
UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2008/2009

November 2008

MAA 101 – Calculus for Sciences Students I
[Kalkulus untuk Pelajar Sains I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all ten** [10] questions.

Arahan : Jawab **semua sepuluh** [10] soalan.]

1. Solve $|x-2|+|x-3|\geq 4$.

[6 marks]

2. Evaluate:

(a) $\lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{x-2}$

(b) $\lim_{x \rightarrow \infty} \frac{3x^3}{2x^2+1}$

(c) $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$

[11 marks]

3. Let $f(x) = \frac{kx+2}{4x-3}$, $x \neq \frac{3}{4}$.

(a) If $(f \circ f)(1) = 1$, find all values of k .

(b) By using $k > 0$ from part (a), find $f^{-1}(x)$.

[8 marks]

4. Show that $f(x) = \begin{cases} x^2 - 2 & , x \leq 1 \\ x - 2 & , x > 1 \end{cases}$ is continuous but not differentiable at $x = 1$.

[10 marks]

5. Find $\frac{dy}{dx}$.

(a) $y = x^2 \arccos\left(\frac{2}{x}\right)$

(b) $y = x^{e^{-x^2}}$

(c) $\sin(x-y) = xy$ at $(0, \pi)$

[14 marks]

6. Let $f(x) = \frac{2x^2 + 3x + 6}{x+2}$. Find

(a) all asymptotes.

(b) the intervals on which f is increasing or decreasing.

(c) the local maximum and minimum values of f if any.

(d) the intervals of concavity and inflection points (if exists).

Then, sketch the graph of f .

[15 marks]

...3/-

1. Selesaikan $|x-2|+|x-3|\geq 4$.

[6 markah]

2. Nilaikan:

(a) $\lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{x-2}$

(b) $\lim_{x \rightarrow \infty} \frac{3x^3}{2x^2+1}$

(c) $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$

[11 markah]

3. Biarkan $f(x) = \frac{kx+2}{4x-3}$, $x \neq \frac{3}{4}$.

(a) Jika $(f \circ f)(1) = 1$, cari semua nilai k .

(b) Dengan menggunakan $k > 0$ dari bahagian (a), cari $f^{-1}(x)$.

[8 markah]

4. Tunjukkan bahawa $f(x) = \begin{cases} x^2-2 & , x \leq 1 \\ x-2 & , x > 1 \end{cases}$ adalah selanjur tetapi tidak terbezakan pada $x = 1$.

[10 markah]

5. Cari $\frac{dy}{dx}$.

(a) $y = x^2 \arccos\left(\frac{2}{x}\right)$

(b) $y = x^{e^{-x^2}}$

(c) $\sin(x-y) = xy$ pada $(0, \pi)$

[14 markah]

6. Biarkan $f(x) = \frac{2x^2+3x+6}{x+2}$. Cari

(a) semua asimtot.

(b) selang di mana f adalah menaik atau menyusut.

(c) nilai maksimum tempatan dan minimum tempatan bagi f jika ada.

(d) selang kecekungan dan titik lengkung balas (jika wujud).

Kemudian lakarkan graf f .

[15 markah]

7. Find the dimension of the largest rectangular that can be put into a circle with diameter 10 cm.
[7 marks]
8. Evaluate the integrals:
(a) $\int \frac{3x^2 + x + 1}{x(x^2 + 1)} dx$
(b) $\int_1^4 \sqrt{x} \ln x dx$
[11 marks]
9. Find an equation for the tangent line to the curve $y = F(x)$ at the point where $x = 1$ and $F(x) = \int_1^{x^2} \frac{t^2}{t-2} dt$.
[6 marks]
10. (a) Find the area of the region bounded by $y^2 = 4 - x$ and $x + 2y - 1 = 0$.
(b) Find the volume of the solid formed by revolving the region enclosed by the curves $y = x^2$ and $y = 8 - x^2$ about the line $x = 3$.
[12 marks]

7. Cari dimensi bagi segiempat terbesar yang dapat diterapkan di dalam bulatan bergarispusat 10 cm.

[7 markah]

8. Nilaikan kamiran:

(a) $\int \frac{3x^2 + x + 1}{x(x^2 + 1)} dx$

(b) $\int_1^4 \sqrt{x} \ln x dx$

[11 markah]

9. Cari persamaan garis tangen ke lengkungan $y = F(x)$ pada titik yang mana

$x = 1$ dan $F(x) = \int_1^{x^2} \frac{t^2}{t-2} dt$.

[6 markah]

10. (a) Cari luas kawasan yang di batasi oleh $y^2 = 4 - x$ dan $x + 2y - 1 = 0$.
- (b) Cari isipadu bongkah kisanan yang terhasil apabila kawasan yang di batasi oleh lengkungan $y = x^2$ dan $y = 8 - x^2$ di kisarkan sekitar garis $x = 3$.

[12 markah]