TERJEMAHAN

UNIVERSITI SAINS MALAYSIA

First Semester Examination
1999/2000 Academic Session

September 1999

ZCT 211 - Vector Analysis

Time : [2 hours]

Please check that the examination paper consists of EIGHT printed pages before you commence this examination.

Answer SIX questions only. Candidates may choose to answer all questions in the Malay Language. If candidates choose to answer in the English Language, it is compulsory to answer at least one question in the Malay Language.

1. Vector $\vec{A}$ is equal to $A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$ in one set of axes and to $A_{x'}\vec{i}' + A_{y'}\vec{j}' + A_{z'}\vec{k}'$ in another. The transformation law between axes is $A_x = c_{xx'}A_{x'} + c_{xy'}A_{y'} + c_{xz'}A_{z'}$ where $c_{xx'}$ is the cosine of the angle between the x and x' axes and so on.

   State the corresponding results for A_y and A_z.

   (20/100)

   Give the explicit form of the transformation law when the primed axes are obtained from the unprimed axes by rotation through an angle θ about the z axis, so that $z' = z$.

   (30/100)

   State the transformation between scalars S and S'. (20/100)

   Write down the scalar product $\vec{A}.\vec{B}$ in terms of the components A_x, B_x etc. and prove that for the rotation through θ $\vec{A}.\vec{B}$ transforms as a scalar.

   (30/100)

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2. $\phi(\vec{r})$ is a scalar field and $\vec{A}(\vec{r})$ is a vector field. State the definitions of $\nabla\phi$, $\nabla.\vec{A}$ and $\nabla\times\vec{A}$.

(30/100)

Prove that for any scalar field $\phi(\vec{r})$ and vector field $\vec{A}(\vec{r})$, $\nabla\times\nabla\phi=0$ and $\nabla.(\nabla\times\vec{A})=0$.

(40/100)

Evaluate $\nabla\phi$ for the scalar fields (a) $\phi(\vec{r})=ax^2+by^2+cz^2$ and (b) $\phi(\vec{r})=(ax^2+by^2+cz^2)^{-1}$

(30/100)

3. The vector field $\vec{A}(\vec{r})$ is $\vec{A}(\vec{r})=\frac{\vec{r}}{r^n}=\frac{1}{r^n}(x,y,z)$, where $r=(x^2+y^2+z^2)^{1/2}$. Evaluate $\nabla.\vec{A}$ and $\nabla\times\vec{A}$.

(60/100)

Find the values of $\nabla.\vec{A}$ and $\nabla\times\vec{A}$ for $n=3/2$ and explain the physical significance of these values.

(40/100)

4. A particle of mass m and charge e moves in a force field $\vec{F}(\vec{r})=eE\cos(\omega t)\vec{i}-mg\vec{k}$ where E and g are constants. Draw a diagram to explain, with reasons, a physical arrangement that would produce this force field.

(30/100)

Write down the equation of motion that gives the acceleration $\vec{f}(t)$.

(20/100)

Integrate the equation of motion twice to find the velocity $\vec{v}(t)$ and position $\vec{r}(t)$, including the constants of integration in general form.

(30/100)

Find $\vec{v}(t)$ and $\vec{r}(t)$ for the initial conditions $\vec{v}=v_0\vec{j}$ and $\vec{r}=-(eE/m\omega^2)\vec{i}$ and describe the motion of the particle in words.

(20/100)

...3/-

5. Evaluate the volume integrals $\int_V f(\vec{r})d^3r$ where $f(\vec{r}) = r^2 = x^2 + y^2 + z^2$ and

(a) V is the box $0 \le x \le a,\ 0 \le y \le a, 0 \le z \le a$ (30/100)

(b) V is the sphere $0 \le r \le R$ (30/100)

(c) V is the cylinder $0 \le \rho \le R,\ 0 \le z \le L$ where $\rho = (x^2 + y^2)^{1/2}$

(40/100)

6. State Gauss's and Stokes's theorems, using suitable diagrams to define carefully the terms you use.

(40/100)

Evaluate both sides of Stokes's theorem for the vector $\vec{F}(\vec{r}) = z^2\vec{i} + 4x\vec{j}$ and the contour C which is the perimeter of the square $0 \le x \le 1,\ 0 \le y \le 1,\ z = 1$.

(60/100)

7. State Gauss's Law relating the gravitational field $\vec{F}(\vec{r})$ to the mass density $\rho(\vec{r})$.

(20/100)

The density of a star depends only on distance r from the origin and is given by $\rho(\vec{r}) = \dfrac{\rho_0 a^6}{(r^3 + a^3)^2}$ where ρ_0 and a are constants.

(a) Integrate $\rho(\vec{r})$ to find the mass $M(R)$ contained in the sphere $0 < r < R$ and hence find the gravitational field at $r = R$. [You may use the substitution $u = r^3$].

(60/100)

(b) Find the total mass M_0 of the star as $M_0 = \lim_{R\to\infty} M(R)$.

(20/100)

8. Explain what is meant by a general orthogonal coordinate system (u_1, u_2, u_3) and define the quantities h_1, h_2, h_3 in the usual notation.

(20/100)

Prove that in spherical polar coordinates $h_r = 1,\ h_\theta = r$ and $h_\phi = r\sin\theta$.

(20/100)

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An electrostatic potential function $V(\vec{r})$ depends only on r and has the form $V(\vec{r}) = r^n$. Find the values of n for which $\nabla^2 V = 0$.

(30/100)

Derive the corresponding values of electric field $\vec{E} = -\nabla V$ and hence explain the physical significance of each of the two values of n.

(30/100)

$$\nabla V = \left(\frac{1}{h_1}\frac{\partial V}{\partial u_1}, \frac{1}{h_2}\frac{\partial V}{\partial u_2}, \frac{1}{h_3}\frac{\partial V}{\partial u_3}\right)$$

$$\nabla^2 V = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial V}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_3 h_1}{h_2}\frac{\partial V}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial V}{\partial u_3}\right)\right]$$

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