UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2003/2004

Februari/Mac 2004

JIM 417 – Persamaan Pembezaan Separa

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab SEMUA soalan.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

...2/-

1. (a) Dapatkan persamaan pembezaan peringkat pertama dengan menggunakan persamaan

$$\psi\left(\frac{u}{x^3}, \frac{2y}{x}\right) = 0.$$

(35 markah)

(b) Tunjukkan bahawa

$$u = f\left(x - \frac{y}{2}\right) + y\ g\left(x - \frac{y}{2}\right)$$

memenuhi persamaan

$$\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0.$$

(30 markah)

(c) Selesaikan persamaan pembezaan separa

$$\frac{\partial u}{\partial x} + (x + y)\frac{\partial u}{\partial y} = xu.$$

(35 markah)

2. Diberi persamaan pembezaan separa berikut:

$$3u_{xx} + 10u_{xy} + 3u_{yy} + 3u_y + u_x = 0.$$

Bagi persamaan ini

(a) tentukan jenis

(10 markah)

(b) dapatkan koordinat cirian dan bentuk berkanun

(70 markah)

(c) cari penyelesaian am.

(20 markah)

...3/-

3. Dengan menggunakan jelmaan Laplace selesaikan masalah nilai awal – sempadan berikut:

(a) $\frac{\partial u}{\partial t}(x, t) = k\frac{\partial^2}{\partial x^2} u(x, t)$, $(x > 0, t > 0)$

dengan syarat awal

$$u(x, 0) = u_0$$

dan syarat sempadan

$$u(0, t) = u_1$$

$$\underset{x\to\infty}{\text{had}}\, u(x, t) = u_0 .$$

(50 markah)

(b) $\frac{\partial u}{\partial t} + x\frac{\partial u}{\partial x} = x,\qquad x > 0,\ \ t > 0$

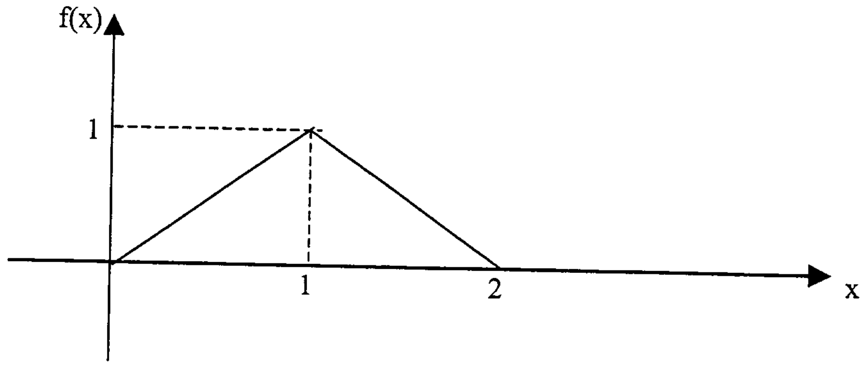
dengan $u = u(x, t)$ dan syarat-syarat awal dan sempadan

$$u(x, 0) = 0$$

$$u(0, t) = 0.$$

(50 markah)

4. Seutas tali kenyal yang panjangnya 2 meter diregangkan dalam keadaan mendatar supaya kedua-dua hujungnya di $x = 0$ dan $x = 2$ ditetapkan. Pada kedudukan $x = 1$, tali ini ditarik sebanyak 1 meter daripada permukaan mendatar dan dilepaskan supaya ia bergetar. Jika halaju pesongan pada $t = 0$ ialah 0. (Sila rujuk Rajah 1).



Rajah 1

(a) Bentukkan model masalah nilai awal-sempadan jika diberi persamaan pembezaan separa yang terbentuk adalah

$$u_{tt}(x,\ t) = c^2 u_{xx}\ ,\ 0 < x < 2,\ t > 0.$$

(20 markah)

(b) Selesaikan masalah nilai awal-sempadan dalam (a).

(80 markah)

5. Selesaikan masalah nilai sempadan

$$\frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} = 0 \qquad , \quad \begin{array}{c} 0 < \theta < \pi \\ 0 < r < 1 \end{array}$$

jika syarat-syarat sempadan diberikan oleh

$$v(r,\ 0) = v(r,\ \pi) = 0\ ,\quad 0 < r < 1$$

$$v(1,\ \theta) = v_o,\ \ 0 < \theta < \pi.$$

(100 markah)

Senarai Rumus

$$u_x = u_\xi \xi_x + u_\eta \eta_x$$

$$u_y = u_\xi \xi_y + u_\eta \eta_y$$

$$u_{xx} = u_{\xi\xi}\xi_x^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}\eta_x^2 + u_\xi \xi_{xx} + u_n \eta_{xx}$$

$$u_{xy} = u_{\xi\xi}\xi_x\xi_y + u_{\xi\eta}(\xi_x\eta_y + \xi_y\eta_x) + u_{\eta\eta}\eta_x\eta_y + u_\xi \xi_{xy} + u_n \eta_{xy}$$

$$u_{yy} = u_{\xi\xi}\xi_y^2 + 2u_{\xi\eta}\xi_y\eta_y + u_{\eta\eta}\eta_y^2 + u_\xi \xi_{yy} + u_\eta \eta_{yy}$$

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty}\left[a_n \operatorname{kos}\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$$

dengan

$$a_o = \frac{1}{L}\int_{-L}^{L} f(x)\, dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x) \operatorname{kos}\left(\frac{n\pi x}{L}\right) dx, \quad n = 1, 2, 3, \ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx, \quad n = 1, 2, 3, \ldots$$

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \operatorname{kos}\left(\frac{n\pi x}{L}\right)$$

dengan

$$a_n = \frac{2}{L}\int_{0}^{L} f(x) \operatorname{kos}\left(\frac{n\pi x}{L}\right) dx, \quad n = 1, 2, 3, \ldots$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

dengan

$$b_n = \frac{2}{L}\int_{0}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx, \quad n = 1, 2, 3, \ldots$$

$$f(x) = \frac{1}{2} \sum_{-\infty}^{\infty} c_n e^{inx}$$

dengan

$$c_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx, \quad n = 0, \pm 1, \pm 2, ...$$

$\frac{d^2y}{dx^2} - \alpha^2 y = 0$ mempunyai penyelesaian

$$y = Ae^{\alpha x} + Be^{-\alpha x}$$

$$y = C \text{ kosh} \, \alpha x + D \sinh \alpha x$$

$\frac{d^2y}{dx^2} + \alpha^2 y = 0$ mempunyai penyelesaian

$$y = A \text{kos} \, \alpha x + B \sin \alpha x$$

$$r^2 \frac{d^2R}{dr^2} + r\frac{dR}{dr} - n^2 R = 0$$

$$R_n = C_n r^n + \frac{D_n}{r^n}.$$

$r \frac{d^2R}{dr^2} + \frac{dR}{dr} = 0$ mempunyai penyelesaian

$$R = A + B \ell\text{n} \, r$$

$$\Im[f(t)] = F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \, e^{i\alpha t} \, dt$$

$$f(x) = \Im^{-1}[F(\alpha)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\alpha) \, e^{-i\alpha x} \, dx$$

...7/-

$$\Im[f(x)] = F_s(n) = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx, \quad n=1,2,...$$

$$f(x) = \Im^{-1}[F_s(n)] = \sum_{n=1}^{\infty} F_s(n)\sin\frac{n\pi x}{L}$$

$$\Im[f(x)] = F_c(n) = \frac{2}{L}\int_0^L f(x)\,\text{kos}\frac{n\pi x}{L}\,dx, \quad n = 1, 2, ...$$

$$f(x) = \Im^{-1}[F_c(n)] = \frac{F_c(0)}{2} + \sum_{n=1}^{\infty} F_c(n)\,\text{kos}\,\frac{n\pi x}{L}$$

$$\Im[f''(x)] = \frac{2n}{\pi}[f(0)-(-1)^n f(\pi)] - n^2 F_s(n)$$

$$\Im[f''(x)] = \frac{2}{\pi}[(-1)^n f'(\pi)-f'(0)]-n^2 F_c(n)$$

$$\mathfrak{L}[f(t)] = F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

$$\mathfrak{L}[e^{\alpha t} f(t)] = F(s-\alpha)$$

Jika $g(t) = \begin{cases} 0 & , t < \alpha \\ f(t-\alpha), & t>0 \end{cases}$

maka

$$\mathfrak{L}[g(t)] = e^{-\alpha s} F(s)$$

$$\mathfrak{L}[f^{(n)}(t)] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - ... - sf^{(n-2)}(0) - f^{(n-1)}(0)$$

$$\mathfrak{L}[tf(t)] = -F'(s) = -\frac{d}{ds}\mathfrak{L}[f(t)]$$

$$\mathfrak{L}\left[\int_0^t f(u)\,du\right] = \frac{F(s)}{s}$$

$$\mathfrak{L}^{-1}[F(s)G(s)] = \int_0^t f(u)\,g(t-u)\,du = f * g$$

...8/-

Jadual Jelmaan Laplace

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1	$\frac{1}{s}$
$t_n,\ n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
kos at	$\frac{s}{s^2+a^2}$
sin at	$\frac{a}{s^2+a^2}$
kosh at	$\frac{s}{s^2-a^2}$
sinh at	$\frac{a}{s^2-a^2}$
t kos bt	$\frac{s^2-a^2}{(s^2+b^2)^2}$
t sin bt	$\frac{2bs}{(s^2+b^2)^2}$
e^{at} kos bt	$\frac{s-a}{(s-a)^2+b^2}$
e^{at} sin bt	$\frac{b}{(s-a)^2+b^2}$

- oooOooo -