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Institution :	University Sains
Institutional Postal Address :	School of Mathematical Sciences, University Sains, Malaysia
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Abstract :	Shape Preserving Rational Spline M. Z. Hussain, J. M. Ali, A. A Majid, A. R. M. Piah School of Mathematical Sciences University Sians Malaysia E-mail:malikzawwar@hotmail.com A rational function has been constructed with free parameters to preserve the shape of data. Simple data dependent constraints are derived on the free parameters to preserve the shape of data.
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Shape Preserving GC^1 Rational Interpolation

A. A Majid, M. Z. Hussain, J. M. Ali, A. R. M. Piah
School of Mathematical Sciences
Universiti Sains Malaysia
majid@cs.usm.my

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Abstract

A GC^1 rational quartic function has been constructed with three parameters to preserve the shape of positive and monotone curve data. Simple data dependent constraints are derived on one of the parameters to preserve the shape of data while other two are free to refine the shape of curve at user choice.

Keywords: Shape preservation; GC^1 rational quartic function; Positive data; Monotone data; Free Parameters.

1 Introduction

Convex, monotone and positive are the fundamental shapes of data. The development of interpolating and approximating shape preserving schemes is a germane area of research. In this study the last two shape properties (monotonicity and positivity) are addressed. Some examples of monotone data are: erythrocyte sedimentation rate (E.S.R.) in cancer patients; blood uric acid level in patients suffering from gout; economic forecasting and data generated from Newton's law of cooling. Rainfall amount [1], probability distribution [1], resistance offered by an electric circuit, area, population growth, density are always positive.

The artistic designs require higher order of smoothness, whereas, the industrial design (Designing of turbine blades, cutting tools) require sharp corners and edges. Hence the development of interpolating schemes which can control magnitudes and directions of end tangents (GC^1) is always desirable.

The problems of monotone and positive data interpolation were discussed by a number of authors. Brodlie and Butt [1] presented a positive curve data interpolation scheme. The piecewise cubic interpolant was used to interpolate the positive data. In a particular interval where shape of the data was lost, the author divided that interval into two subintervals by inserting an extra knot to preserve the positive shape of data. Fahr and Kallay [2] used a C^1 rational

B-spline of degree one for monotone curve data interpolation. Goodman, Ong and Unsworth [3] presented two schemes to preserve the shape of data lying on either side of straight line. The first scheme adopted the method of scaling weights and the second scheme refers to insert some extra point to preserve the shape of data. Goodman [4] provided a comprehensive review of the existing curve data interpolation schemes. Lamberti and Manni [7] used parametric cubic Hermite interpolant for the shape preserving curve data interpolation. The constraints were developed on subinterval's length for shape preservation of data. The first order derivatives at the knots were estimated by a tri-diagonal system of equations which assured C^1 continuity at the knots. Sarfraz *et al.* [10] have developed a GC^1 cubic function with one shape parameter to preserve the shape of data. The simple sufficient data dependent constraints were developed on shape parameter to preserve the shape of data while no parameter is available for shape refinement. Wang and Tan [11] developed a rational quartic function with two parameters.

The remainder of the paper is organized as follows: In Section 2, a GC^1 rational quartic function with three parameters is developed. The problems of positive and monotone curve data interpolation are discussed in Section 3 and 4 respectively. The effect of parameters on the shape of the curve is demonstrated in Section 3.1 and 4.1. Section 5 concludes the paper.

2 GC^1 Rational Quartic Function

In this section a GC^1 rational quartic function with three parameters in quartic/quadratic form is developed.
Let $\{(x_i, f_i),\ i = 0, 1, 2, \ldots, n\}$ be the given set of data points defined over the interval $[a, b]$, where $a = x_0 < x_1 < x_2 < \cdots < x_n = b$. The GC^1 piecewise rational quartic function with three parameters α_i, β_i and γ_i is defined over each subinterval $I_i = [x_i, x_{i+1}],\ i = 0, 1, 2, \cdots, n-1$ as

$$S(x) \equiv S_i(x) = \frac{p_i(\theta)}{q_i(\theta)}, \tag{1}$$

where

$$\begin{aligned} p_i(\theta) &= A_0(1-\theta)^4 + A_1(1-\theta)^3\theta + A_2(1-\theta)^2\theta^2 + A_3(1-\theta)\theta^3 + A_4\theta^4, \\ q_i(\theta) &= \alpha_i(1-\theta)^2 + 2(1-\theta)\theta + \beta_i\theta^2, \end{aligned}$$

$$\begin{aligned} A_0 &= \alpha_i f_i, \\ A_1 &= (2\alpha_i + 2)f_i + \frac{h_i\alpha_i d_i}{\gamma_i}, \\ A_2 &= (\alpha_i + 2)f_{i+1} + (\beta_i + 2)f_i, \\ A_3 &= (2\beta_i + 2)f_{i+1} - \frac{h_i\beta_i d_{i+1}}{\gamma_i}, \\ A_4 &= \beta_i f_{i+1}, \end{aligned}$$

$$h_i = x_{i+1} - x_i, \ \theta = \frac{x - x_i}{h_i},$$

$S_i(x_i) = f_i, \ S_i(x_{i+1}) = f_{i+1}, \ S_i^{(1)}(x_i) = \frac{d_i}{\gamma_i}, \ S_i^{(1)}(x_{i+1}) = \frac{d_{i+1}}{\gamma_i}.$
$S_i^{(1)}(x)$ denotes the derivative with respect to x and d_i denotes derivative values estimated or provided. It is noted that when $\alpha_i = 1, \ \beta_i = 1$ and $\gamma_i = 1$, the GC^1 rational quartic function (1) reduces to C^1 quartic function. In this paper the parameters $\alpha_i, \ \beta_i$ and γ_i are assumed positive real numbers.

3 Positivity Preserving Interpolation

In this section, sufficient conditions are derived on parameter γ_i for positive interpolation of curve data.
Let $\{(x_i, f_i), \ i = 0, 1, 2, \cdots, n\}$ be the positive data defined over the interval $[a, b]$ as

$$f_i, \ i = 0, 1, 2, \cdots, n. \tag{2}$$

The piecewise GC^1 rational quartic function defined in (1) is positive if $p_i(\theta) > 0$ and $q_i(\theta) > 0$.
$q_i(\theta) > 0$ if

$$\alpha_i > 0, \ \beta_i > 0, \ \gamma_i > 0.$$

$p_i(\theta) > 0$ if

$$A_i, \ i = 0, 1, 2, 3, 4.$$

$A_0 > 0$, if

$$\alpha_i > 0.$$

$A_1 > 0$ if

$$\alpha_i > 0,$$

$$\gamma_i > Max\left\{0, \frac{-h_i \alpha_i d_i}{f_i}\right\}.$$

$A_2 > 0$ if

$$\alpha_i > 0, \ \beta_i > 0.$$

$A_3 > 0$ if

$$\beta_i > 0,$$

$$\gamma_i > Max\left\{0, \frac{h_i \beta_i d_{i+1}}{f_{i+1}}\right\}.$$

$A_4 > 0$, if

$$\beta_i > 0.$$

The above discussion can be summarized as:

Theorem 3.1

The piecewise GC^1 rational quartic interpolant $S(x)$, defined over the interval $[a, b]$, in (1), is positive if in each sub interval $I_i = [x_i, x_{i+1}]$ the following sufficient conditions are satisfied:

$$\alpha_i > 0,\ \beta_i > 0,$$
$$\gamma_i > Max\left\{0, \frac{-h_i\alpha_i d_i}{f_i}, \frac{h_i\beta_i d_{i+1}}{f_{i+1}}\right\}.$$

The above constraints can be rearranged as:

$$\alpha_i > 0,\ \beta_i > 0,$$
$$\gamma_i = l_i + Max\left\{0, \frac{-h_i\alpha_i d_i}{f_i}, \frac{h_i\beta_i d_{i+1}}{f_{i+1}}\right\},\ l_i > 0.$$

3.1 Numerical Examples

A positive data set is taken in Table 1.

Table 1: A positive data set.

x	0	2	3	9	11
y	0.5	1.5	7	9	13

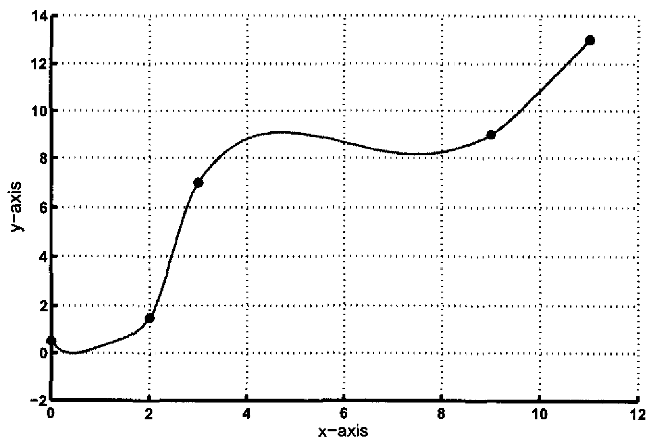

Fig. 1: Cubic Hermite spline.

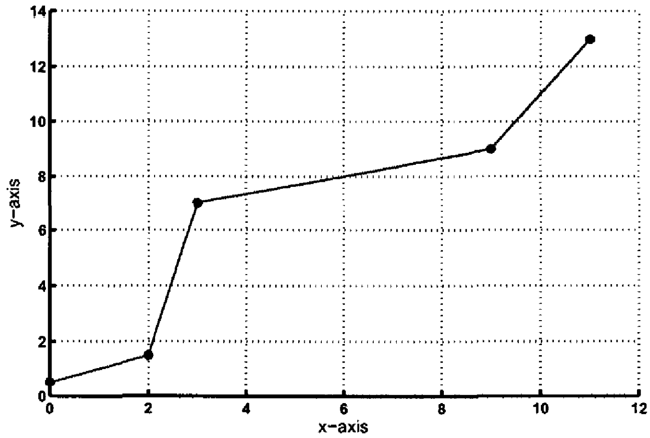

Fig. 2: Rational quartic function with $\alpha_i = 0.001$ and $\beta_i = 0.001$.

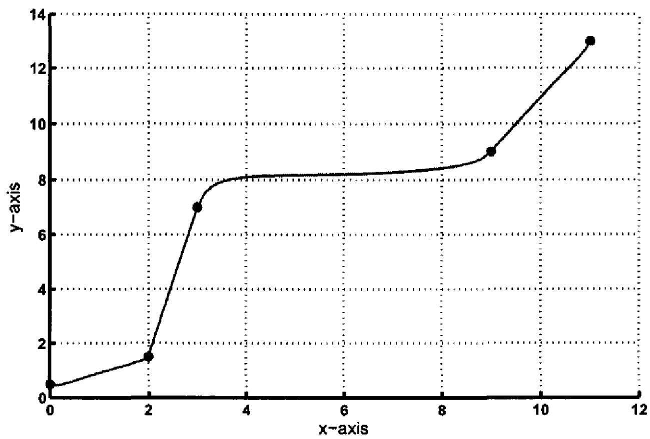

Fig. 3: Rational quartic function with $\alpha_i = 0.1$ and $\beta_i = 0.1$.

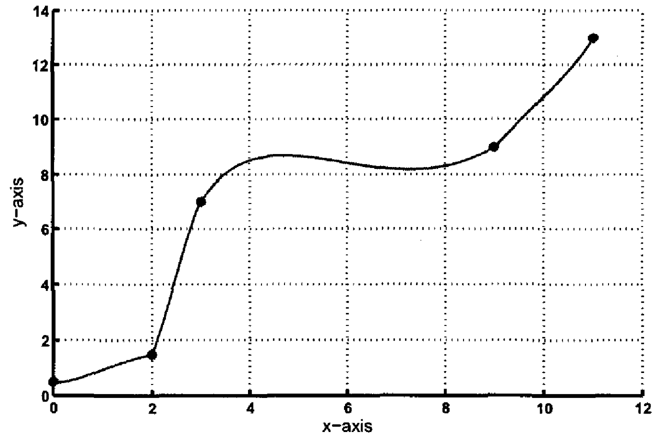

Fig. 4: Rational quartic function with $\alpha_i = 0.6$ and $\beta_i = 0.7$.

Fig. 1 is produced by interpolating the positive data set taken in Table 1 using C^1 quartic function. Positive curve in Fig. 2 is produced by the positivity preserving scheme developed in Section 3 with $\alpha_i = 0.001$ and $\beta_i = 0.001$. It is clear from Fig. 2 that although the shape of the data has been preserved but it is tense. Fig. 3 is produced from the same scheme with $\alpha_i = 0.1$ and $\beta_i = 0.1$. The positive curve is further refined in Fig. 4 by choosing $\alpha_i = 0.6$ and $\beta_i = 0.7$.

4 Monotonicity Preserving Interpolation

Let $\{(x_i, f_i),\ i = 0, 1, 2, \cdots, n\}$ be the monotone data defined over the interval $[a, b]$ such that

$$f_i < f_{i+1},\ \Delta_i = \frac{f_{i+1} - f_i}{h_i} > 0,\ i = 0, 1, 2, \cdots, n-1,$$

$$d_i > 0,\ i = 0, 1, 2, \cdots, n.$$

The piecewise GC^1 rational quartic function defined in (1) is monotone if

$$S_i^{(1)}(x) > 0, \quad \forall\, x \in [x_i, x_{i+1}], \quad i = 0, 1, 2, \cdots, n-1 \tag{3}$$

where

$$S_i^{(1)}(x) \quad = \quad \frac{\sum_{i=0}^{5} (1-\theta)^{5-i} \theta^i B_i}{(q_i(\theta))^2},$$

with

$$B_0 \quad = \quad \frac{\alpha_i^2 d_i}{\gamma_i},$$

$$
\begin{aligned}
B_1 &= \alpha_i(2\alpha_i + 4)\Delta_i - \frac{\alpha_i^2 d_i}{\gamma_i}, \\
B_2 &= (6\alpha_i\beta_i + 8\alpha_i + 4)\Delta_i - \frac{3\alpha_i\beta_i d_{i+1}}{\gamma_i} - \frac{(\alpha_i\beta_i + 4\alpha_i)d_i}{\gamma_i}, \\
B_3 &= (6\alpha_i\beta_i + 8\beta_i + 4)\Delta_i - \frac{3\alpha_i\beta_i d_i}{\gamma_i} - \frac{(\alpha_i\beta_i + 4\beta_i)d_{i+1}}{\gamma_i}, \\
B_4 &= \beta_i(2\beta_i + 4)\Delta_i + \frac{\beta_i^2 d_{i+1}}{\gamma_i}, \\
B_5 &= \frac{\beta_i^2 d_{i+1}}{\gamma_i}.
\end{aligned}
$$

$S_i^{(1)}(x) > 0$ if

$$B_i > 0,\ i = 0, 1, 2, 3, 4, 5.$$

$B_i > 0,\ i = 0, 5$ if

$$\gamma_i > 0.$$

$B_1 > 0$ if

$$\gamma_i > Max\left\{0,\ \frac{\alpha_i d_i}{\Delta_i}\right\}.$$

$B_2 > 0$ if

$$\gamma_i > Max\left\{0,\ \frac{d_i}{\Delta_i},\ \frac{3d_{i+1} + d_i}{6\Delta_i}\right\}.$$

$B_3 > 0$ if

$$\gamma_i > Max\left\{0,\ \frac{d_{i+1}}{\Delta_i},\ \frac{3d_i + d_{i+1}}{6\Delta_i}\right\}.$$

$B_4 > 0$ if

$$\gamma_i > Max\left\{0,\ \frac{-\beta_i d_{i+1}}{\Delta_i}\right\}.$$

All this discussion is summarized in the following theorem:

Theorem 4.1

The piecewise GC^1 rational quartic interpolant $S(x)$, defined over the interval $[a, b]$, in (1), is monotone if in each sub interval $I_i = [x_i, x_{i+1}]$ the following sufficient conditions are satisfied:

$$
\begin{aligned}
&\alpha_i > 0,\ \beta_i > 0, \\
\gamma_i &> Max\left\{0,\ \frac{d_i}{\Delta_i},\ \frac{d_{i+1}}{\Delta_i},\ \frac{\alpha_i d_i}{\Delta_i},\ \frac{-\beta_i d_{i+1}}{\Delta_i},\ \frac{3d_{i+1} + d_i}{6\Delta_i},\ \frac{3d_i + d_{i+1}}{6\Delta_i}\right\}.
\end{aligned}
$$

The above constraints can be rearranged as:

$$
\begin{aligned}
&\alpha_i > 0,\ \beta_i > 0, \\
\gamma_i &= m_i + Max\left\{0,\ \frac{d_i}{\Delta_i},\ \frac{d_{i+1}}{\Delta_i},\ \frac{\alpha_i d_i}{\Delta_i},\ \frac{-\beta_i d_{i+1}}{\Delta_i},\ \frac{3d_{i+1} + d_i}{6\Delta_i},\ \frac{3d_i + d_{i+1}}{6\Delta_i}\right\},\ m_i > 0.
\end{aligned}
$$

4.1 Numerical Examples

A monotone data set is taken in Table 2.

Table 2: A monotone data set.

x	0	6	10	29.5	30
y	0	15	15	25	30

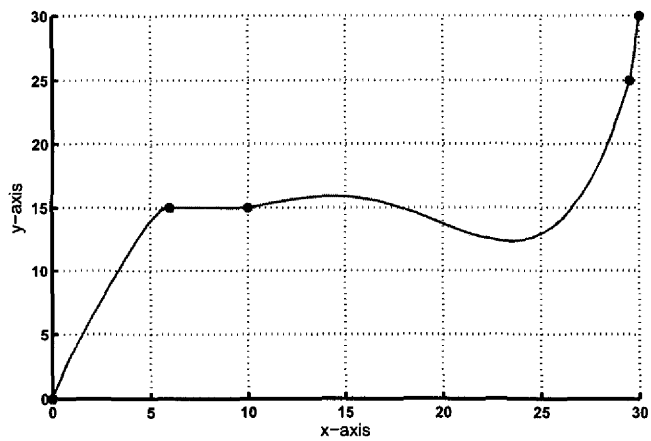

Fig. 5: Cubic Hermite spline.

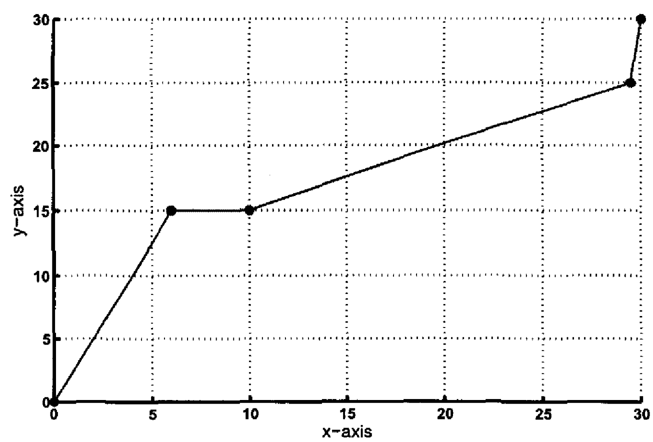

Fig. 6: Rational quartic function with $\alpha_i = 0.001$ **and** $\beta_i = 0.001$.

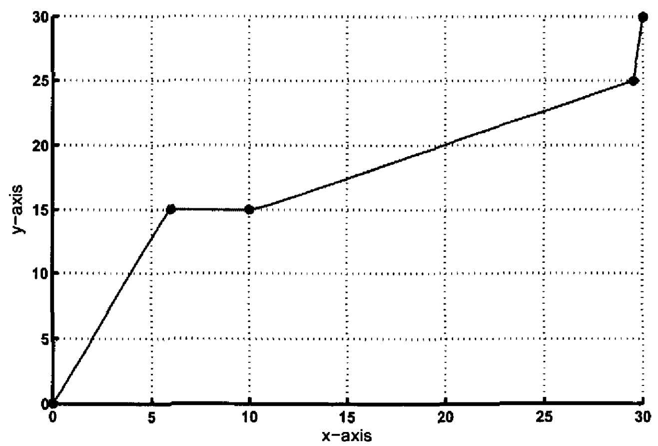

Fig. 7: Rational quartic function with $\alpha_i = 0.5$ and $\beta_i = 0.5$.

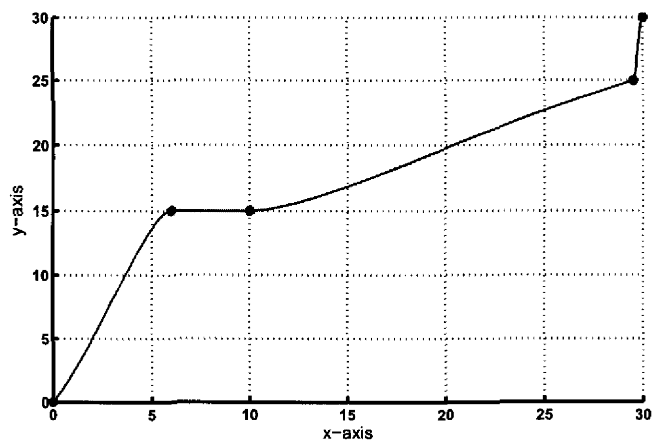

Fig. 8: Rational quartic function with $\alpha_i = M_1$ and $\beta_i = M_2$.

Fig. 5 is produced by interpolating the monotone data set in Table 2 using C^1 quartic function. It is clear from Fig. 5 that C^1 quartic function does not preserve the monotone shape of data. Monotone curves in Fig. 6 and Fig. 7 are produced by the monotonicity preserving scheme developed in Section 4 with $(\alpha_i,\ \beta_i) = (0.001,\ 0.001)$ and $(\alpha_i,\ \beta_i) = (0.5,\ 0.5)$. It is clear from Fig. 6 and Fig. 7 that although the monotonicity is preserved but curve is not smooth. The monotone curve is further refined in Fig. 8 by assigning the following values to parameters $\alpha_i = M_1 = [0.5, 3, 0.5, 5]$ and $\beta_i = M_2 = [0.5, 3, 0.5, 5.2]$.

5 Conclusion

In this study, the problems of positive and monotone curve data interpolation are addressed. A piecewise rational quartic GC^1 function with three parameters is developed to preserve the shape of positive and monotone curve data. Some sufficient data dependent constraints are developed on one of the parameters to preserve the shape of positive and monotone curve data while other two are free to refine the shape of curve at user choice. The effect of free parameters on the shape of curve is demonstrated in Fig. 2-Fig. 4 and Fig. 6-Fig. 8.

GC^1 cubic function [10] had only one parameter which was constrained to preserve the shape of data and no free parameter to refine the shape of curve. The scheme developed in this paper has constrained only one parameter while other two are free to refine the shape of curve.

Unlike [1, 7], it does not need to refine the interval's length to preserve the shape of data.

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