UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2009/2010

November 2009

# EAS 663/4-Dynamics And Stability Of Structures

Duration : 3 hours

Please check that this examination paper consists of **ELEVEN (11)** pages including appendix before you begin the examination.

**Instructions:**

This paper consists of **SIX (6)** questions. Answer **FIVE (5)** questions only.

All question should be answered in English.

Write the answered question numbers on the cover sheet of the answer script.

1. a) Define viscous damping. Sketch the displacement response, (v) versus (t) of undamped and damped SDOF systems for free vibration. Does the natural period of vibration, T, change with the presence of damping?

( 6 marks)

b) Figure 1 shows one-story building which is idealized as a rigid girder to support a rotating machine. A horizontal force , F(t)=800 cos 5.3 t N is exerted on the girder. Assuming that the damping of the system is equal to 5% of critical damping and the value of E = 200 x $10^3$ MPa, determine:

(i) the natural circular frequency

(ii) the frequency ratio, $r$

(iii) the static deflection, $V_o$

(iv) the steady state amplitude of vibration, given $V = D_s\ V_o$ where

$$D_s = \frac{1}{\sqrt{(1-r^2)+(2\zeta r)^2}};$$

(v) the maximum shear force in the column

(vi) the maximum bending moment in the column

( 8 marks)

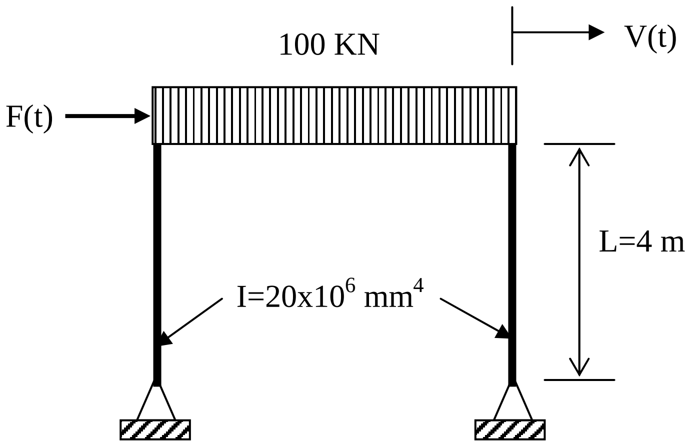



**Figure 1**

c) If frame in Figure 1 is subjected to sinusoidal ground motion Vs(t)=8.0x$10^{-3}$ cos 5.3 t N is exerted instead of F(t), on the girder. Assume the damping of the system is equal to 5% of critical damping and the value of E = 200 x $10^3$ MPa. Determine:

i) the maximum shear force in the column; given $u_{\max} = \dfrac{r^2\, Vo}{\sqrt{(1-r^2)+(2\zeta r)^2}}$;

ii) the maximum bending moment in the column

( 6 marks)

2. a) Define response spectra in structural dynamic problems.

( 4 marks)

b) Figure 2a shows a model of column-mass SDOF system subjected to **TWO (2)** triangular blast loads, p(t) as shown in Figure 2b . The weight of the mass block is 3000 kN and the column stiffness, k = 1800 kN/mm. Assume it is an undamped system. Predict the maximum displacement response, $v_{\max} = R_{\max}(\dfrac{Po}{K})$ and the maximum total elastic forces developed in the system for both p(t) . The value of the maximum response ratio, $R_{max}$ can be obtained from the displacement response spectra as shown in Figure 2c. Give comments on your observations of the results for both blast loads.

( 6 marks)

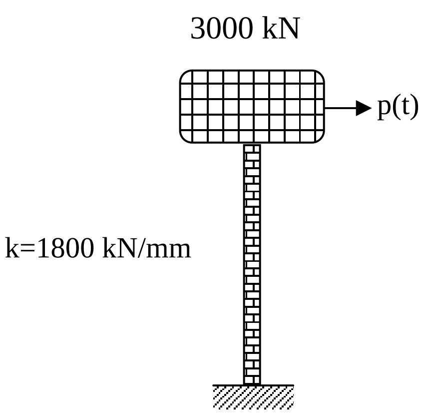



**Figure 2a**

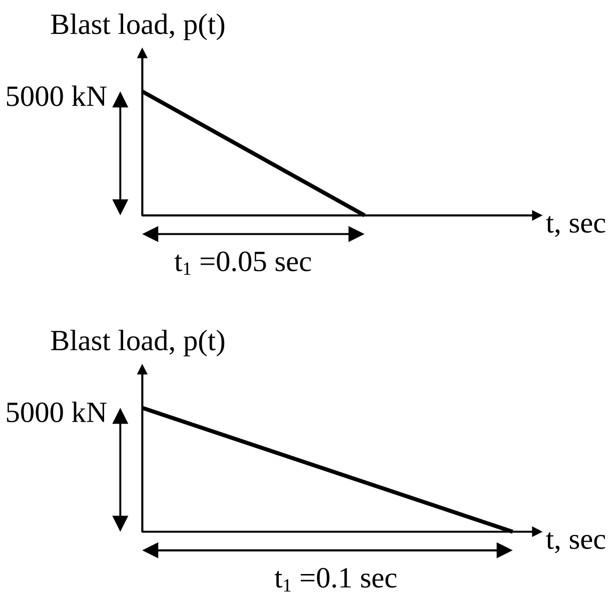



**Figure 2b**

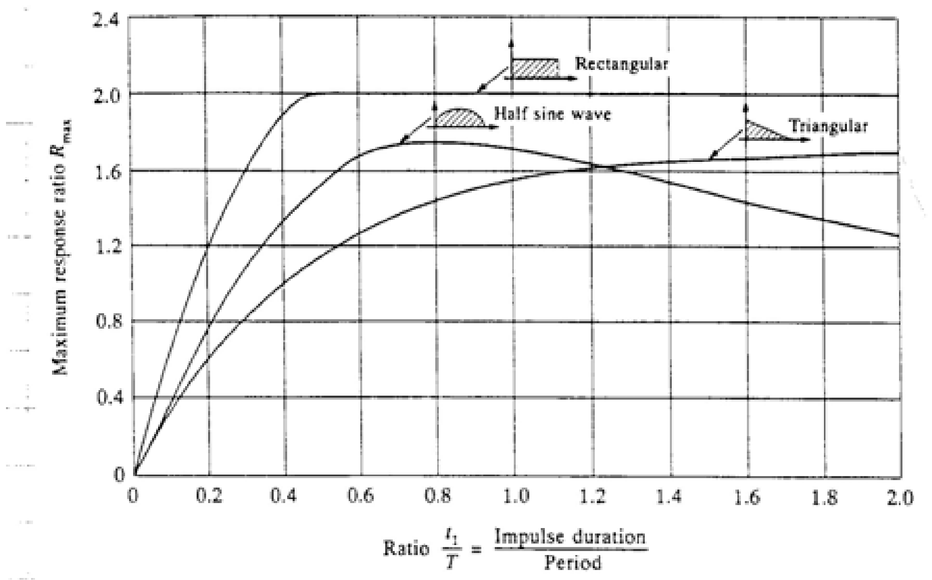



**Figure 2c**

c) Duhamel Integral is normally used for the evaluation of a linear SDOF system subjected to arbitrary time varying force. Define the underline term with the help of the graph Force (P) versus (t).

( 5 marks)

d) Figure 2d shows a spring-mass model for 3DOF system under free vibration. Derive the equations of motion for the system.

(5 marks)

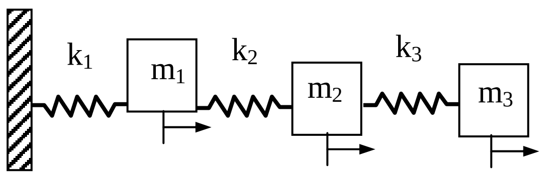



**Figure 2d**

3. a) Classify **THREE (3)** cases of free vibration of viscous-damped SDOF systems with regards to the magnitude of damping factor.

(3 marks)

b) Figure 3a shows a dynamic response of undamped SDOF system to step force with finite rise time with static solution shown by dashed lines. Explain the pattern of the dynamic response with respect to all **SIX (6)** values of $t_r / T_n$.

(7 marks)

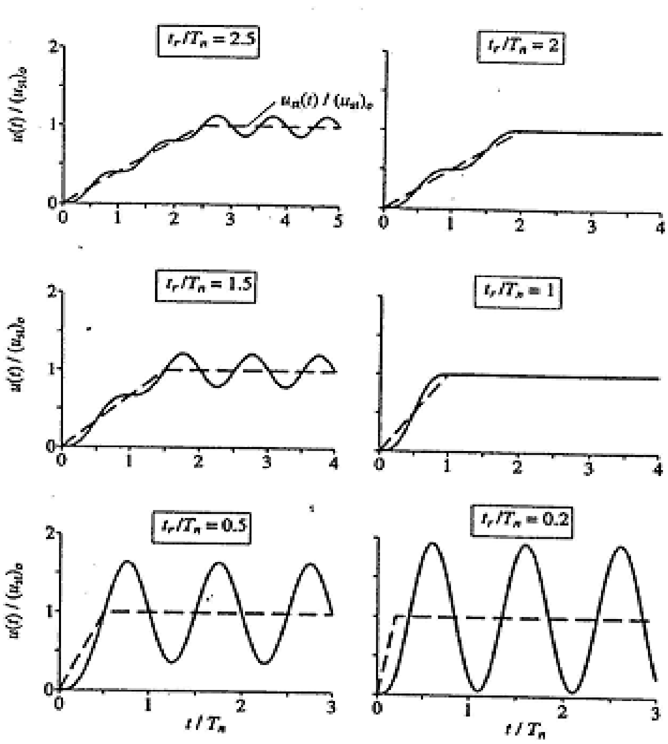
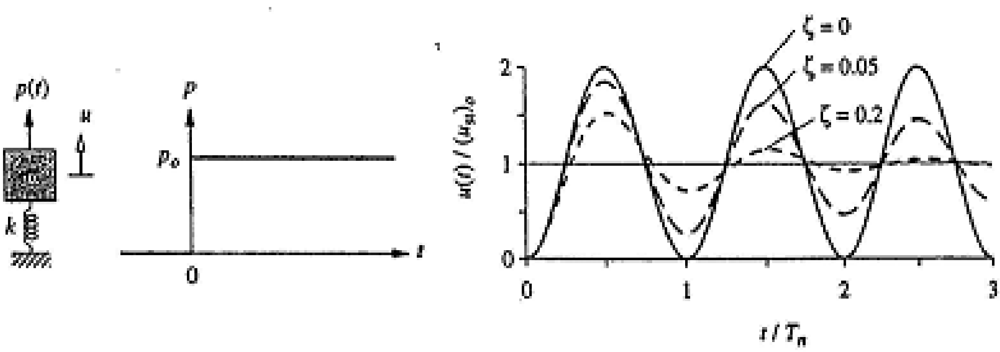



**Figure 3a**

c) Figure 3b shows a dynamic response to step force with sudden jump from zero to $p_o$ and stay constant at that value. Prove that the response is given by the following expression, $v(t) = \frac{P_o}{k}[1 - \cos \varpi t]$

(10 marks)




(i) (ii) (iii)

**Figure 3b: (i) SDOF system (ii) step force (iii) dynamic response**

4. a) Explain the concept of neutral equilibrium using the example of a pinned-pinned column. Subsequently, derive the second-order differential equation for the bending of the column shown in Figure 4 and use the equation to obtain the critical load of the column. The lower end of column is fixed and the upper end is prevented from rotating but free to translate laterally. Given flexural rigidity of the column is *EI*.

(8 marks)

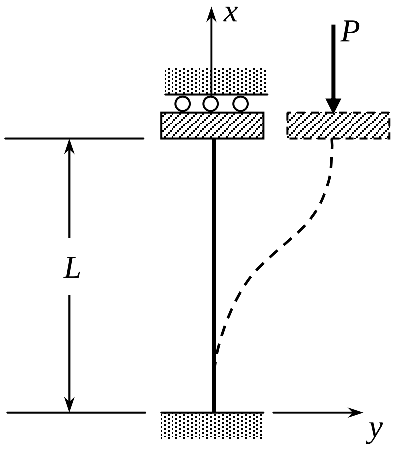



**Figure 4**

b) The second order differential equation for the imperfect column shown in Figure 5 is given as follows:

$$y'' + k^2 y = -k^2 a \sin\frac{\pi x}{L}$$

where $k^2 = P/EI$ and $a$ is the amplitude of the initial deformation at mid-height of the column.

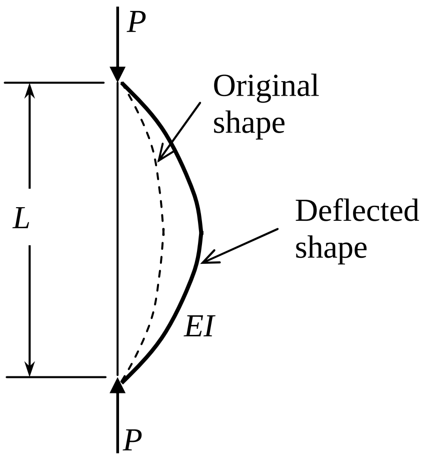



**Figure 5**

Show using the above equation that the relation between mid-height deflection δ and ratio $P/P_E$ where $P_E$ : Euler buckling load ($=\pi^2 EI/L^2$ ) is given by the following equation:

$$\delta = \frac{a}{1 - P/P_E}$$

Next, sketch a plot of $P/P_E$ versus δ for three different values of *a* and comment on the behavior of initially bent column versus initially straight column.

(12 marks)

5. a) Obtain the critical load of the tapered compression members where *I* varies linearly as shown in Figure 6(a) and (b) using Rayleigh-Ritz method. Lower end of each of the member is fixed and the upper end is free. Assume the deflection for both cases is given by:

$$y = y_0 x^2$$

where $y_0$ : amplitude of lateral displacement of the compression member at the free end. Give one reason for the difference in the two critical loads calculated.

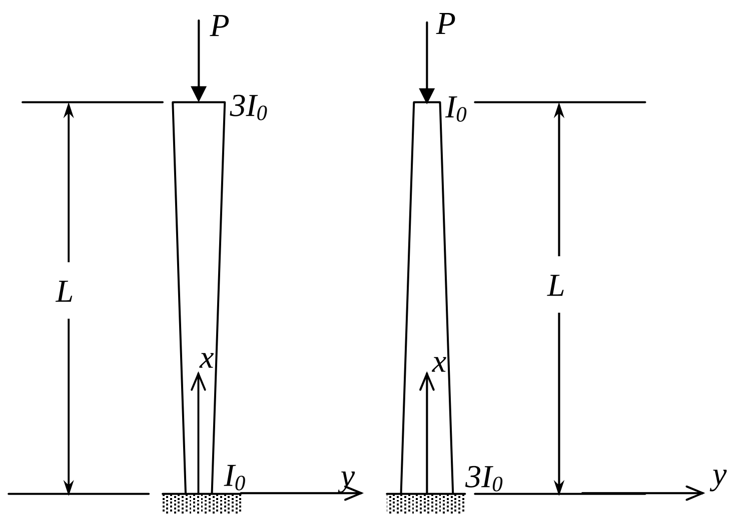



**Figure 6a** **Figure 6b**

(12 marks)

b) Mid-span deflection of the simply supported beam subjected to a lateral load *W* at mid-span and axial load *P* as shown in Figure 7 is given by the following equation:

$$\delta = \delta_0 \frac{1}{1-(P/P_{cr})}$$

where $\delta_0$ : the mid-span deflection in the absence of axial load and $P_{cr}$:critical axial load of for the beam.

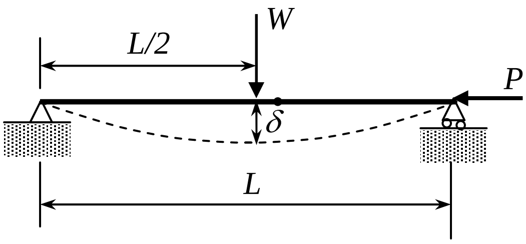



**Figure 7**

Using the equation as given above, explain the effect *P* on the mid-span deflection of the beam. Next, derive the following relation for maximum bending moment occurring in the beam:

$$M_{\max} = M_0 \frac{1-(0.18P/P_{cr})}{1-(P/P_{cr})}$$

where $M_0=WL/4$ : maximum bending in the absence of axial load.

6. a) Using slope-deflection equations, determine the critical load for the frame shown in Figure 8. Refer Appendix A for the slope deflection equations.

(8 marks)

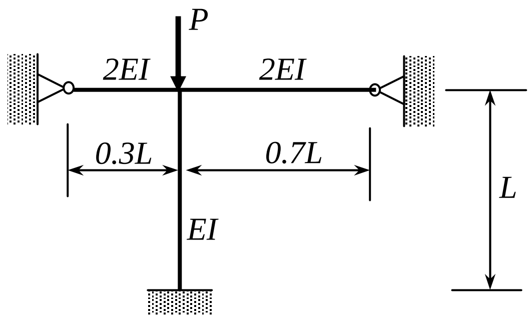



**Figure 8**

b) Show the sidesway and symmetric buckling modes for the frame shown in Figure 9.

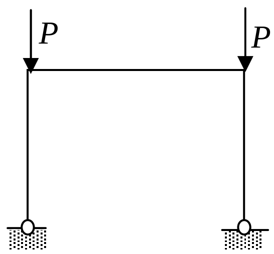



**Figure 9**

Evaluate the critical load for the case of symmetric buckling. Use matrix method. Given that length of all members is $L$ and that $EI$ of all members is the same. Subsequently determine the effective length factor $K$ ($L_e=KL$) for the column. Justify why the effective length factor $K$ of the column in the frame above must be within the following range for the case of symmetric buckling:

$0.7 < K < 1.0$

(12 marks)

**APPENDIX A**

Slope deflection equations for a beam-column are given as follows :

$$M_A = \frac{EI}{L}\left(s_{ii}\theta_A + s_{ij}\theta_B\right)$$
$$M_B = \frac{EI}{L}\left(s_{ji}\theta_A + s_{jj}\theta_B\right)$$

where $s_{ii}$, $s_{ij}(=s_{ji})$, $s_{jj}$ are stability functions :

$$s_{ij} = s_{ji} = \frac{(kL)^2 \cosh kL - kL \sinh kL}{2 - 2\cosh kL + kL \sinh kL}; \qquad k = \sqrt{\frac{P}{EI}}$$

$$s_{ii} = s_{jj} = \frac{kL \sinh kL - (kL)^2}{2 - 2\cosh kL + kL \sinh kL}$$

and $M_A$, $M_B$, $\theta_A$ and $\theta_B$ are as shown in Figure A1.

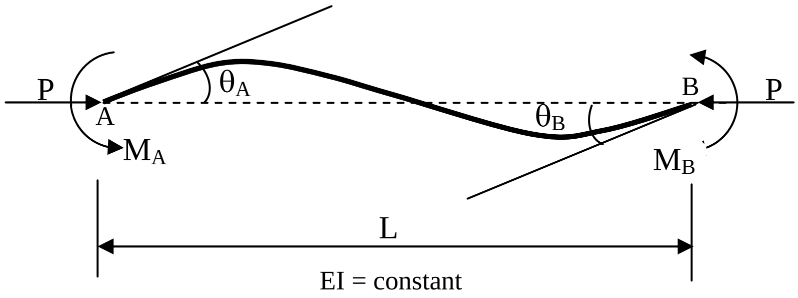



**Figure A1**

**APPENDIX B**

Stiffness matrix for a beam column member

$$[k] = EI\begin{bmatrix} \frac{12}{L^3} & -\frac{6}{L^2} & -\frac{12}{L^3} & -\frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{4}{L} & \frac{6}{L^2} & \frac{2}{L} \\ -\frac{12}{L^3} & \frac{6}{L^2} & \frac{12}{L^3} & \frac{6}{L^2} \\ -\frac{6}{L^2} & \frac{2}{L} & \frac{6}{L^2} & \frac{4}{L} \end{bmatrix} - P\begin{bmatrix} \frac{6}{5L} & -\frac{1}{10} & -\frac{6}{5L} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{2L}{15} & \frac{1}{10} & -\frac{1}{30} \\ -\frac{6}{5L} & \frac{1}{10} & \frac{6}{5L} & \frac{1}{10} \\ -\frac{1}{10} & -\frac{1}{30} & \frac{1}{10} & \frac{2L}{15} \end{bmatrix}$$

where *EI*:flexural rigidity of member; *L*:length of member; *P*:axial force of member

**….oooOOooo….**