

---

UNIVERSITI SAINS MALAYSIA

PEPERIKSAAN KURSUS SEMASA CUTI PANJANG  
ACADEMIC SESSION 2008/2009

JUNE 2009

**JIK 317 – QUANTUM CHEMISTRY AND GROUP THEORY**  
**[KIMIA KUANTUM DAN TEORI KUMPULAN]**

Duration : 3 hours  
[Masa : 3 jam]

---

Please ensure that this examination paper contains **FIVE** printed pages together with a 20 printed pages of **Appendix JIK 317** before you begin the examination.

Answer **FIVE** questions. You may answer either in Bahasa Malaysia or English.

All answers must be written in the answer booklet provided.

Each question is worth 20 marks and the mark for each sub question is given at the end of that question.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA** muka surat yang bercetak bersama dengan **Lampiran JIK 317** sebanyak 20 muka surat bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **LIMA** soalan. Anda dibenarkan menjawab soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.*

*Setiap jawapan mesti dijawab di dalam buku jawapan yang disediakan.*

*Setiap soalan bernilai 20 markah dan markah subsoalan diperlihatkan di penghujung subsoalan itu.*

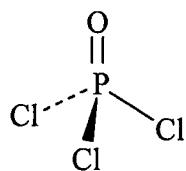
1. List the symmetry elements of the following molecules and classify each molecule into specific point group.

*Senaraikan unsur-unsur simetri bagi molekul-molekul berikut dan kelaskan setiap molekul kepada kumpulan titik tertentu.*

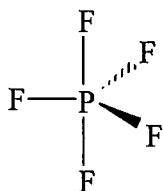
(a)



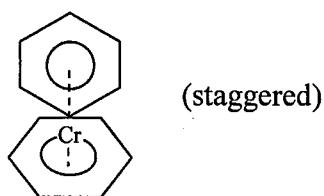
(b)



(c)



(d)



(e)      O = C = S

[20 marks/markah]

2. (a) Reduce the following reducible representations.

*Turunkan perwakilan-perwakilan terturunkan berikut :*

(i)

$C_{2h}$	E	$C_2$	i	$\sigma_h$
$\Gamma_{10}$	8	0	6	2

(ii)

$C_{3v}$	E	$2C_3$	$3\sigma_v$
$\Gamma_{11}$	6	0	-2

(iii)

$C_{2v}$	E	$C_2$	$\sigma_{(xz)}$	$\sigma_{(yz)}$
$\Gamma_{12}$	3	-3	1	-1

[10 marks/markah]

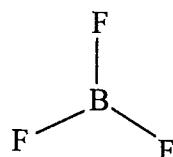
- (b) Set up the correlation diagram for  $CH_4$  molecule. Consider the 2s and 2p orbitals of carbon and the 1s orbital of each hydrogen atom.

*Lukis gambar rajah korelasi bagi molekul  $CH_4$ . Atom karbon melibatkan orbital 2s dan 2p manakala setiap atom hidrogen melibatkan orbital 1s.*

[10 marks/markah]

3. (a) Find the number, and symmetry species of the Raman and infrared active vibrations of boron trifluoride ( $D_{3h}$ ). Show your calculation.

*Kira bilangan dan spesies simetri spektrum Raman dan inframerah daripada getaran aktif molekul boron trifluorida ( $D_{3h}$ ). Tunjukkan pengiraan anda.*



[10 marks/markah]

- (b) With appropriate calculation predict the hybrid orbital responsible for  $\sigma$ -bond at carbon atom in ethylene molecule,  $\text{CH}_2 = \text{CH}_2$ .

*Dengan menunjukkan pengiraan yang berkaitan ramalkan orbital hibrid yang terlibat dengan ikatan- $\sigma$  pada atom karbon dalam molekul etilena,  $\text{CH}_2 = \text{CH}_2$ .*

[10 marks/markah]

4. Write out the results of experiment that produce the phenomenon known as the photoelectric effect. Which of those results cannot be explained using classical wave theory? How could the photoelectric effect phenomenon be explained by Einstein's hypothesis of quantization of light?

*Tuliskan keputusan-keputusan eksperimen yang menghasilkan fenomena yang dikenali sebagai kesan fotoelektrik. Antara keputusan-keputusan itu, manakah yang tidak dapat dijelaskan dengan penggunaan teori klasik gelombang? Bagaimanakah hipotesis Einstein yang menyatakan bahawa cahaya adalah terkuantum berjaya menjelaskan fenomena ini?*

[20 marks/markah]

5. (a) Write out the postulates of quantum mechanics.

*Nyatakan postulat-postulat mekanik kuantum.*

[15 marks/markah]

- (b) A state of quantum system is given by

$$\psi = \phi_1 + 2\phi_2 + 3\phi_3 + 4\phi_4$$

Calculate the expectation value for measurement of energy, given that

$$\hat{H}\phi_1 = h\nu\phi_1$$

$$\hat{H}\phi_2 = 2h\nu\phi_2$$

$$\hat{H}\phi_3 = 3h\nu\phi_3$$

$$\hat{H}\phi_4 = 4h\nu\phi_4$$

*Keadaan satu sistem kuantum diberikan oleh*

$$\psi = \phi_1 + 2\phi_2 + 3\phi_3 + 4\phi_4$$

*Kirakan nilai jangkaan bagi pengukuran tenaga, diberi bahawa*

$$\hat{H}\phi_1 = h\nu\phi_1$$

$$\hat{H}\phi_2 = 2h\nu\phi_2$$

$$\hat{H}\phi_3 = 3h\nu\phi_3$$

$$\hat{H}\phi_4 = 4h\nu\phi_4$$

[5 marks/markah]

6. A particle of mass  $m$  is confined to a one-dimensional box of length  $L$ .

*Satu zarah berjisim  $m$  dihadkan dalam satu kotak satu dimensi yang panjangnya  $L$ .*

- (a) Find the wavefunction of the particle in the box by solving the relevant Schroedinger equation, with appropriate boundary conditions.

*Dapatkan fungsi gelombang untuk zarah dalam kotak tersebut dengan menyelesaikan persamaan Schroedinger yang berkaitan, dengan syarat-syarat sempadan yang bersesuaian.*

[12 marks/markah]

- (b) Calculate the probability of finding the particle in the regions  $0 \leq x \leq L/2$  and  $L/2 \leq x \leq 3L/4$ . Evaluate the expressions for general values of quantum number  $n$  obtained from 3(a).

*Kirakan kebarangkalian untuk mencari zarah tersebut dalam ruang  $0 \leq x \leq L/2$  dan  $L/2 \leq x \leq 3L/4$ . Tuliskan jawapan untuk nilai nombor kuantum  $n$  yang umum daripada 3(a).*

[8 marks/markah]



**APPENDIX JIK 317**

**QUANTUM CHEMISTRY  
AND GROUP THEORY**

**[CHARACTER TABLE]**

## Appendix

### Character Tables for Some Chemically Important Symmetry Groups

$C_s$	E	$\sigma_h$		$C_i$	E	i	
A'	1	1	T <sub>x</sub> , T <sub>y</sub> , R <sub>z</sub>	x <sup>2</sup> , y <sup>2</sup>	A <sub>g</sub>	1	R <sub>x</sub> , R <sub>y</sub> , R <sub>z</sub>
A''	1	-1	T <sub>z</sub> , R <sub>x</sub> , R <sub>y</sub>	z <sup>2</sup> , xy yz, zx	A <sub>u</sub>	-1	T <sub>x</sub> , T <sub>y</sub> , T <sub>z</sub>
							x <sup>2</sup> , y <sup>2</sup> , z <sup>2</sup> xy, zx, yz
<i>The C<sub>n</sub> Groups:</i>							
$C_2$	E	$C_2$					
A	1	1	T <sub>z</sub> , R <sub>z</sub>	x <sup>2</sup> , y <sup>2</sup> , z <sup>2</sup> , xy			
B	1	-1	T <sub>x</sub> , T <sub>y</sub> , R <sub>x</sub> , R <sub>y</sub>	yz, zx			
$C_3$	E,	$C_3$	$C_3^2$				
A	1	1	1	T <sub>z</sub> , R <sub>z</sub>	$\epsilon = \exp(2\pi i/3)$		
	{1 1}	$\epsilon^*$	$\epsilon^*$	(T <sub>x</sub> , T <sub>y</sub> ), (R <sub>x</sub> , R <sub>y</sub> )	x <sup>2</sup> +y <sup>2</sup> , z <sup>2</sup>		
					(x <sup>2</sup> -y <sup>2</sup> , xy), (yz, zx)		

INTRODUCTORY GROUP THEORY FOR CHEMISTS

The  $C_n$  Groups (continued)

$C_4$	E	$C_4$	$C_2$	$C_{4^3}$	$T_z, R_z$	$x^2+y^2, z^2$
A	1	1	1	1	$(T_x, T_y),$ $(R_x, R_y)$	$x^2-y^2, xy$
B	1	-1	1	-1		$(yz, zx)$
E	{1 1}	i	-1	-i		
			-i	i		
$C_5$	E	$C_5$	$C_{5^2}$	$C_{5^3}$	$C_{5^4}$	$\epsilon = \exp(2\pi i/5)$
A	1	1	1	1	$T_z, R_z$	$x^2+y^2, z^2$
	{1 1}	$\epsilon$	$\epsilon^2$	$\epsilon^{2*}$	$(T_x, T_y), (R_x, R_y)$	$(yz, zx)$
E <sub>1</sub>	{1 1}	$\epsilon^*$	$\epsilon^{2*}$	$\epsilon^2$		
	{1 1}	$\epsilon^2$	$\epsilon^*$	$\epsilon$		
E <sub>2</sub>	{1 1}	$\epsilon^{2*}$	$\epsilon$	$\epsilon^*$		
$C_6$	E	$C_6$	$C_3$	$C_2$	$C_{3^2}$	$C_{6^5}$
A	1	1	1	1	1	$T_z, R_z$
B	1	-1	1	-1	-1	$x^2+y^2, z^2$
	{1 1}	$\epsilon$	$-\epsilon^*$	-1	$-\epsilon$	$(T_x, T_y),$ $(R_x, R_y)$
E <sub>1</sub>	{1 1}	$\epsilon^*$	$-\epsilon$	-1	$-\epsilon^*$	$(yz, zx)$
	{1 1}	$-\epsilon^*$	$-\epsilon$	1	$-\epsilon$	$(x^2-y^2, xy)$
E <sub>2</sub>	{1 1}	$-\epsilon$	$-\epsilon^*$	1	$-\epsilon$	$(x^2-y^2, xy)$

*The  $C_n$  Groups (continued)*

$C_7$	E	$C_7$	$C_{7^2}$	$C_{7^3}$	$C_{7^4}$	$C_{7^5}$	$C_{7^6}$	$\epsilon = \exp(2\pi i/7)$
A	1	1	1	$\epsilon^3$	$\epsilon^{3*}$	1	1	$T_z, R_z$ $(T_x, T_y),$ $(R_x, R_y)$
$E_1$	{1 1 1 1 1 1 1}	$\epsilon$ $\epsilon^*$ $\epsilon^{2*}$ $\epsilon^3*$ $\epsilon^2*$ $\epsilon^3$ $\epsilon^*$	$\epsilon^2$ $\epsilon^3$ $\epsilon^3*$ $\epsilon^*$ $\epsilon^3$ $\epsilon^2$ $\epsilon$	$\epsilon^3$ $\epsilon^3*$ $\epsilon^2$ $\epsilon^*$ $\epsilon^2$ $\epsilon^2*$ $\epsilon^2$	$\epsilon^{2*}$ $\epsilon^2$ $\epsilon^3*$ $\epsilon^3*$ $\epsilon^2*$ $\epsilon^2$ $\epsilon^*$	$\epsilon$ $\epsilon^2$ $\epsilon^3*$ $\epsilon^3$	$T_z, R_z$ $(T_x, T_y),$ $(R_x, R_y)$	$x^2 + y^2, z^2$ $(yz, zx)$ $(x^2 - y^2, xy)$
$E_2$	{1 1 1 1 1 1 1}	$\epsilon^2*$ $\epsilon^3$ $\epsilon^*$ $\epsilon^3*$ $\epsilon^2*$ $\epsilon^2$ $\epsilon$	$\epsilon^3$ $\epsilon^2$ $\epsilon$ $\epsilon^3$ $\epsilon^2*$ $\epsilon^2$ $\epsilon^*$	$\epsilon^3*$ $\epsilon^2$ $\epsilon^*$ $\epsilon^2*$ $\epsilon^2$ $\epsilon^*$ $\epsilon^3*$	$\epsilon^2*$ $\epsilon$ $\epsilon^3*$ $\epsilon^2$ $\epsilon^*$ $\epsilon^2$ $\epsilon^3$	$\epsilon$ $\epsilon^2$ $\epsilon^3*$ $\epsilon^3$	$T_z, R_z$ $(T_x, T_y),$ $(R_x, R_y)$	$x^2 + y^2, z^2$ $(yz, zx)$ $(x^2 - y^2, xy)$
$C_8$	E	$C_8$	$C_4$	$C_2$	$C_{4^3}$	$C_{8^3}$	$C_{8^5}$	$C_{8^7}$
A	1	1	1	1	1	1	1	$\epsilon = \exp(2\pi i/8)$
B	-1	1	1	1	-1	-1	-1	$x^2 + y^2, z^2$
$E_1$	{1 1 1 1 1 1 1 1}	$\epsilon$ $\epsilon^*$ $i$ $i$ $-i$ $-i$ $-i$ $-i$	$i$ $-i$ $-i$ $1$ $-1$ $1$ $-1$ $1$	$-1$ $-i$ $i$ $-1$ $-1$ $1$ $-1$ $1$	$-\epsilon^*$ $-\epsilon$ $-\epsilon^*$ $-\epsilon$ $-\epsilon^*$ $-\epsilon$ $-\epsilon^*$ $-\epsilon$	$\epsilon^*$ $\epsilon$ $\epsilon^*$ $\epsilon$ $\epsilon^*$ $\epsilon$ $\epsilon^*$ $\epsilon$	$T_z, R_z$ $(T_x, T_y),$ $(R_x, R_y)$	$x^2 + y^2, z^2$ $(yz, zx)$ $(x^2 - y^2, xy)$
$E_2$	{1 1 1 1 1 1 1 1}	$-i$ $-i$ $-i$ $-i$ $-i$ $-i$ $-i$ $-i$	$1$ $1$ $1$ $1$ $1$ $1$ $1$ $1$	$-1$ $-1$ $-1$ $-1$ $-1$ $-1$ $-1$ $-1$	$i$ $i$ $i$ $i$ $i$ $i$ $i$ $i$	$-i$ $-i$ $-i$ $-i$ $-i$ $-i$ $-i$ $-i$	$T_z, R_z$ $(T_x, T_y),$ $(R_x, R_y)$	$x^2 + y^2, z^2$ $(yz, zx)$ $(x^2 - y^2, xy)$
$E_3$	{1 1 1 1 1 1 1 1}	$-\epsilon$ $-\epsilon^*$ $-i$ $-i$ $-1$ $-1$ $-1$ $-1$	$i$ $i$ $-i$ $-i$ $-1$ $-1$ $-1$ $-1$	$1$ $1$ $1$ $1$ $1$ $1$ $1$ $1$	$\epsilon^*$ $\epsilon$ $\epsilon^*$ $\epsilon^*$ $\epsilon$ $\epsilon$ $\epsilon^*$ $\epsilon^*$	$\epsilon$ $\epsilon^*$ $\epsilon$ $\epsilon^*$ $\epsilon$ $\epsilon$ $\epsilon^*$ $\epsilon^*$	$T_z, R_z$ $(T_x, T_y),$ $(R_x, R_y)$	$x^2 + y^2, z^2$ $(yz, zx)$ $(x^2 - y^2, xy)$

INTRODUCTORY GROUP THEORY FOR CHEMISTS

*The  $D_n$  Groups*

$D_2$	E	$C_2(z)$	$C_2(y)$	$C_2(x)$		
A	1	1	1	1	$T_z, R_z$	$x^2, y^2, z^2$
B <sub>1</sub>	1	1	-1	-1	$T_y, R_y$	$xy$
B <sub>2</sub>	1	-1	1	-1	$T_x, R_x$	$zx$
B <sub>3</sub>	1	-1	-1	1		$yz$
$D_3$	E	$2C_3$	$3C_2$			
A <sub>1</sub>	1	1	1	$T_z, R_z$	$x^2+y^2, z^2$	
A <sub>2</sub>	1	1	-1	$(T_x, T_y), (R_x, R_y)$	$(x^2-y^2, xy)$	
E	2	-1	0		$(yz, zx)$	
$D_4$	E	$2C_4$	$C_2 (= C_4^2)$	$2C'_2$	$2C''_2$	
A <sub>1</sub>	1	1	1	1	1	$x^2+y^2, z^2$
A <sub>2</sub>	1	1	1	-1	-1	$T_z, R_z$
B <sub>1</sub>	1	-1	1	1	-1	$x^2-y^2$
B <sub>2</sub>	1	-1	1	-1	1	$xy$
E	2	0	-2	0	0	$(yz, zx)$

APPENDIX

*The  $D_n$  Groups (continued)*

$D_5$	E	$2C_5$	$2C_{5^2}$	$5C_2$	
$A_1$	1	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	
$E_1$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	
$E_2$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	

$D_6$	E	$2C_6$	$2C_3$	$C_2$	$3C_2'$	$3C_2''$	
$A_1$	1	1	1	1	1	1	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	-1	-1	
$B_1$	1	-1	1	-1	1	-1	
$B_2$	1	-1	1	-1	-1	1	
$E_1$	2	1	-1	-2	0	0	
$E_2$	2	-1	-1	2	0	0	

## INTRODUCTORY GROUP THEORY FOR CHEMISTS

*The  $C_{nv}$  Groups*

$C_{2v}$	E	$C_2$	$\sigma_v(xz)$	$\sigma_v(yz)$		
$A_1$	1	1	1	1	$T_z$	$x^2, y^2, z^2$
$A_2$	1	1	-1	-1	$R_z$	$xy$
$B_1$	1	-1	1	-1	$T_x, R_y$	$zx$
$B_2$	1	-1	-1	1	$T_y, R_x$	$yz$
$C_{3v}$	E	$2C_3$	$3\sigma_v$			
$A_1$	1	1	1	$T_z$	$x^2 + y^2, z^2$	
$A_2$	1	1	-1	$R_z$		
$E$	2	-1	0	$(T_x, T_y), (R_x, R_y)$	$(x^2 - y^2, xy), (yz, zx)$	
$C_{4v}$	E	$2C_4$	$C_2$	$2\sigma_v$	$2\sigma_d$	
$A_1$	1	1	1	1	$T_z$	$x^2 + y^2, z^2$
$A_2$	1	1	1	-1	$R_z$	
$B_1$	1	-1	1	1	-1	$x^2 - y^2$
$B_2$	1	-1	1	-1	1	$xy$
$E$	2	0	-2	0	0	$(yz, zx)$

*The  $C_{nv}$  Groups (continued)*

$C_{5v}$	E	$2C_5$	$2C_5^2$	$3\sigma_v$	
$A_1$	1	1	1	1	$T_z$
$A_2$	1	1	1	-1	$R_z$
$B_1$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(T_x, T_y), (R_x, R_y)$
$E_2$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	$(yz, zx)$ $(x^2 - y^2, xy)$

$C_{6v}$	E	$2C_6$	$2C_3$	$C_2$	$3\sigma_v$	$3\sigma_d$	
$A_1$	1	1	1	1	1	1	$T_z$
$A_2$	1	1	1	1	-1	-1	$R_z$
$B_1$	1	-1	1	-1	1	-1	
$B_2$	1	-1	1	-1	-1	1	$(T_x, T_y), (R_x, R_y)$
$E_1$	2	1	-1	-2	0	0	$(yz, zx)$ $(x^2 - y^2, xy)$
$E_2$	2	-1	-1	2	0	0	

APPENDIX

## INTRODUCTORY GROUP THEORY FOR CHEMISTS

The  $C_{nh}$  Groups

$C_{2h}$	E	$C_2$	i	$\sigma_h$	$R_z$	$R_{x,y}$	$x^2, y^2, z^2, xy$
$A_g$	1	1	1	1	$R_z$	$R_{x,y}$	$y, z$
$B_g$	1	-1	1	-1	$R_{x,y}$	$R_z$	$x, y$
$A_u$	1	1	-1	-1	$T_z$	$T_x, T_y$	$x, y$
$B_u$	1	-1	-1	1	$T_x, T_y$	$T_z$	$x, y$

$C_{3h}$	E	$C_3$	$C_{3^2}$	$\sigma_h$	$S_3$	$S_{3^2}$	$\epsilon = \exp(2\pi i/3)$
$A'$	1	1	1	1	1	$\epsilon^*$	$x^2 + y^2, z^2$
$E'$	{1}	$\epsilon^*$	$\epsilon^*$	1	$\epsilon^*$	$\epsilon$	$(x^2 - y^2, xy)$
$A''$	{1}	1	1	-1	-1	-1	$(x^2, y)$
$E''$	{1}	$\epsilon^*$	$\epsilon^*$	-1	$\epsilon$	$\epsilon^*$	$(yz, zx)$

$C_{4h}$	E	$C_4$	$C_2$	$C_{4^3}$	$i$	$S_{4^3}$	$\sigma_h$	$S_4$	$\epsilon = \exp(2\pi i/4)$
$A_g$	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$
$B_g$	1	-1	1	-1	1	-1	1	-1	$x^2 - y^2, xy$
$E_g$	{1}	$i$	-1	$-i$	1	1	-1	$-i$	$(yz, zx)$
$A_u$	{1}	1	1	1	-1	-1	-1	-1	$(R_x, R_y)$
$B_u$	{1}	-1	1	-1	-1	1	-1	1	$T_z$
$E_u$	{1}	$i$	-1	$-i$	-1	1	1	$-i$	$(T_x, T_y)$

APPENDIX

The  $C_{nh}$  Groups (continued)

$C_{5h}$	E	$C_5$	$C_{5^2}$	$C_{5^3}$	$C_{5^4}$	$\sigma_h$	$S_5$	$S_{5^7}$	$S_{5^3}$	$S_{5^9}$	$R_z$	$\epsilon = \exp(2\pi i/5)$
$A'$	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$	
$E'_1$	{1}	$\epsilon$	$\epsilon^2$	$\epsilon^{2*}$	$\epsilon$	$\epsilon^*$	$\epsilon$	$\epsilon^2$	$\epsilon^{2*}$	$\epsilon$	$(T_x, T_y)$	
$E'_2$	{1}	$\epsilon^*$	$\epsilon^{2*}$	$\epsilon^2$	$\epsilon$	$\epsilon^{2*}$	$\epsilon^2$	$\epsilon^*$	$\epsilon$	$\epsilon^{2*}$	$(x^2 - y^2, xy)$	
$A''$	{1}	1	1	1	1	-1	-1	-1	-1	-1	$T_z$	
$E''_1$	{1}	$\epsilon$	$\epsilon^2$	$\epsilon^{2*}$	$\epsilon^2$	$\epsilon^{2*}$	$\epsilon$	$\epsilon^*$	$\epsilon$	$\epsilon^{2*}$	$(R_x, R_y)$	
$E''_2$	{1}	$\epsilon^*$	$\epsilon^{2*}$	$\epsilon^2$	$\epsilon$	$\epsilon$	-1	$\epsilon^2$	$\epsilon^{2*}$	$\epsilon$	$(yz, zx)$	
											$y^2, z^2$	

$C_{6h}$	E	$C_6$	$C_3$	$C_2$	$C_{3^2}$	$C_{6^5}$	i	$S_{3^5}$	$S_{6^5}$	$\sigma_h$	$S_6$	$S_3$	$\epsilon = \exp(2\pi i/6)$	
$A_g$	1	1	1	1	1	1	1	1	1	1	1	1	$x^2 + y^2, z^2$	
$B_g$	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	-1	$R_z$	
$E_{1g}$	{1}	$\epsilon$	$\epsilon^*$	$\epsilon^{2*}$	$\epsilon^{-1}$	$\epsilon^*$	$\epsilon^*$	$\epsilon$	$\epsilon^*$	$\epsilon$	$\epsilon^*$	$\epsilon$	$(yz, zx)$	
$E_{2g}$	{1}	$\epsilon^*$	$\epsilon^{-1}$	$\epsilon^{-1}$	$\epsilon^{-1}$	$\epsilon^*$	$\epsilon^*$	$\epsilon^*$	$\epsilon^*$	$\epsilon$	$\epsilon^*$	$\epsilon$	$(x^2 - y^2, xy)$	
$A_u$	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	$T_z$	
$B_u$	-1	1	-1	-1	-1	1	-1	1	-1	1	-1	1	$(Tx, Ty)$	
$E_{1u}$	{1}	$\epsilon$	$\epsilon^*$	$\epsilon^{-1}$	$\epsilon^{-1}$	$\epsilon^*$	$\epsilon^*$	$\epsilon$	$\epsilon^*$	$\epsilon$	$\epsilon^*$	$\epsilon$	$(y^2, z^2)$	
$E_{2u}$	{1}	$\epsilon^*$	$\epsilon^{-1}$	$\epsilon^{-1}$	$\epsilon^{-1}$	$\epsilon^*$	$\epsilon$	$\epsilon^*$	$\epsilon$	$\epsilon^*$	$\epsilon$	$\epsilon$	$x^2 + y^2, z^2$	

INTRODUCTORY GROUP THEORY FOR CHEMISTS

The  $D_{nh}$  Groups

$D_{2h}$	E	$C_2(z)$	$C_2(y)$	$C_2(x)$	i	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$	
$A_g$	1	1	1	1	1	1	1	1	$x^2, y^2, z^2$
$B_{1g}$	1	1	-1	-1	1	1	-1	-1	$xy$
$B_{2g}$	1	-1	1	-1	1	-1	1	-1	$zx$
$B_{3g}$	1	-1	-1	1	1	-1	-1	1	$yz$
$A_u$	1	1	1	1	-1	-1	-1	-1	$R_z$
$B_{1u}$	1	1	-1	-1	-1	-1	1	1	$R_y$
$B_{2u}$	1	-1	1	-1	-1	1	1	1	$R_x$
$B_{3u}$	1	-1	-1	1	-1	1	-1	1	$T_z$
									$T_y$
									$T_x$

$D_{3h}$	E	$2C_3$	$3C_2$	$\sigma_h$	$2S_3$	$3\sigma_v$		
$A'_1$	1	1	1	1	1	1	$x^2+y^2, z^2$	
$A''_2$	1	1	-1	1	-1	-1		
$E'$	2	-1	0	2	-1	0	$(T_x T_y)$	
$A''_1$	1	1	1	-1	-1	-1	$(x^2-y^2, xy)$	
$A''_2$	1	1	-1	-1	-1	1	$T_z$	
$E''$	2	-1	0	-2	1	0	$(R_x, R_y)$	$(yz, zx)$

APPENDIX

The  $D_{nh}$  Groups (continued)

$D_{4h}$	E	$2C_4$	$C_2$	$2C_2'$	$2C_2''$	i	$2S_4$	$\sigma_h$	$2\sigma_v$	$2\sigma_d$	
$A_{1g}$	1	1	1	1	1	1	1	1	1	1	$x^2+y^2, z^2$
$A_{2g}$	1	1	-1	-1	1	1	-1	-1	-1	-1	$x^2-y^2$
$B_{1g}$	1	-1	1	-1	1	-1	1	-1	-1	-1	$xy$
$B_{2g}$	1	-1	1	-1	1	-1	1	-1	0	0	$(yz, zx)$
$E_g$	2	0	-2	0	0	2	-2	0	-1	-1	$(R_x, R_y)$
$A_{1u}$	1	1	1	*1	1	-1	-1	-1	-1	-1	$T_z$
$A_{2u}$	1	1	1	-1	-1	-1	-1	-1	1	1	
$B_{1u}$	1	-1	1	1	-1	-1	1	-1	-1	1	
$B_{2u}$	1	-1	1	-1	1	-1	1	-1	-1	-1	
$E_u$	2	0	-2	0	0	-2	0	2	0	0	$(T_x, T_y)$

$D_{5h}$	E	$2C_5$	$2C_5^2$	$5C_2$	$\sigma_h$	$2S_5$	$2S_5^3$	$2S_5^5$	$5\sigma_v$	$5\sigma_d$	
$A_1'$	1	1	1	1	1	1	1	1	1	1	$x^2+y^2, z^2$
$A_2'$	1	1	1	-1	1	1	1	1	-1	-1	$R_z$
$E_1'$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	$2 \cos 144^\circ$	0	0	$(T_x, T_y)$
$E_2'$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	$2 \cos 72^\circ$	0	0	$(x^2-y^2, xy)$
$A_1''$	1	1	1	1	-1	-1	-1	-1	-1	-1	$T_z$
$A_2''$	1	1	1	-1	-1	-1	-1	-1	1	1	$(R_x, R_y)$
$E_1''$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	-2	$-2 \cos 72^\circ$	$-2 \cos 144^\circ$	$-2 \cos 144^\circ$	0	0	$(yz, zx)$
$E_2''$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	-2	$-2 \cos 144^\circ$	$-2 \cos 72^\circ$	$-2 \cos 72^\circ$	0	0	

## INTRODUCTORY GROUP THEORY FOR CHEMISTS

<i>D<sub>6h</sub></i>	E	2C <sub>6</sub>	2C <sub>3</sub>	C <sub>2</sub>	3C <sub>2'</sub>	3C <sub>2''</sub>	i	2S <sub>3</sub>	2S <sub>6</sub>	σ <sub>h</sub>	3σ <sub>d</sub>	3σ <sub>u</sub>	
A <sub>1g</sub>	1	1	1	1	1	-1	1	1	1	1	1	1	R <sub>z</sub>
A <sub>2g</sub>	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	
B <sub>1g</sub>	1	-1	1	-1	-1	1	-1	1	-1	-1	1	-1	
B <sub>2g</sub>	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1	
E <sub>1g</sub>	2	1	-1	-2	0	0	2	1	-1	-2	0	0	(R <sub>x</sub> , R <sub>y</sub> )
E <sub>2g</sub>	2	-1	-1	2	0	0	2	-1	-1	2	0	0	(yz, zx) (x <sup>2</sup> -y <sup>2</sup> , xy)
A <sub>1u</sub>	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	(R <sub>x</sub> , R <sub>y</sub> )
A <sub>2u</sub>	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	T <sub>z</sub>
B <sub>1u</sub>	1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	
B <sub>2u</sub>	1	-1	1	-1	-1	1	-1	1	-1	1	-1	-1	
E <sub>1u</sub>	2	1	-1	-2	0	0	-2	-1	1	2	0	0	(T <sub>x</sub> , T <sub>y</sub> )
E <sub>2u</sub>	2	-1	-1	2	0	0	-2	1	1	-2	0	0	

The D<sub>nh</sub> Groups (continued)

APPENDIX

The  $D_{nd}$  Groups

$D_{2d}$	E	$2S_4$	$C_2$	$2C_2'$	$2\sigma_d$		
$A_1$	1	1	1	1	1	$R_z$	$x^2+y^2, z^2$
$A_2$	1	1	-1	-1	-1		
$B_1$	1	-1	1	-1	-1	$T_z$	$x^2-y^2$
$B_2$	1	-1	1	-1	1	$(T_x, T_y)$	$xy$
$E$	2	0	-2	0	0	$(R_x, R_y)$	$(yz, zx)$
$D_{3d}$	E	$2C_3$	$3C_2 \cdot i$	$2S_6$	$3\sigma_d$		
$A_{1g}$	1	1	1	1	1	$R_z$	$x^2+y^2, z^2$
$A_{2g}$	1	1	-1	1	-1	$(R_x, R_y)$	
$E_g$	2	-1	0	2	-1		
$A_{1u}$	1	1	1	-1	-1	$T_z$	$(x^2-y^2, xy) (yz, zx)$
$A_{2u}$	1	1	-1	-1	1	$(T_x, T_y)$	
$E_u$	2	-1	0	-2	1	$(R_x, R_y)$	
$D_{4d}$	E	$2S_8$	$2C_4$	$2S_83$	$C_2$	$4C_2'$	$4\sigma_d$
$A_1$	1	1	1	1	1	1	$x^2+y^2, z^2$
$A_2$	1	1	1	1	-1	-1	
$B_1$	1	-1	1	-1	1	-1	$R_z$
$B_2$	1	-1	1	-1	1	-1	
$E_1$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	0	$T_z$
$E_2$	2	0	-2	0	2	0	$(T_x, T_y)$
$E_3$	2	$-\sqrt{2}$	0	$\sqrt{2}$	-2	0	$(R_x, R_y)$
						0	$(x^2-y^2, xy) (yz, zx)$

INTRODUCTORY GROUP THEORY FOR CHEMISTS

The  $D_{nd}$  Groups (continued)

$D_{5d}$	E	$2C_5$	$2C_{5^2}$	$5C_2$	i	$2S_{10^3}$	$2S_{10}$	$5\sigma_d$	
$A_{1g}$	1	1	1	1	1	1	1	1	$x^2+y^2, z^2$
$A_{2g}$	1	1	-1	1	1	1	-1	$R_z$	$(R_x, R_y)$
$E_{1g}$	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	2	$2 \cos 72^\circ$	$2 \cos 144^\circ$	0	$(yz, zx)$
$E_{2g}$	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	2	$2 \cos 144^\circ$	$2 \cos 72^\circ$	0	$(x^2-y^2, xy)$
$D_{6d}$	E	$2S_{12}$	$2C_6$	$2S_4$	$2C_3$	$2S_{12^5}$	$C_2$	$6C_2'$	$6\sigma_d$
$A_1$	1	1	1	1	1	1	1	1	$x^2+y^2, z^2$
$A_2$	1	1	1	1	1	1	-1	-1	$R_z$
$B_1$	1	-1	1	-1	1	-1	1	-1	$T_z$
$B_2$	1	-1	1	-1	1	-1	-1	1	$(T_x, T_y)$
$E_1$	2	$\sqrt{3}$	1	0	-1	$-\sqrt{3}$	-2	0	$(x^2-y^2, xy)$
$E_2$	2	1	-1	-2	-1	1	2	0	$(x^2-y^2, xy)$
$E_3$	2	0	-2	0	2	0	-2	0	$(x^2-y^2, xy)$
$E_4$	2	-1	-1	2	-1	-1	2	0	$(R_x, R_y)$
$E_5$	2	$-\sqrt{3}$	1	0	-1	$\sqrt{3}$	-2	0	$(yz, zx)$

*The  $S_n$  Groups*

$S_4$	E	$S_4$	$C_2$	$S_{4^3}$						
A	1	1	1	1				$R_z$	$x^2+y^2, z^2$	
B	1	-1	1	-1				$T_z$	$x^2-y^2, xy$	
	{1}	i	-1	-i					$(T_x, T_y), (R_x, R_y)$	
E	{1}	-i	-1	i					$(yz, zx)$	
$S_6$	E	$C_3$	$C_{3^2}$	i	$S_6^5$	$S_6$			$\epsilon = \exp(2\pi i/3)$	
$A_g$	1	1	1	1	1	1		$R_z$	$x^2+y^2, z^2$	
$E_g$	{1}	$\epsilon^*$	$\epsilon^*$	1	$\epsilon^*$	$\epsilon^*$		$(R_x, R_y)$	$(x^2-y^2, xy)$	
$A_u$	{1}	1	1	-1	-1	-1		$T_z$	$(yz, zx)$	
$E_u$	{1}	$\epsilon^*$	$\epsilon^*$	-1	$-\epsilon$	$-\epsilon^*$		$(T_x, T_y)$	$(x^2-y^2, xy)$	
$S_8$	E	$S_8$	$C_4$	$S_8^3$	$C_2$	$S_8^5$	$C_4^3$	$S_8^7$	$\epsilon = \exp(2\pi i/8)$	
A	1	1	1	1	1	1	1	1	$x^2+y^2, z^2$	
B	1	-1	1	-1	1	-1	1	-1	$(T_x, T_y), (R_x, R_y)$	
$E_1$	{1}	$\epsilon^*$	i	$-\epsilon^*$	-1	$-\epsilon$	$-\epsilon^*$	$\epsilon^*$	$(R_x, R_y)$	
$E_2$	{1}	i	-1	-i	-1	i	i	-i	$(x^2-y^2, xy)$	
$E_3$	{1}	-i	-1	i	$\epsilon^*$	-1	$\epsilon^*$	i	$(yz, zx)$	

INTRODUCTORY GROUP THEORY FOR CHEMISTS

The Cubic Groups

$T_d$	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$		$x^2+y^2+z^2$
$A_1$	1	1	1	1	1		
$A_2$	1	1	1	-1	-1		
E	2	-1	2	0	0		$(2z^2-x^2-y^2, x^2-y^2)$
$T_1$	3	0	-1	1	-1	$(R_x, R_y, R_z)$	
$T_2$	3	0	-1	-1	1	$(T_x, T_y, T_z)$	$(xy, yz, zx)$
$O_h$	E	$8C_3$	$6C_2$	$3C_2 (= C_4^2)$	i	$6S_4$	$8S_6$
$A_{1g}$	1	1	1	1	1	1	1
$A_{2g}$	1	1	-1	-1	-1	1	-1
$E_g$	2	-1	0	0	2	0	-1
$T_{1g}$	3	0	-1	1	-1	3	1
$T_{2g}$	3	0	1	-1	-1	3	-1
$A_{1u}$	1	1	1	1	-1	-1	-1
$A_{2u}$	1	1	-1	-1	1	-1	1
$E_u$	2	-1	0	0	2	-2	0
$T_{1u}$	3	0	-1	1	-1	-3	0
$T_{2u}$	3	0	1	-1	-1	-3	1
						1	0
						0	1
						-1	-1

$x^2+y^2+z^2$	$(2z^2-x^2-y^2, x^2-y^2)$	$(R_x, R_y, R_z)$	$(xy, yz, zx)$	$(T_x, T_y, T_z)$

*The Groups  $C_{\infty v}$  and  $D_{\infty h}$  for Linear Molecules*

$C_{\infty v}$	E	$2C_{\infty}^{\Phi}$	...	$\infty \sigma_v$	$T_z$	$R_z$	$x^2+y^2, z^2$
$A_1 = \Sigma +$	1	1	...	1	1	...	
$A_2 = \Sigma -$	1	1	...	-1	1	...	
$E_1 = \Pi$	2	$2 \cos \Phi$	...	0	-2 cos $\Phi$	...	$(yz, zx)$
$E_2 = \Delta$	2	$2 \cos 2\Phi$	...	0	$2 \cos 2\Phi$	...	$(x^2-y^2, xy)$
$E_3 = \Phi$	2	$2 \cos 3\Phi$	...	0	$2 \cos 3\Phi$	...	
...	...	...	...	...	...	...	

$D_{\infty h}$	E	$2C_{\infty}^{\Phi}$	...	$\infty \sigma_v$	i	$2S_{\infty}^{\Phi}$	...	$\infty C_2$	$x^2+y^2, z^2$
$\Sigma_g +$	1	1	...	1	1	1	...	1	
$\Sigma_g^-$	1	1	...	-1	1	1	...	1	
$\Pi_g$	2	$2 \cos \Phi$	...	0	2	-2 cos $\Phi$	...	0	$(yz, zx)$
$\Delta_g$	2	$2 \cos 2\Phi$	...	0	2	$2 \cos 2\Phi$	...	0	$(x^2-y^2, xy)$
...	...	...	...	...	...	...	...	...	
$\Sigma_u +$	1	1	...	1	-1	-1	...	-1	
$\Sigma_u^-$	1	1	...	-1	-1	-1	...	1	
$\Pi_u$	2	$2 \cos \Phi$	...	0	-2	2 cos $\Phi$	...	0	$(T_x, T_y)$
$\Delta_u$	2	$2 \cos 2\Phi$	...	0	-2	$2 \cos 2\Phi$	...	0	$(T_x, T_y)$
...	...	...	...	...	...	...	...	...	

**Reducing Formula**

$$\mathbf{a}_i = 1/h \sum \mathbf{X}^s_R \mathbf{X}^s_i \mathbf{N}^s$$

$h$  ; total number of operations in certain point group.

$\mathbf{X}^s_R$  ; Character ( $\mathbf{X}$ ) for reducible representation.

$\mathbf{X}^s_i$  ; Character ( $\mathbf{X}$ ) for reducible representation (from the character Table)

$\mathbf{N}^s$  ; Number of symmetry operation for each type or class of operation.

**Contribution for the Character,  $\chi(R)$ , for each unshifted atom in  $\Gamma_{3N}$**

---

R	$\chi(R)$
E	+3
i	-3
$\sigma$	+1
$C_2$	-1
$C_3^1, C_3^2$	0
$C_4^1, C_4^3$	+1
$C_6^1, C_6^5$	+2
$S_3^1, S_3^5$	-2
$S_4^1, S_4^3$	-1
$S_6^1, S_6^5$	0

## Notations of the Character Table

a	b		
f	c	d	e

- a. Schoenflies symbols for point group
- b. lists the symmetry operations (by classes) for that group
- c. lists all the characters, for all irreducible representations, of each class of each operation
- d. shows the irreducible representations for which the six vectors ,Tx, Ty, Tz, Rx, Ry, Rz, provide the bases
- e. shows the funtions which are binary combinations of x, y, z (e.g. xy,  $z^2$ ) provide bases for certain irreducible representations
- f. lists conventional symbols for the irreducible representations called *Mulliken symbols*. All one -dimentional irreducible rep. Are labelled as A or B, all two-dimentional as E, all three-dimentional as T (in certain texts it is given the label F), four-dimentional as G and five-dimentional as H.

In addition to the letter, most Mulliken symbols possess certain subscripts and/or superscripts. For two- and higher – dimentional irreducible representations they can be regarded as labels. For one-dimentional representations, they have the following specifications.

A : One-dimentional irreducible rep. if it is symmetry about Cn axis, i.e. ( $\chi = +1$ )

B :       "       "       "       "       antisymm.       "       ( $\chi = -1$ )

Sub.<sub>1</sub> : Irr. Rep is symmetry with respect to  $C_2 \perp C_n$  (if no  $C_2$ ), then

Irr. Rep. Is symmetry with respect to  $\sigma_v$

Sub.<sub>2</sub> : Irr. Rep is antisymmetry under conditions as those in Sub<sub>1</sub> of above.

Sub.<sub>g</sub> : (gerade) irr. rep. are symm. With respect to inversion at an i

Sub.<sub>u</sub> : (ungerade) irr. rep. are antisymm. with respect to an i

' : irr. Rep are symm with respect to reflection in a  $\sigma_h$

'' : irr. Rep. Are antisymm with respect to reflection in a  $\sigma_h$

