

UNIVERSITI SAINS MALAYSIA
Second Semester Examination
Academic Session 1997/98

February 1998

AGW615 ADVANCED BUSINESS STATISTICS

Time: [3 hours]

INSTRUCTION:

Please ensure that this examination paper consists of 11 printed pages before you begin.

Answer all questions in SECTION A and any ONE question from SECTION B

1. During the economic crisis, a bank manager collected the following data of all 350 defaulting loans.

Nature of Business	Collateral Value	
	Sufficient	Insufficient
Exporters	33	109
Importers	63	145

If an auditor is to select a defaulting loan at random from this set of loans, what is the probability that the loan

- a. had sufficient collateral?
- b. was taken by an importer?
- c. had insufficient collateral if it was taken by an importer?
- d. was taken by an exporter if the loan collateral value is sufficient?

(10 marks)

2. Royal Malaysian Air Force (RMAF) regularly purchases a particular spare part used in its aircraft's navigation systems that need replacement regularly. RMAF made purchases in lots of 1000 pieces each time. When a shipment of parts arrives, RMAF will carry out acceptance sampling, whereby it takes a random sample from the lot, and accepts the lot if not more than a certain number (acceptance number) of defectives were found in the selected sample; otherwise the lot will be rejected and returned to the supplier.

If past records show that the supplies contains 1% defectives, what is the probability of accepting a lot that was just received, if

- b. the sample was 20 pieces and the acceptance number was 1?
- b. the sample was 50 pieces and the acceptance number was 2?

...2/-

If the supplier supplies at a quality level of 2% defectives, what is the producer's risk (risk of rejecting the lot and returning it to the supplier) using sampling plan (a) above?

(10 marks)

3. As the managing director of a company (with 10,000 employees) that claims to be charitable to the less fortunate, you were told that 80% of your staff, recently contributed to the North Korean Famine Fund (NKFF). You then randomly select 36 workers and found only 5 had contributed to NKFF.
 - a. What would you conclude? Explain using probabilities.
 - b. What if you had found 25 contributors?
 - c. How many workers do you have to ask if you are to estimate to within 2% of the true proportion of your staff who had contributed to NKFF with 98% confidence?

(15 marks)

4. Malaysian Dental Surgeon Association (MDSA) carried out tests to determine whether there are differences in three major brands of toothpaste sold in Malaysia. To achieve this objective, 15 primary school children were monitored throughout their primary school years and the number of cavities each had during this period were noted as follows.

Observation	Type of Toothpaste Used		
	1	2	3
1	19	20	18
2	15	25	12
3	22	22	16
4	17	19	17
5	19	23	15
Total	92	109	78

- a. MDSA has assigned the 15 schoolchildren randomly to the three major brands of toothpaste. What conclusions can you draw at 5% significance level? Explain and state all assumptions you use in arriving at your conclusions.
- b. Suppose MDSA believes that dental health depends on dietary habits and that the habits of the major races -- Malay, Chinese, Indians, Orang Asli and Others differ. What design would you suggest MDSA use, for it to come out with more valid conclusions? How would you then analyse the above data, if each of the rows in the above table represents one race? Use significance level of 5%. State the hypotheses that you are testing.

(30 marks)

...3/-

SECTION B

Answer any ONE (1) of the following questions.

5. A marketing executive for computers wishes to test market a range of new computers to determine the right combination of price and accessories that will induce purchase of the computer. Intent to purchase is measured on a scale 0 (very unlikely to purchase) to 100 (most likely to purchase). Three levels of prices (P1, P2 and P3) and three accessories packages were tested. Each price-accessories package were judged by three potential customers who then state their likelihood of purchase. The description of the subpopulations are as follows:

- - Description of Subpopulations - -					
Summaries of By levels of	LSALES AP PRICE	Likelihood of Sales (0-100) Accessories Package Price			
Variable	Value	Label	Mean	Std Dev	Cases
For Entire Population			46.2222	24.9990	27
AP	1.00	Accessories Package	36.6667	7.2284	9
PRICE	1.00	Price Level 1	28.0000	3.6056	3
PRICE	2.00	Price Level 2	39.3333	3.5119	3
PRICE	3.00	Price Level 3	42.6667	2.5166	3
AP	2.00	Accessories Package	58.5556	25.6959	9
PRICE	1.00	Price Level 1	69.0000	2.6458	3
PRICE	2.00	Price Level 2	25.3333	3.2146	3
PRICE	3.00	Price Level 3	81.3333	5.1316	3
AP	3.00	Accessories Package	43.4444	32.1874	9
PRICE	1.00	Price Level 1	84.3333	3.0551	3
PRICE	2.00	Price Level 2	12.3333	2.5166	3
PRICE	3.00	Price Level 3	33.6667	5.0332	3
Total Cases = 27					

- a. Based upon the description above, which combination of price level and accessories package should be selected for the computer for marketing purposes?
- b. A 2-way ANOVA was carried out with the following results. Interpret the solution, bearing in mind that the ultimate objective of the exercise is to determine the best combination of price and accessories package that will induce purchase.

...4/-

UNIQUE sums of squares
All effects entered simultaneously

Source of Variation	Sum of Squares	DF	Mean Square	F	Sig. of F
Main Effects	8244.444	4	2061.111	159.456	.000
AP	2260.222	2	1130.111	87.430	.000
PRICE	5984.222	2	2992.111	231.481	.000
2-Way Interactions	7771.556	4	1942.889	150.309	.000
AP PRICE	7771.556	4	1942.889	150.309	.000
Explained	16016.000	8	2002.000	154.883	.000
Residual	232.667	18	12.926		
Total	16248.667	26	624.949		

- c. Suppose that the above analysis was carried out using regression analysis with dummy variables whereby DAP1=1 for accessories package 1 and 0 otherwise; DAP2 = 1 for accessories package 2 and 0 otherwise; dprice1 = 1 for price level 1 and 0 otherwise; dprice2 = 1 for price level 2 and 0 otherwise; and interactions DAP11 between DAP1 and dprice1; DAP12 between DAP1 and dprice2; DAP21 between DAP2 and dprice2; and DAP22 between DAP2 and dprice2. Interpret the solution and discuss the similarities and differences with the 2-Way ANOVA results above.

(35 marks)

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      * * * *  M U L T I P L E  R E G R E S S I O N  * * * *
Listwise Deletion of Missing Data

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Equation Number 1 Dependent Variable.. LSALES Likelihood of
Sales (0-100)

```
Block Number 1. Method: Enter
      DAP1      DAP11      DAP12      DAP2      DAP21      DAP22      DPRICE1
DPRICE2
```

Variable(s) Entered on Step Number

- ```
1.. DPRICE2
2.. DAP2
3.. DAP11
4.. DAP21
5.. DAP12
6.. DPRICE1
7.. DAP22
8.. DAP1
```

...5/-

Multiple R .99281  
R Square .98568  
Adjusted R Square .97932  
Standard Error 3.59526

Analysis of Variance

|            | DF | Sum of Squares | Mean Square |
|------------|----|----------------|-------------|
| Regression | 8  | 16016.00000    | 2002.00000  |
| Residual   | 18 | 232.66667      | 12.92593    |

F = 154.88252 Signif F = .0000

----- Variables in the Equation -----

| Variable   | B          | SE B     | Beta     | Tolerance | VIF   | T       | Sig T |
|------------|------------|----------|----------|-----------|-------|---------|-------|
| DAP1       | 9.000000   | 2.935521 | .172946  | .250000   | 4.000 | 3.066   | .0067 |
| DAP11      | -65.333333 | 4.151454 | -.836971 | .281250   | 3.556 | -15.737 | .0000 |
| DAP12      | 18.000000  | 4.151454 | .230594  | .281250   | 3.556 | 4.336   | .0004 |
| DAP2       | 47.666667  | 2.935521 | .915971  | .250000   | 4.000 | 16.238  | .0000 |
| DAP21      | -63.000000 | 4.151454 | -.807079 | .281250   | 3.556 | -15.175 | .0000 |
| DAP22      | -34.666667 | 4.151454 | -.444107 | .281250   | 3.556 | -8.350  | .0000 |
| DPRICE1    | 50.666667  | 2.935521 | .973619  | .250000   | 4.000 | 17.260  | .0000 |
| DPRICE2    | -21.333333 | 2.935521 | -.409945 | .250000   | 4.000 | -7.267  | .0000 |
| (Constant) | 33.666667  | 2.075727 |          |           |       | 16.219  | .0000 |

Collinearity Diagnostics

| Number | Eigenval | Cond Index | Variance Proportions                     |
|--------|----------|------------|------------------------------------------|
|        |          |            | Constant DAP1 DAP11 DAP12 DAP2 DAP21     |
| 1      | 3.29966  | 1.000      | .00842 .00631 .00474 .00474 .0063 .00474 |
| 2      | 1.81650  | 1.348      | .00000 .00015 .01445 .01995 .0001 .01995 |
| 3      | 1.81650  | 1.348      | .00000 .02279 .01995 .01445 .0227 .01445 |
| 4      | 1.00000  | 1.816      | .00000 .00000 .06 .06250 .0000 .06250    |
| 5      | .333333  | 3.146      | .00489 .12133 .00275 .00275 .1213 .00275 |
| 6      | .333333  | 3.146      | .16177 .00367 .09100 .09100 .0036 .09100 |
| 7      | .18350   | 4.240      | .00000 .00010 .17760 .16300 .0001 .16300 |
| 8      | .18350   | 4.240      | .00000 .22696 .16300 .17760 .2269 .17760 |
| 9      | .03367   | 9.899      | .82491 .61869 .46401 .46401 .6186 .46401 |

|   |        |         |         |
|---|--------|---------|---------|
|   | DAP22  | DPRICE1 | DPRICE2 |
| 1 | .00474 | .00631  | .00631  |
| 2 | .01445 | .02279  | .02279  |
| 3 | .01995 | .00015  | .00015  |
| 4 | .06250 | .00000  | .00000  |
| 5 | .00275 | .12133  | .12133  |
| 6 | .09100 | .00367  | .00367  |
| 7 | .17760 | .22696  | .22696  |
| 8 | .16300 | .00010  | .00010  |
| 9 | .46401 | .61869  | .61869  |

End Block Number 1 All requested variables entered.

Residuals Statistics:

|         | Min     | Max     | Mean    | Std Dev | N  |
|---------|---------|---------|---------|---------|----|
| *PRED   | 12.3333 | 84.3333 | 46.2222 | 24.8193 | 27 |
| *RESID  | -4.6667 | 5.6667  | .0000   | 2.9914  | 27 |
| *ZPRED  | -1.3654 | 1.5355  | .0000   | 1.0000  | 27 |
| *ZRESID | -1.2980 | 1.5761  | .0000   | .8321   | 27 |

Total Cases = 27

Durbin-Watson Test = 2.40879

...6/-

6. Quarterly data for retail car sales (RCS) for the years 1987 to 1996 were related to Disposable Personal Income (DPI) and Prime Interest Rate (PR). A portion of the data is shown below.

| TIME    | RCS      | DPI     | MR    |
|---------|----------|---------|-------|
| 1996.00 | 273792.0 | 2742.90 | 13.73 |
| 1996.00 | 261643.0 | 2692.00 | 14.43 |
|         |          |         |       |
|         |          |         |       |
| 1987.00 | 372414.0 | 3466.90 | 10.00 |
| 1987.00 | 369017.0 | 3493.00 | 9.82  |

- a. SPSS regression results with RCS as the dependent and DPI and PR as the explanatory variables are as follow:

- How good is the model? (Use the usual criteria for judging goodness of model)
- Are the assumptions of the regression models satisfied.
- What is the regression equation?
- If Quarter 1 1997 has DPI=3450 and PR=10.20, what is your 95% prediction interval for retail car sales (RCS)

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* * * * * M U L T I P L E R E G R E S S I O N * * * * *
Listwise Deletion of Missing Data
Equation Number 1 Dependent Variable.. RCS Retail car sales
Block Number 1. Method: Enter DPI MR

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Variable(s) Entered on Step Number
1.. MR Prime mortgage rate
2.. DPI Real Disposable Income

```

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Multiple R .99357
R Square .98717
Adjusted R Square .98648
Standard Error 4476.99657

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Analysis of Variance
 DF Sum of Squares Mean Square
Regression 2 57068532922.52700 28534266461.2635
Residual 37 741609435.44798 20043498.25535

```

F = 1423.61708      Signif F = .0000

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----- Variables in the Equation -----
Variable B SE B Beta Tolerance VIF T
DPI 125.399641 5.077089 .841618 .298609 3.349 24.699
MR -2773.581235 537.385823 -.175869 .298609 3.349 -5.161
(Constant) -38992.61095 21745.81210

```

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----- in -----
Variable Sig T
DPI .0000
MR .0000
(Constant) .0811

```

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Collinearity Diagnostics

| Number | Eigenval | Cond Index | Variance Constant | Proportions DPI | MR     |
|--------|----------|------------|-------------------|-----------------|--------|
| 1      | 2.96363  | 1.000      | .00012            | .00023          | .00116 |
| 2      | .03570   | 9.111      | .00112            | .01951          | .17955 |
| 3      | .00066   | 66.821     | .99876            | .98026          | .81929 |

End Block Number 1 All requested variables entered.

Residuals Statistics:

|          | Min         | Max         | Mean        | Std Dev    | N  |
|----------|-------------|-------------|-------------|------------|----|
| *PRED    | 258560.4531 | 371791.7813 | 313416.4750 | 38253.0485 | 40 |
| *ZPRED   | -1.4340     | 1.5260      | .0000       | 1.0000     | 40 |
| *SEPRD   | 731.0881    | 2024.0631   | 1189.3041   | 301.8203   | 40 |
| *ADJPRED | 258156.8281 | 372060.3125 | 313401.1509 | 38286.5022 | 40 |
| *RESID   | -8090.5645  | 9008.9521   | .0000       | 4360.6911  | 40 |
| *ZRESID  | -1.8071     | 2.0123      | .0000       | .9740      | 40 |
| *SRESID  | -1.8559     | 2.0691      | .0016       | 1.0110     | 40 |
| *DRESID  | -8574.6426  | 9525.1553   | 15.3241     | 4700.2953  | 40 |
| *SDRESID | -1.9223     | 2.1704      | .0022       | 1.0318     | 40 |
| *MAHAL   | .0650       | 6.9965      | 1.9500      | 1.5326     | 40 |
| *COOK D  | .0001       | .1171       | .0261       | .0320      | 40 |
| *LEVER   | .0017       | .1794       | .0500       | .0393      | 40 |

Total Cases = 40

Durbin-Watson Test = 1.43008

- b. The retail car sales may be seasonal; i.e. the changes in sales may be due to the time of the year. For this reason, dummy variables were introduced with Q1 taking value 1 for Quarter 2 and 0 otherwise; Q2 taking value 1 for Quarter 3 and 0 otherwise; and Q3 taking value 1 for Quarter 4 and 0 otherwise. For quarter 1 all three dummy variables take on value 0. Interpret the solution given below and interpret the impact of quarters on the retail car sales.

(35 marks)

\*\*\*\*\* MULTIPLE REGRESSION \*\*\*\*\*  
Listwise Deletion of Missing Data  
Equation Number 1 Dependent Variable.. RCS Retail car sales  
Block Number 1. Method: Enter  
DPI MR Q1 Q2 Q3

Variable(s) Entered on Step Number

1.. Q3  
2.. MR Prime mortgage rate  
3.. Q2  
4.. Q1  
5.. DPI Real Disposable Income

Multiple R .99394  
R Square .98793  
Adjusted R Square .98615  
Standard Error 4530.90420

...8/-

# Analysis of Variance

|            | DF | Sum of Squares    | Mean Square      |
|------------|----|-------------------|------------------|
| Regression | 5  | 57112153199.62950 | 11422430639.9259 |
| Residual   | 34 | 697989158.34550   | 20529092.89251   |

F = 556.40211 Signif F = .0000

| ----- Variables in the Equation ----- |              |            |          |           |       |        |
|---------------------------------------|--------------|------------|----------|-----------|-------|--------|
| Variable                              | B            | SE B       | Beta     | Tolerance | VIF   | T      |
| DPI                                   | 124.765919   | 5.178581   | .837365  | .293973   | 3.402 | 24.093 |
| MR                                    | -2843.217449 | 547.085692 | -.180284 | .295094   | 3.389 | -5.197 |
| Q1                                    | 1735.037869  | 2032.16671 | .019762  | .662811   | 1.509 | .854   |
| Q2                                    | 2881.581194  | 2036.90119 | .032822  | .659734   | 1.516 | 1.415  |
| Q3                                    | 1038.219241  | 2039.88535 | .011825  | .657805   | 1.520 | .509   |
| (Constant)                            | -37562.71675 | 22065.6792 |          |           |       | -1.702 |

----- in -----  
Variable Sig T

|            |       |
|------------|-------|
| DPI        | .0000 |
| MR         | .0000 |
| Q1         | .3992 |
| Q2         | .1663 |
| Q3         | .6141 |
| (Constant) | .0978 |

## Collinearity Diagnostics

| Number | Eigenval | Cond Index | Variance Proportions |        |        |        |        |        |
|--------|----------|------------|----------------------|--------|--------|--------|--------|--------|
|        |          |            | Constant             | DPI    | MR     | Q1     | Q2     | Q3     |
| 1      | 3.76760  | 1.000      | .00007               | .00014 | .00069 | .00992 | .00987 | .00979 |
| 2      | 1.00009  | 1.941      | .00000               | .00000 | .00000 | .17866 | .01574 | .30026 |
| 3      | 1.00000  | 1.941      | .00000               | .00000 | .00000 | .15241 | .31378 | .02923 |
| 4      | .19621   | 4.382      | .00040               | .00064 | .00460 | .65544 | .65219 | .64558 |
| 5      | .03544   | 10.310     | .00123               | .01985 | .17486 | .00161 | .00304 | .00810 |
| 6      | .00066   | 75.660     | .99830               | .97937 | .81984 | .00196 | .00537 | .00704 |

## Residuals Statistics:

|          | Min         | Max         | Mean        | Std Dev    | N  |
|----------|-------------|-------------|-------------|------------|----|
| *PRED    | 259014.5469 | 371362.4688 | 313416.4750 | 38267.6650 | 40 |
| *ZPRED   | -1.4216     | 1.5142      | .0000       | 1.0000     | 40 |
| *SEPPRED | 1445.4250   | 2454.6794   | 1741.2522   | 220.4996   | 40 |
| *ADJPRED | 258400.2188 | 371789.3750 | 313402.8236 | 38291.6227 | 40 |
| *RESID   | -8462.2305  | 8617.0020   | .0000       | 4230.5033  | 40 |
| *ZRESID  | -1.8677     | 1.9018      | .0000       | .9337      | 40 |
| *SRESID  | -2.0024     | 2.0416      | .0014       | 1.0113     | 40 |
| *DRESID  | -9726.9951  | 9930.4912   | 13.6514     | 4966.6764  | 40 |
| *SDRESID | -2.1004     | 2.1473      | .0018       | 1.0344     | 40 |

Total Cases = 40  
Durbin-Watson Test = 1.36537

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# AGW 615 ADVANCED BUSINESS STATISTICS FORMULAE

## 1. Mean

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} \dots \text{or} \dots \bar{x} = \frac{\sum_{i=1}^N x_i}{N} \dots$$

## 2. Variance

$$s^2 = \frac{\sum (x - \bar{x})^2}{n-1} = \frac{\sum x^2 - n\bar{x}^2}{n-1}$$

## 3. Sampling Distribution

$$\mu_{\bar{x}} = \mu \dots \text{and} \dots \mu_{\hat{p}} = p$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \dots \text{and} \dots \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

## 4. Confidence Interval

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \dots \text{or} \dots \bar{x} \pm t_{\alpha/2, n-1} \cdot \frac{\sigma}{\sqrt{n}} \dots \text{or} \dots \hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

## 5. Sample Sizes

$$n = \left( \frac{z_{\alpha/2} \sigma}{B} \right)^2 \dots \text{or} \dots n = \left( \frac{t_{\alpha/2, n-1} s}{B} \right)^2 \dots \text{or} \dots n = \left( \frac{z_{\alpha/2} \sqrt{p(1-p)}}{B} \right)^2$$

## 6. Sampling distribution of Differences

$$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2 \dots \text{or} \dots \mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \dots \text{or} \dots s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \dots$$

$$df = n_1 + n_2 - 2 \dots \text{or} \dots df = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\left( \frac{s_1^2/n_1}{n_1-1} + \frac{s_2^2/n_2}{n_2-1} \right)}$$

$$s_{\bar{x}_1 - \bar{x}_2} = s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \dots \text{and} \dots s_p = \sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1 + n_2 - 2}}$$

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

## WILCOXON RANK SUM TEST

$$E(T) = \frac{n_1(n_1 + n_2 + 1)}{2} \dots \text{and} \dots \sigma_T = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

## WILCOXON SIGNED RANK SUM TEST

$$E(T) = \frac{n(n+1)}{2} \dots \text{and} \dots \sigma_T = \sqrt{\frac{n(n+1)(2n+1)}{24}}$$

## KRUSKAL-WALLIS

$$H = \left[ \frac{12}{n(n+1)} \sum \frac{T_j^2}{n_j} \right] - 3(n+1)$$

## FRIEDMAN

$$F_r = \left[ \frac{12}{bk(k+1)} \sum T_j^2 \right] - 3b(k+1)$$

## CHI-SQUARED

$$\chi^2 = \sum \frac{(o_i - e_i)^2}{e_i} \dots \text{where} \dots e_i = np_i$$

$$\chi^2 = \sum \sum \frac{(o_{ij} - e_{ij})^2}{e_{ij}} \dots \text{where} \dots e_{ij} = \frac{T_i T_j}{N}$$

and  $\dots T_i = \text{row} \dots i \dots \text{total}, \dots \text{and} \dots T_j = \text{column} \dots j \dots \text{total}$

## ANALYSIS OF VARIANCE

$$\bar{x}_j = \frac{\sum_{i=1}^{n_j} x_{ij}}{n_j} \dots \bar{\bar{x}}_j = \frac{\sum_{j=1}^k \sum_{i=1}^{n_j} x_{ij}}{n}$$

$$SST = \sum_{j=1}^k n_j (\bar{x}_j - \bar{\bar{x}})^2 \dots SSE = \sum_{j=1}^k \sum_{i=1}^{n_j} n_j (x_{ij} - \bar{x}_j)^2$$

$$SS(\text{Total}) = SSE + SST \dots MST = \frac{SST}{k-1} \dots MSE = \frac{SSE}{n-k} \dots F = \frac{MST}{MSE}$$

$$SS(\text{Total}) = \sum_{j=1}^k \sum_{i=1}^b (x_{ij} - \bar{\bar{x}})^2 \dots SST = b \sum_{j=1}^k (\bar{x}_{T_j} - \bar{\bar{x}})^2$$

$$SSB = k \sum_{i=1}^b (\bar{x}_{B_i} - \bar{\bar{x}})^2 \dots SSE = \sum_{j=1}^k \sum_{i=1}^b (x_{ij} - \bar{x}_{T_j} - \bar{x}_{B_i} + \bar{\bar{x}})^2$$

$$MST = \frac{SST}{k-1} \dots MSB = \frac{SSB}{b-1} \dots MSE = \frac{SSE}{n-k-b+1}$$

$$SS(Total) = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^r (x_{ijk} - \bar{x})^2 \dots \dots SS(A) = rb \sum_{i=1}^a (\bar{x}[A]_i - \bar{x})^2$$

$$SS(B) = ra \sum_{j=1}^b (\bar{x}[B]_j - \bar{x})^2 \dots \dots SS(AB) = r \sum_{j=1}^b \sum_{i=1}^a (\bar{x}[AB]_{ij} - \bar{x}[A]_i - \bar{x}[B]_j + \bar{x})^2$$

$$SSE = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^r (x_{ijk} - \bar{x}[AB]_{ij})^2$$

$$MS(A) = \frac{SS(A)}{a-1} \dots \dots MS(B) = \frac{SS(B)}{b-1}$$

$$MS(AB) = \frac{SS(AB)}{(a-1)(b-1)} \dots \dots MSE = \frac{SSE}{n-ab}$$

### REGRESSION

$$y = \beta_0 + \beta_1 x \dots \dots \hat{\beta}_1 = \frac{SS_{xy}}{SS_x} \dots \dots \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x}$$

$$SS_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) \dots \dots SS_x = \sum (x_i - \bar{x})^2 = \sum x_i^2 - n\bar{x}^2$$

$$SS_y = \sum (y_i - \bar{y})^2 = \sum y_i^2 - n\bar{y}^2 \dots \dots SSE = SS_y - \frac{SS_{xy}^2}{SS_x}$$

Standard Error of Estimate & Coefficient of Determination,

$$S_e = \sqrt{\frac{SS}{n-2}} \quad R^2 = 1 - \frac{SSE}{SS_y}$$

Prediction and Estimation Interval at  $x_g$

$$\hat{y} \pm t_{\alpha/2, n-2} \sqrt{1 + \frac{1}{n} + \frac{(x_g - \bar{x})^2}{SS_x}} \dots \dots \hat{y} \pm t_{\alpha/2, n-2} \sqrt{\frac{1}{n} + \frac{(x_g - \bar{x})^2}{SS_x}}$$

Coefficient of Correlation

$$r = \frac{SS_{xy}}{\sqrt{SS_x \cdot SS_y}} \dots \dots t = \sqrt{\frac{n-2}{1-r^2}}$$

Spearman Rank Correlation

$$r_s = \frac{SS_{ab}}{\sqrt{SS_a \cdot SS_b}}$$

Standard Deviation of i-th residual

$$s_{r_i} = s_e \cdot \sqrt{1 - h_i} \dots \dots h_i = \frac{1}{n} + \frac{(x_g - \bar{x})^2}{SS_x}$$

