^2$ dari $(0,0)$ ke $(1,2)$.

(c) *Pertimbang medan vektor* $\underset{\sim}{F}$, *dengan permukaan berarah* S_1 *dan* S_2 *dan suatu lengkung berarah C.*

$$\underset{\sim}{F}(x,y,z) = < z, -xy, 2z - x >$$

S_1 *sebahagian paraboloid* $z = 1 - x^2 - y^2$ *ke atas satah-xy,*

S_2 *sebahagiansatah-xy yang diliputi oleh paraboloid* $z = 1 - x^2 - y^2$, *dan C adalah sempadan supunya* S_1 *dan* S_2



(i) *Cari keikalan dan kecapahan* $\underset{\sim}{F}$.

(ii) *Nilaikan kamiran bergaris* $\oint_C \underset{\sim}{F} \cdot d\underset{\sim}{r}$

(iii) *Nilaikan kamiran permukaan* $\iint_{S_1} \text{curl}\, \underset{\sim}{F} \cdot d\underset{\sim}{s}$

(iv) *Guna teorem kecapahan*

Tunjuk $\iint_{S_1} \underset{\sim}{F} \cdot ds = \iint_{S_2} \underset{\sim}{F} \cdot ds$

[30 markah]

3. (a) Consider the function

$$f(x,y) = y - y^2 - x^2$$

(i) Find and classify all critical points of $f(x,y)$

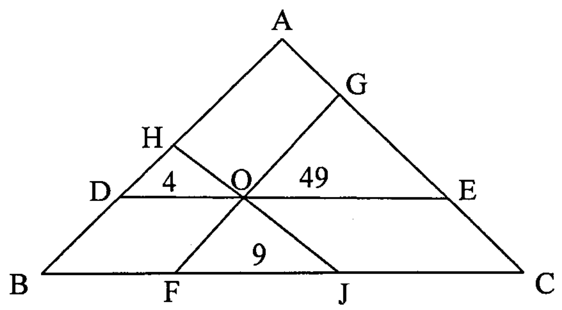
(ii) Find the absolute maximum and absolute minimum value of $f(x,y)$ over the domain

$$D = \{(x,y) : x^2 + y^2 \leq 1\}$$

(b) (i) Show that if $\underset{\sim}{u} = (\underset{\sim}{b} \cdot \underset{\sim}{c})\underset{\sim}{a} - (\underset{\sim}{a} \cdot \underset{\sim}{c})\underset{\sim}{b}$, then $\underset{\sim}{u} \cdot \underset{\sim}{c} = 0$.

(ii) Show that the curve $\underset{\sim}{r}(t) = < t^2, t, t^3 >$ intersects the plane $2x + y + z - 4 = 0$ at a single point only. Find the point of intersection.

(c) Given triangle ABC and lines DE, FG, HJ so that DE is parallel to BC, FG is parallel to BA and HJ is parallel to AC. The lines DE, FG, and HJ meet at O. Given that the area of triangles DOH, FOJ, and EOG are 4, 9, and 49 respectively. Compute the area of triangle ABC by using vector approach.



[30 marks]

3. (a) *Pertimbang fungsi*

$$f(x,y) = y - y^2 - x^2$$

(i) *Cari dan kelaskan titik genting* $f(x,y)$

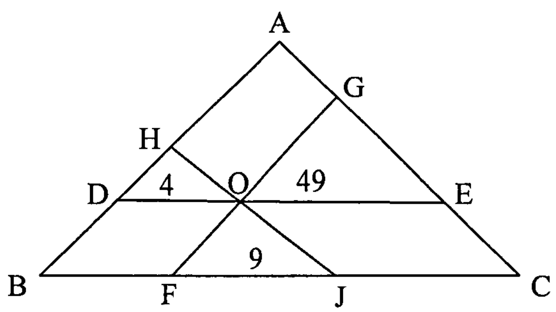
(ii) *Cari nilai minimum dan nilai maksimum mutlak* $f(x,y)$ *pada doman*

$$D = \{(x,y) : x^2 + y^2 \le 1\} .$$

(b) (i) *Tunjukkan jika* $\underset{\sim}{u} = (\underset{\sim}{b} \cdot \underset{\sim}{c})\underset{\sim}{a} - (\underset{\sim}{a} \cdot \underset{\sim}{c})\underset{\sim}{b}$, *maka* $\underset{\sim}{u} \cdot \underset{\sim}{c} = 0$.

(ii) *Tunjukkan bahawa lengkung* $\underset{\sim}{r}(t) = < t^2, t, t^3 >$ *menyilang satah* $2x + y + z - 4 = 0$ *pada satu titik sahaja. Cari titik persilangan tersebut.*

(c) *Diberi segitiga ABC dan garis DE, FG, HJ supaya DE selari dengan BC, FG selari dengan BA dan HJ selari dengan AC. Garis DE, FG dan HJ bertemu di O. Diberi luas segitiga DOH, FOJ dan EOG adalah masing-masing 4, 9 dan 49. Dengan kaedah vektor kira luas segitiga ABC.*



[30 markah]

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