
UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2008/2009

November 2008

MAT 101 – Calculus
[Kalkulus]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all seventeen** [17] questions in Part A and Part B.

Arahan : Jawab **semua tujuh belas** [17] soalan di Bahagian A dan Bahagian B.]

Part A: (60 marks) True or False. If it is false, give a counterexample.

1. If $\lim_{x \rightarrow a} f(x)$ exists and $\lim_{x \rightarrow a} g(x)$ does not exist, then $\lim_{x \rightarrow a} [f(x) + g(x)]$ does not exist.
2. If $\lim_{x \rightarrow a} f(x)$ exists, then f is continuous at a .
3. If $\lim_{x \rightarrow a} f(x)$ exists, then f is differentiable at a .
4. The integral $\int_1^2 \sqrt{1+y^2} dy$ can be solved using the substitution $y = \tan x$.
5. If $\lim_{x \rightarrow -2} \frac{f(x)}{x+2} = L$, then $\lim_{x \rightarrow -2} f(x) = 0$.
6. If f has an inflection point at c , then c is a critical number of f .
7. All continuous functions have derivatives.
8. If f and g are continuous and $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x) dx \geq \int_a^b g(x) dx$.
9. If f is integrable on $[a, b]$, then f is continuous on $[a, b]$.
10. Suppose f is continuous on $[a, b]$, then there exists c in (a, b) such that $(b-a)f(c) = \int_a^b f(x) dx$.

Bahagian A: (60 markah) Benar atau Palsu. Jika palsu, beri satu contoh lawan.

1. Jika $\lim_{x \rightarrow a} f(x)$ wujud dan $\lim_{x \rightarrow a} g(x)$ tak wujud, maka $\lim_{x \rightarrow a} [f(x) + g(x)]$ tak wujud.
2. Jika $\lim_{x \rightarrow a} f(x)$ wujud, maka f selanjar pada a .
3. Jika $\lim_{x \rightarrow a} f(x)$ wujud, maka f terbezakan pada a .
4. Kamiran $\int_1^2 \sqrt{1+y^2} dy$ dapat diselesaikan dengan menggunakan gantian $y = \tan x$.
5. Jika $\lim_{x \rightarrow -2} \frac{f(x)}{x+2} = L$, maka $\lim_{x \rightarrow -2} f(x) = 0$.
6. Jika f mempunyai titik lengkok balas di c , maka c ialah nombor genting untuk f .
7. Semua fungsi selanjar mempunyai terbitan.
8. Jika f dan g adalah selanjar dan $f(x) \geq g(x)$ untuk $a \leq x \leq b$, maka $\int_a^b f(x) dx \geq \int_a^b g(x) dx$.
9. Jika f terkamirkan pada $[a, b]$, maka f selanjar pada $[a, b]$.
10. Andaikan f selanjar pada $[a, b]$, maka wujud c dalam (a, b) sedemikian $(b-a)f(c) = \int_a^b f(x) dx$.

Part B: Answer all the questions in this section.

1. Find the following limit.

(a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$

(b) $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + 9}}{x}$

[12 marks]

2. (a) State the Intermediate Value Theorem.

(b) Prove that the equation $x^5 = x^2 + 4$ has *at least* one real root.

[14 marks]

3. (a) Show that if f is differentiable at a , then f is continuous at a .

(b) Show that $f(x) = \begin{cases} 2x, & x \geq 0 \\ x^2, & x < 0 \end{cases}$ is continuous at $x = 0$ but is not differentiable at $x = 0$.

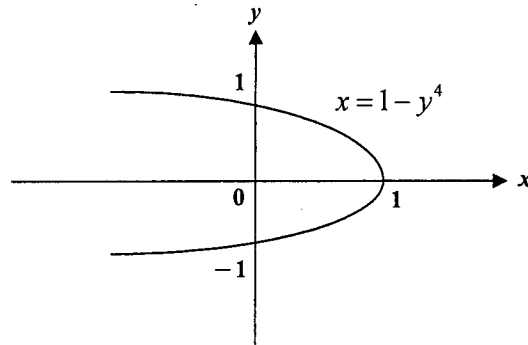
[28 marks]

4. Consider $f(t) = \int_1^{t^2} \frac{e^{(u^2+1)}}{u} du$, $t \geq 1$. If $F(x) = \int_1^x f(t) dt$, find $F''(2)$.

(Hint: Use Fundamental Theorem of Calculus)

[7 marks]

5. The region bounded by the curves $x = 1 - y^4$, $x = 0$, $y = 0$ (graph given below)



is rotated about the specified axis. Setup, but **do not** evaluate, the integral for finding the volume of the resulting solid using the stated method.

(a) y -axis ; disk/washer

(b) $y = 2$; cylindrical shell

[12 marks]

...5/-

Bahagian B: Jawab semua soalan dalam seksyen ini.

1. Cari had berikut.

(a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$

(b) $\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 + 9}}{x}$

[12 markah]

2. (a) Nyatakan Teorem Nilai Pertengahan.

(b) Buktikan bahawa persamaan $x^5 = x^2 + 4$ mempunyai sekurang-kurangnya satu punca nyata.

[14 markah]

3. (a) Tunjukkan bahawa jika f terbezakan pada a , maka f selanjur pada a .

(b) Tunjukkan bahawa $f(x) = \begin{cases} 2x, & x \geq 0 \\ x^2, & x < 0 \end{cases}$ selanjur pada $x = 0$ tetapi tak terbezakan pada $x = 0$.

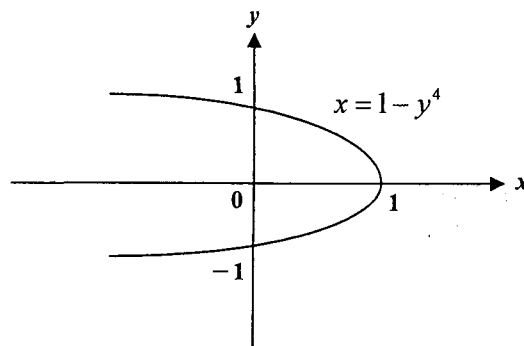
[28 markah]

4. Pertimbangkan $f(t) = \int_1^{t^2} \frac{e^{(u^2+1)}}{u} du$, $t \geq 1$. Jika $F(x) = \int_1^x f(t) dt$, cari $F''(2)$.

(Hint: Guna Teorem Asasi Kalkulus)

[7 markah]

5. Rantau yang dibatasi oleh lengkung-lengkung $x = 1 - y^4$, $x = 0$, $y = 0$ (graf diberi di bawah)



diputarakan sekitar paksi tertentu. Bentuk, **tanpa** menilai, kamiran untuk mencari isipadu pepejal yang dijanakan dengan menggunakan kaedah yang dinyatakan.

(a) paksi- y ; cakera/washer

(b) $y = 2$; kerangka silinder

[12 markah]

...6/-

6. The curve with equation $y^2 = x^3 + 3x^2$ is called the *Tschirnhausen cubic*.

(a) Find $\frac{dy}{dx}$.

(b) Find an equation of the tangent line to this curve at the point $(1, -2)$.

[10 marks]

7. Suppose f is a differentiable function on the real line. Find

(a) $\frac{d}{dx} f(\sqrt{x})$

(b) $\frac{d}{dx} [\sqrt{f(x)}]$

[7 marks]

6. Lengkung dengan persamaan $y^2 = x^3 + 3x^2$ disebut kubik Tschirnhausen.
- (a) Cari $\frac{dy}{dx}$.
- (b) Cari suatu persamaan untuk garis tangen kepada lengkung ini pada titik $(1, -2)$.

[10 markah]

7. Andaikan f ialah fungsi terbezakan pada garis nyata. Cari

(a) $\frac{d}{dx} f(\sqrt{x})$

(b) $\frac{d}{dx} [\sqrt{f(x)}]$

[7 markah]