UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 1994/95

April 1995

ZCC 542/4 - Teori Keadaan Pepejal II

Masa : [3 jam]

Sila pastikan bahawa kertas peperiksaan ini mengandungi SEPULUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab MANA-MANA EMPAT soalan sahaja.
Kesemuanya wajib dijawab dalam **Bahasa Inggeris**.

1. **EITHER**

(a) (i) An external charge +q is introduced into a free electron gas moving in the presence of a uniform neutralising positive charge (jellium). Give a qualitative description of the modification of the electron distribution that results in screening of the potential due to the external charge.

(10 marks)

(ii) In the Thomas-Fermi analysis of screening the k-space expression for the screened potential is

$$\phi(k) = (e/\varepsilon_o)(k^2 + k_s^2)^{-1}$$

where k_s is a constant. By taking spherical polar coordinates along **r** show that the Fourier transform is

$$\phi(r) = (e/4\pi\varepsilon_o r) \exp(-k_s r)$$

(25 marks)

(iii) Find the expression for the corresponding force $\mathbf{F}(r) = -\nabla\phi(r)$.

(15 marks)

(iv) Draw sketches to compare $\phi(r)$ and $\mathbf{F}(r)$ with the corresponding unscreened forms.

(20 marks)

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(v) Briefly describe the Mott transition and outline one piece of experimental evidence in which a Mott transition is seen.

(20 marks)

(vi) What is the relationship between the Mott transition and screening?

(10 marks)

[The Fourier transform is defined by

$$\phi(\mathbf{r}) = (2\pi)^{-3} \int d^3k\phi(k) \exp(i\mathbf{k.r})$$

Tables of integrals give

$$\int_0^\infty x \sin(mx)(a^2 + x^2)^{-1} dx = (\pi/2) \exp(-ma)]$$

OR

(b) (i) An rf electric field $\mathbf{E} \exp(-i\omega t)$ is applied to jellium. Assuming that the equation of motion of the electron gas is

$$m(d\mathbf{v}/dt + \mathbf{v}/\tau) = -e\mathbf{E} \exp(-i\omega t)$$

with current density $j = -n_o e\mathbf{v}$

prove that the dielectric constant of the jellium is

$$\varepsilon(\omega) = 1 - \omega_p{}^2/(\omega^2 + i\omega/\tau)$$

where $\omega_p{}^2 = n_o e^2/\varepsilon_o m$.

(40 marks)

(ii) For the case of no damping, $1/\tau = 0$, draw a sketch to show $\varepsilon(\omega)$ as a function of ω.

(10 marks)

(iii) Fig. 1 shows the infrared reflectivity of InSb doped with a carrier concentration of 4.10^{18} cm^{-3}. Give a discussion of the data in terms of the dielectric function.

(40 marks)

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(iv) Describe and explain the corresponding result for an alkali metal like Na.

(10 marks)

[The reflectance R is given in terms of $\varepsilon(\omega)$ by

$$R = |(1 - \varepsilon^{1/2})/(1 + \varepsilon^{1/2})|^2$$]

2. Fig. 2a shows the resistivity as a function of temperature for Na samples of differing purity. Figs. 2b and 2c show conductivity and Hall coefficient as functions of temperature for a number of doped InSb samples. Give a detailed discussion of the data, quoting appropriate formulae from elementary transport theory.

(metal 30 marks, semiconductor 70 marks)

3. **EITHER**

(a) (i) Explain what is meant by a magnetic hysteresis loop.

(10 marks)

(ii) Draw sketches to show typical hysteresis loops of [a] a soft and [b] a hard magnetic material.

(20 marks)

(iii) Discuss the differences between the two classes of magnetic material in terms of domain-wall movement.

(20 marks)

(iv) What are the sources of loss in transformer cores and in what ways can the designer attempt to overcome losses?

(20 marks)

(v) A ring of a hard magnetic material is opened to produce a gap as shown in Fig. 3. By means of a magnetostatic analysis, prove that the B and H fields in the magnet, B_m and H_m, are related by

$$B_m = -\mu_o L_m H_m / L_g$$

(20 marks)

...4/-

(vi) Using this relation together with the fact that B_m and H_m lie on the hysteresis loop, explain the significance of the parameter $(BH)_{max}$ in hard magnets.

(10 marks)

[You may assume that the magnetic energy W in the two gap regions is given by $W = -B_m H_m L_m A$, where A is the pole area].

OR

(b) (i) Write an account of magnetic recording. You should cover at least the following:

Physical principles of inductive and magnetoresistive heads, their advantages and disadvantages.

Materials requirements for recording media.

Materials requirements for inductive and magnetoresistive heads.

(70 marks)

(ii) Describe and explain the phenomenon of giant magnetoresistance that has been observed in transition-metal films coupled across a thin non-magnetic film.

(30 marks)

4. (a) The Heisenberg exchange Hamiltonian in a simple model of a magnet is

$$H = -(J/2) \Sigma \mathbf{S}_i . \mathbf{S}_j$$

where the sum is over nearest-neighbour magnetic sites i and j. Describe the forms of magnetic ordering observed at T=0 K for [a] positive J and [b] negative J. Describe in simple terms what happens to the magnetic order as T increases.

(30 marks)

(b) Explain how the idea of a mean field is used to give an approximate account of the statistical mechanics.

(10 marks)

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(c) The mean-field equation for the magnetization M of a ferromagnet in an external field B_o is

$$M = N\mu \tanh [\mu(B_o + \lambda M)/k_B T]$$

where μ is the magnetic moment of one spin and λ is the mean-field constant. Derive expressions from this for [a] the Curie temperature T_C and [b] the magnetic susceptibility χ for $T > T_C$.

(40 marks)

(d) Sketch the temperature dependence of χ and explain the form of the graph in physical terms.

(20 marks)

5. (a) Explain what is meant by a macroscopic wave function in superconductivity.

(20 marks)

(b) Describe briefly the phenomenon of flux quantization in superconductors and explain how it lends support to the idea of a macroscopic wave function.

(20 marks)

(c) A Josephson junction is a weak link between two superconductors 1 and 2. Describe two possible structures that behave as Josephson junctions.

(20 marks)

(d) Assume that the junction is described by the coupled time-dependent wave equations

$$i\hbar\, \partial\psi_1/\partial t = \mu_1\psi_1 + K\psi_2$$

$$i\hbar\, \partial\psi_2/\partial t = \mu_2\psi_2 + K\psi_1$$

where ψ_1 and ψ_2 are the two wave functions. By substituting $\psi_1 = n_1^{1/2} \exp(iS_1)$ and a similar expression for ψ_2, prove that

$$\hbar\partial n_1/\partial t = -\hbar\partial n_2/\partial t = 2Kn_1^{1/2}n_2^{1/2} \sin(S_1 - S_2)$$

(20 marks)

...6/-

(e) Interpret this result in terms of the Josephson supercurrent and describe a simple experiment in which this supercurrent is observed. Explain carefully what happens to the phase difference $S_1 - S_2$ as the current through the weak link is increased.

(20 marks)

6. (a) Describe nematic ordering in a liquid crystal and define the director **n**.

(10 marks)

(b) Explain how the orientation of **n** is affected by [a] pinning at a boundary and [b] an applied magnetic or electric field. Explain the notion of a director profile.

(20 marks)

(c) Fig. 6 shows the expected director profile in the vicinity of a wall where **n** is pinned vertical at y=0 but for large y the preferred orientation is horizontal because of an applied horizontal field. Assume that the angle θ, defined in the figure, satisfies

$$Kd^2\theta/dy^2 + \chi_a H^2 \sin\theta \cos\theta = 0$$

What is the physical significance of the parameters K and χ_a in this equation?

(20 marks)

(d) Multiply by $d\theta/dy$ to show that a first integral of this equation is

$$\xi^2(d\theta/dy)^2 = A + \cos^2\theta$$

(10 marks)

(e) Define ξ in terms of K, χ_a and H and use the boundary condition for $y \to \infty$ to show that A=0. Integrate a second time to show that the director profile is given by

$$\tan(\pi/4 + \theta/2) = \exp(y/\xi)$$

(30 marks)

...7/-

(f) Draw sketches to show that the solution of this equation for θ versus y does have the expected form.

(10 marks)

[If $t = \tan(x/2)$ then

$dx = 2dt/(1 + t^2)$

$\cos x = (1 - t^2)/(1 + t^2)$]

...8/-

Figures

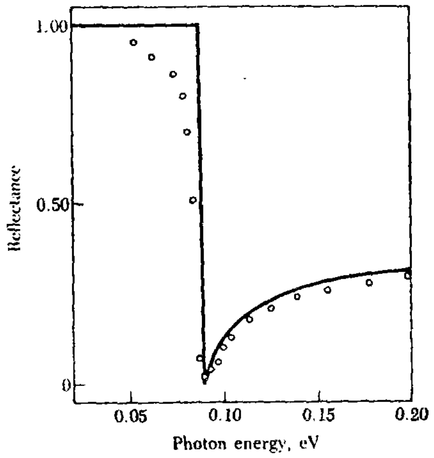

Fig. 1: Reflectance of InSb.

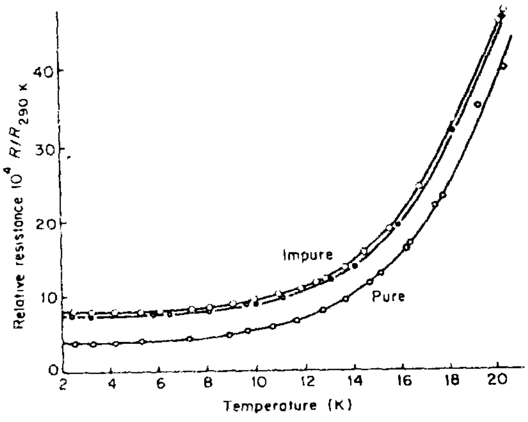

Fig. 2a: Resistivity-temperature curves for Na samples of differing purity.

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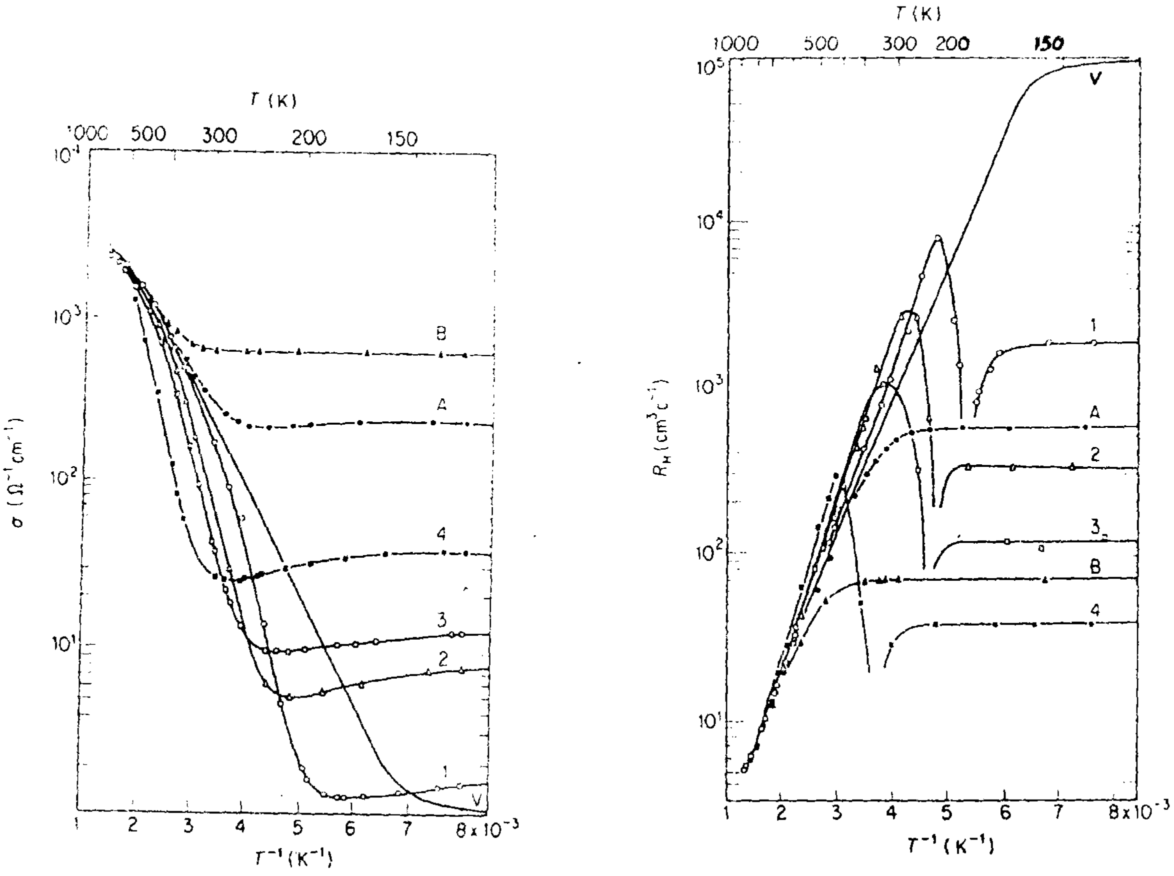

(b) (c)

Fig. 2b Conductivity and 2c Hall coefficient versus temperature for InSb samples. Samples A, B and V are n type and 1 to 4 are p type.

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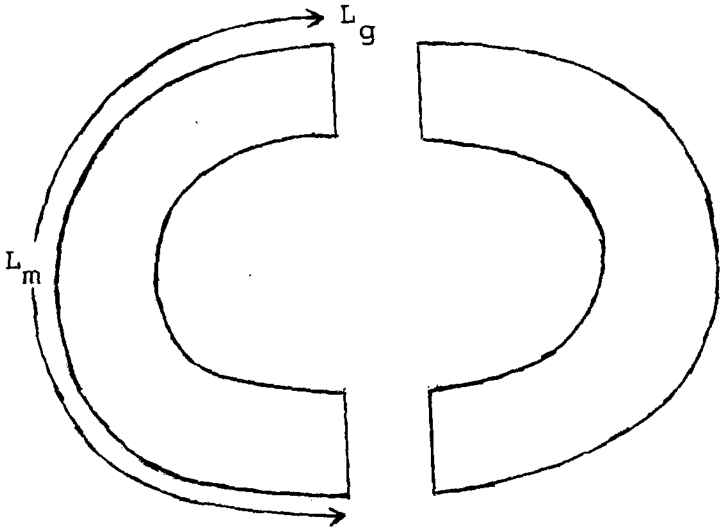

Fig. 3: Hard magnet with gap.

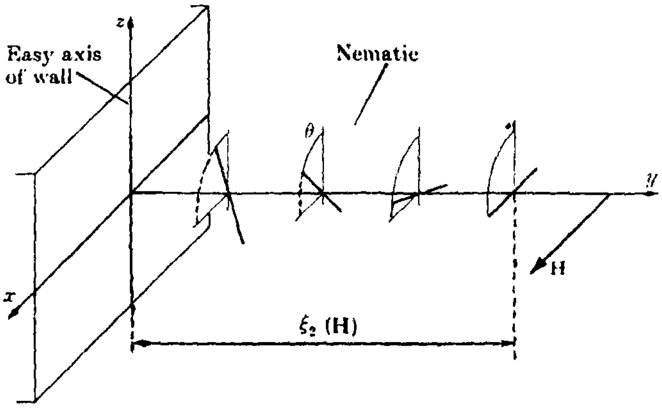

Fig. 6: Director profile with competition between wall pinning and field orientation.

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