UNIVERSITI SAINS MALAYSIA

First Semester Examination
2003/2004 Academic Session

September - October 2003

ZCT 211E - Vector Analysis

Time : 2 hours

Please check that the examination paper consists of **SIX** printed pages before you commence this examination.

Answer all FIVE questions. Students are allowed to answer all questions in English OR Bahasa Malaysia OR combinations of both.

1. (a) Two particles emitting from a source have displacements $\mathbf{r}_1 = 4\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ and $\mathbf{r}_2 = 2\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}$ at any time. Find the displacement of second particle relative to first.

(3/15)

(b) Prove that $(\mathbf{a} \times \mathbf{a}') + (\mathbf{b} + \mathbf{b}') + (\mathbf{c} \times \mathbf{c}') = O$, where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors and $\mathbf{a}', \mathbf{b}', \mathbf{c}'$ their reciprocals.

(6/15)

(c) A particle is moving in a circular orbit of radius 10 cm. If its frequency of motion is 60 cycles/sec., find the time period, velocity and acceleration of the particle.

(6/15)

2. (a) If $\vec{\mathbf{r}} = t^2\vec{\mathbf{i}} - t^2\vec{\mathbf{j}} + (2t+1)\vec{\mathbf{k}}$. Find the value of

$$\frac{d\mathbf{r}}{dt}, \frac{d^2\mathbf{r}}{dt^2}, \left|\frac{d\mathbf{r}}{dt}\right|, \left|\frac{d^2\mathbf{r}}{dt^2}\right| \text{ at } t=0 .$$

(3/15)

...5/-

(b) If $\mathbf{r} = \mathbf{a}e^{\omega t} - \mathbf{b}e^{-\omega t}$, show that $\frac{d^2\mathbf{r}}{dt^2} - \omega^2\mathbf{r} = 0$; **a, b** are constant vectors and ω being a constant.

(6/15)

(c) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$, where t is the time

(i) Determine its velocity and acceleration at any time.
(ii) Find the magnitudes of velocity and acceleration at t = 0.

(6/15)

3. (a) If $\mathbf{A} = x^2yz\mathbf{i} - 2xz^3\mathbf{j} + xz^3\mathbf{k}$, $\mathbf{B} = 2z\mathbf{i} + y\mathbf{j} - x^2\mathbf{k}$ find $\frac{\partial^2}{\partial x \partial y}(\mathbf{A} \times \mathbf{B})$ at (1, 0, -2).

(5/20)

(b) If **r** is the position vector of a point, deduce the value of grad $\left(\frac{1}{r}\right)$.

(5/20)

(c) If $\mathbf{V} = x^2z\mathbf{i} - 2y^3z^2\mathbf{j} + xy^2z\mathbf{k}$

(i) find $\nabla.\mathbf{V}$ at the point (1, -1, 1).

(ii) If $\mathbf{V} = \frac{x\mathbf{i} + y\mathbf{j}}{x + y}$ find $\nabla.\mathbf{V}$

(iii) If $\mathbf{V} = x\cos z\,\mathbf{i} + y\log x\,\mathbf{j} - z^2\mathbf{k}$, evaluate $\nabla.\mathbf{V}$

(10/20)

4. Prove the following:

(i) div grad $\phi = \nabla^2\phi$ (6/25)

(ii) curl grad $\phi = \nabla \times (\nabla\phi) = 0$ (6/25)

(iii) div curl $\mathbf{f} = \nabla.(\nabla \times \mathbf{f}) = 0$ (6/25)

(iv) curl curl $\mathbf{f} = \nabla \times (\nabla \times \mathbf{f})$ = grad div $\mathbf{f}$ - $\nabla^2\mathbf{f}$ (7/25)

...6/-

5. (a) Given that $\mathbf{r}(t) = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ when t = 2 and $\mathbf{r}(t) = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ when t = 3.

Show that $\int_2^3 \mathbf{r} . \frac{d\mathbf{r}}{dt} \, dt = 10$

(10/25)

(b) Let $\phi = 45\, x^2 y$ and let V denote the closed region bounded by the planes

$$4x + 2y + z = 8, \ x = 0, \ y = 0, \ z = 0$$

evaluate $\iiint_V \phi \, dv$.

(15/25)

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