UNIVERSITI SAINS MALAYSIA

First Semester Examination
2009/2010 Academic Session

November 2009

EAS 661/4 – Advanced Structural Mechanics

Time : 3 hours

Please check that this examination paper consists of **NINE (9)** pages of printed material before you begin the examination.

Instruction :

This paper consists of **SIX (6)** questions. Answer **FIVE (5)** questions only. All questions carry the same mark.

ALL questions should be answered in English.

All questions **MUST BE** answered on new sheets.

Write the answered question numbers on the cover page of the answer script.

1. (a) A state of stress will be set up in an elastic body under the action of some system of external forces. Show all the stress components (in Cartesian coordianate system) acting on it using the infinitesimal volume dxdydz shown Figure 1.

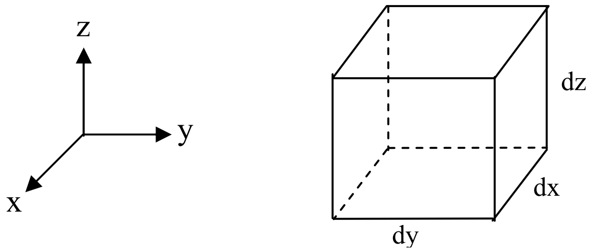

Figure 1

Derive the equilibrium equation in x-direction for the infinitesimal volume shown in Figure 1 above. It is given that body forces R_x, R_y and R_z act on the infinitesimal volume in x, y and z-direction respectively.

Making use of the derived equilibrium equation with proper specialization, obtain the governing equation in terms of u for a prismatic bar subjected to a uniformly distributed load w and a concentrated load P at the free end as shown in Figure 2. State clearly the specialization made in the process of obtaining the governing equation. It is given that the material of the bar is linearly elastic with modulus of elasticity E.

(10 marks)

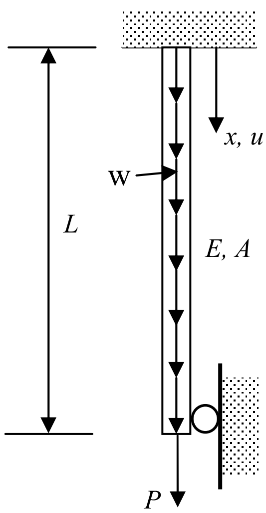

Figure 2

(b)(i) Figure 3 shows a thin wall structure loaded by a uniformly distributed load w in z-direction. Justify why this problem can be solved as a plane stress problem.

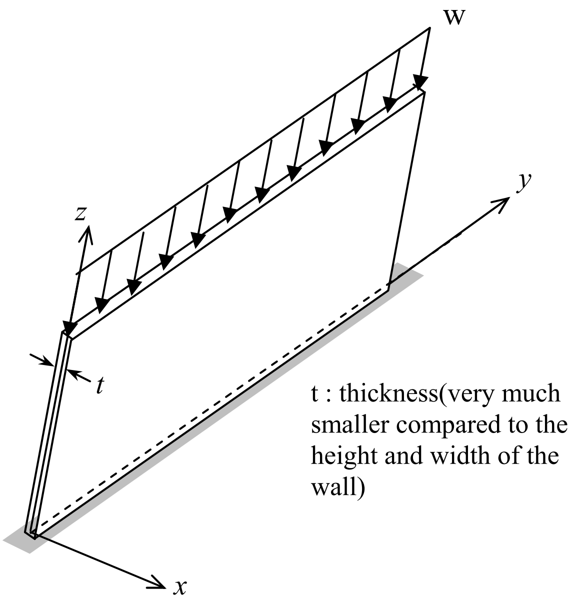

Figure 3

ii) Figure 4 shows an infinitesimal volume taken from the interior of the wall shown in Figure 3. Indicate the non-zero stress components on the infinitesimal volume.

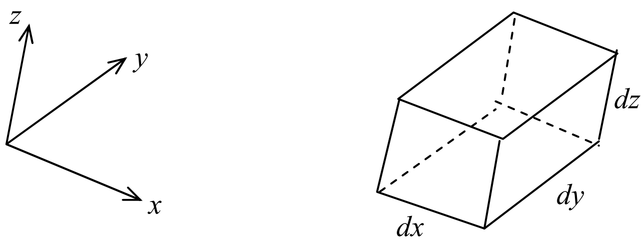

Figure 4

iii) Using the following general stress-strain relation, derive the stress-strain relation, $\boldsymbol{\sigma}$=$\boldsymbol{C\varepsilon}$, for the plane stress problem shown in Figure 3, where $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$: vector of components of non-zero stress and the corresponding strains respectively and $\boldsymbol{C}$:the elasticity matrix for plane stress problem.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{Bmatrix}$$

(E:elastic modulus, G:shear modulus, ν:Possion's ratio)

(10 marks)

2. (a) The strain energy U_p stored in an elastic body can be obtained using the following equation :

$$U_p = \int_{vol} v_p dvol$$

where v_p is strain energy density and *vol* is volume of the elastic body. Using the above equation, show that the strain energy stored in an elastic bar of length L with arbitrary variation of cross-section A can be represented using the following equation :

$$U_p = \frac{1}{2} E \int_0^L A \left(\frac{du}{dx} \right)^2 dx$$

Specialize the above derived equation of U_p to the case of the prismatic elastic bar as shown Figure 5.

(10 marks)

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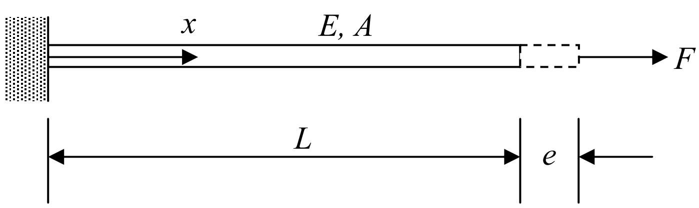

Figure 5

(b) Figure 6 shows a cantilever column with height H subjected to a linearly distributed load from W at bottom end until 0 at the upper end. Two linear springs with spring constant k are located at points A and B. The following expression for lateral displacement field, v, has been suggested :

$$v = V_0\left(1-\cos\left(\frac{\pi x}{2H}\right)\right)$$

where V_0 is a constant. Show that the above displacement field is admissible. Next, solve for the constant V_0 by applying the principle of minimum potential energy (PMPE). Flexural rigidity of the column is EI.

(10 marks)

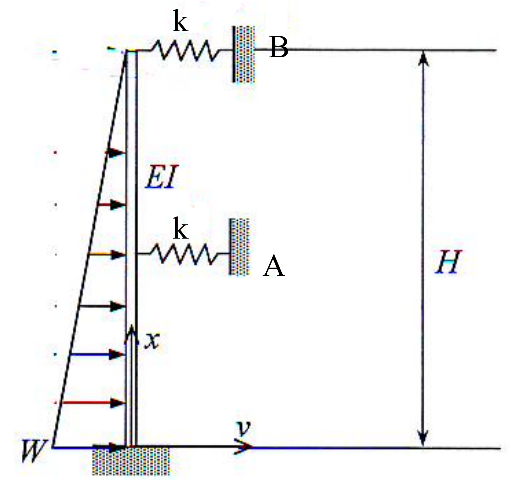

Figure 6

3. (a) State the three basic relations that a structural mechanics problem must satisfy in order that an exact solution is obtained. Then, write down the corresponding three basic relations for the case of a simple 1D linearly elastic prismatic bar subjected to end force P as shown in Figure 7, where Δ : elongation of bar due to force P, u : axial displacement in the direction of bar axis x, E : elastic modulus of material of bar, A : cross-sectional area and L : original length of bar.

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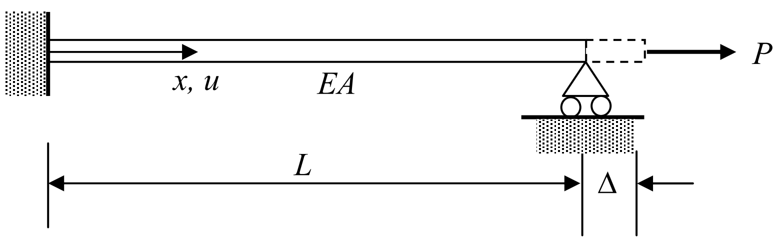

Figure 7

(b) Figure 8 shows a prismatic bar which is fixed at both ends. Length of the bar is *L*, elastic modulus is *E* and cross-sectional area is *A*. The bar is subjected to a uniformly distributed load *w* per unit length acting in the direction of x-axis and two concentrated loads *P* and *Q* at points *L/3* and *2L/3* from the upper end, respectively. Using piece-wise Rayleight-Ritz method, derive the expression for axial displacement, *u*. Divide the bar into three equal portions and assume linear displacement field for each portion. Plot distribution of *u* and axial stress, σ along the bar. Show clearly all the steps involved in the derivation and state clearly the meanings of all symsbols used in the derivation.

(17 marks)

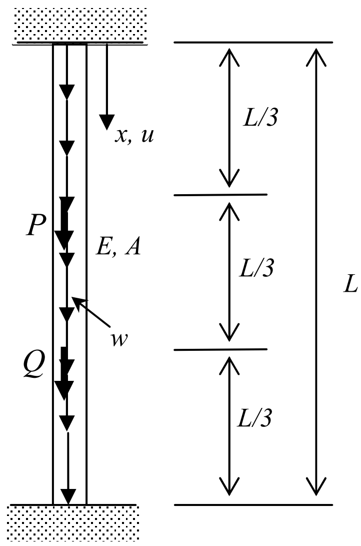

Figure 8

4. (a) Write down the element stiffness matrices and global matrix for the three bar assembly which is loaded with force 2P and constrained at the two ends as shown in Figure 9(a) in terms of E,A and L.

(5 marks)

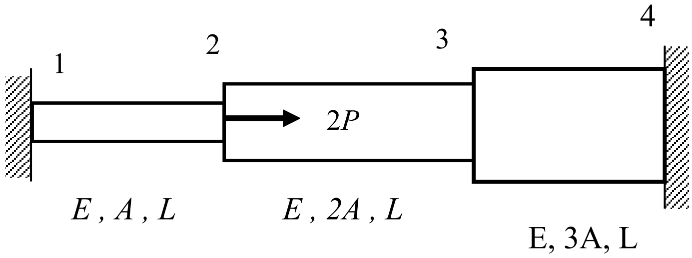

Figure 9(a)

(b) Clearly define the difference between a bar and a beam element in the analysis using Finite Element Method.

(5 marks)

(c) Figure 9(b) shows a system of two beams labeled as node 1,2 and 3 and a spring labeled as node 3 and 4 subjected to a nodal force of P = 50 kN at node 3. The beam is fixed at node 1, simply supported at node 2 and spring supported at node 3. The spring system can only displace in axial direction and is supported at node 4. Given that k = 200 kN/m, $L_1 = L_2 = 3$m, E = 210GPa and I = $2\text{x}10^{-4}$ m^4,

i) Obtain the element stiffness matrix for the beam and the spring.

ii) Derive the global stiffness matrix for the system.

iii) Evaluate the deflection V_3 , θ_2 and θ_3 in unit metre and rad respectively.

(10 marks)

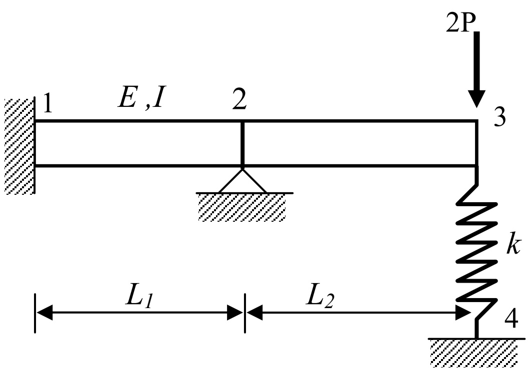

Figure 9(b)

Given the stiffness of the beam element in 2D :

$$k = \frac{EI}{L^3} \begin{array}{c} \begin{array}{cccc} v_i & \theta_i & v_j & \theta_j \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \end{array} \text{ for the beam element}$$

$$k = \begin{array}{c} \begin{array}{cc} u_i & u_j \end{array} \\ \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \end{array} \text{ for the spring element}$$

5. (a) Explain the assumptions made in the modeling procedures for materials properties and loading conditions in Finite Element Method.

(5 marks)

(b) Figure 10(a) shows a cantilever beam carrying a concentrated load of 20MN at point B. The beam is modeled with linear four-noded rectangular elements (□) and three noded triangular elements (△). Given elastic modulus E = 200 GPa, thickness, t = 10 mm and the Poisson's ratio v = 0.3. Sketch the results for the maximum stress in xx and yy direction for the beam especially at points A and B.

(10 marks)

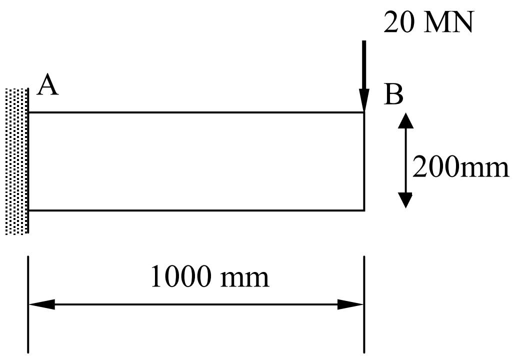

Figure 10(a)

(c) Derive the stiffness matrix for the element shown in Figure 10(b) in terms of applied axial loads F_1, F_2, displacements u_1, u_2 , axial rigidity EA and initial length L.

(5 marks)

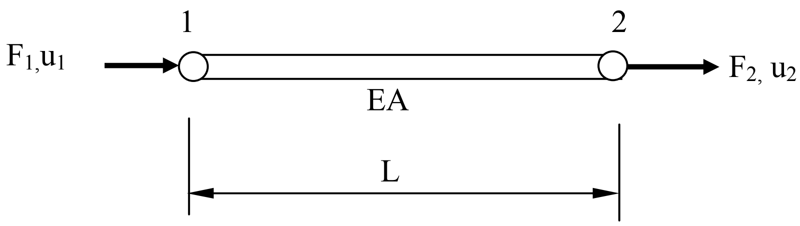

Figure 10(b)

6. (a) Clearly define the difference between a triangular and a rectangular finite element in plane elasticity.

(5 marks)

(b) Show clearly in step by step manner the development process of the stiffness matrix, $[K]^e$, for a rectangular element in a state of plane stress as shown in Figure 11. Given $E = 200\ GN/m^2$, $\nu = 0.3$ and $t = 2$ cm.

(15 marks)

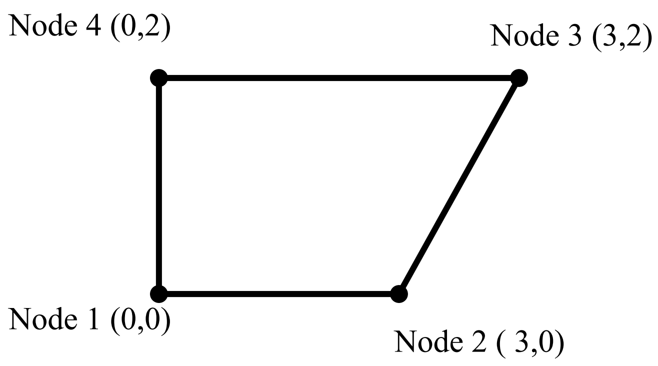

Figure 11

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