UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

MSS 212 – Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all nine** [9] questions.

[Arahan: *Jawab* ***semua sembilan*** *[9] soalan.]*

1. Solve the simultaneous equations using Cramer's Rule.

$$x_1 + x_2 + x_3 + x_4 = 2$$
$$x_1 + 2x_2 + 3x_3 + 4x_4 = 2$$
$$2x_1 + 3x_2 + 5x_3 + 9x_4 = 2$$
$$x_1 + x_2 + 2x_3 + 7x_4 = 2$$

[20 marks]

2. (a) Let

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 4 & 2 & -3 \\ 5 & 3 & 1 \end{pmatrix}$$

(i) Find all the cofactors of A.

[9 marks]

(ii) Find A^{-1}.

[6 marks]

(b) If B is an $n \times n$ nonsingular matrix, show that $|B^{-1}| = |B|^{-1}$.

[6 marks]

3. Determine whether the following sets are subspaces of the vector space $\mathbb{R}^3$.

(a) $W_1 = \{a, b, -a \mid a, b \in \mathbb{R}\}$

(b) $W_2 = \{(a+1, b, 0) \mid a, b \in \mathbb{R}\}$

(c) $W_3 = \{(a, a, 0) \mid a \in \mathbb{R}\}$

(d) $W_4 = \{a+b, b, c \mid a, b, c \in \mathbb{R}\}$

(e) $W_5 = W_3 \cap W_4$

[20 marks]

4. Suppose $f : V \to U$ is a linear transformation. Suppose V has finite dimension and $\dim(V) = \dim(U)$. Show that F is an isomorphism if and only if $Ker(F) = \{\underset{\sim}{0}\}$.

[14 marks]

5. Let $M_{3\times2}(\mathbb{C})$ be a vector space of 3×2 matrices over $\mathbb{C}$ and let $P_5(\mathbb{C})$ be a vector space of polynomials of degree at most 5 over $\mathbb{C}$. Construct an isomorphism to show that $M_{3\times2}(\mathbb{C})$ is isomorphic to $P_5(\mathbb{C})$.

[14 marks]

1. *Selesaikan persamaan serentak dengan menggunakan Petua Cramer.*

$$x_1 + x_2 + x_3 + x_4 = 2$$
$$x_1 + 2x_2 + 3x_3 + 4x_4 = 2$$
$$2x_1 + 3x_2 + 5x_3 + 9x_4 = 2$$
$$x_1 + x_2 + 2x_3 + 7x_4 = 2$$

[20 markah]

2. *(a) Biar*

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 4 & 2 & -3 \\ 5 & 3 & 1 \end{pmatrix}$$

(i) Cari kesemua kofaktor bagi A.

[9 markah]

(ii) Cari A^{-1}.

[6 markah]

(b) Jika B suatu matriks $n \times n$ tidak singular, tunjukkan $\left|B^{-1}\right| = |B|^{-1}$.

[6 markah]

3. *Tentukan samada set-set berikut merupakan subruang kepada ruang vektor* $\mathbb{R}^3$

(a) $W_1 = \{a, b, -a \mid a, b \in \mathbb{R}\}$

(b) $W_2 = \{(a+1, b, 0) \mid a, b \in \mathbb{R}\}$

(c) $W_3 = \{(a, a, 0) \mid a \in \mathbb{R}\}$

(d) $W_4 = \{a+b, b, c \mid a, b, c \in \mathbb{R}\}$

(e) $W_5 = W_3 \cap W_4$

[20 markah]

4. *Andai $f : V \to U$ suatu transformasi linear. Andai V mempunyai dimensi terhingga dan* $\dim(V) = \dim(U)$. *Tunjukkan bahawa F adalah suatu isomorfisma jika dan hanya jika Inti* $(F) = \{\underset{\sim}{0}\}$.

[14 markah]

5. *Biar $M_{3\times2}(\mathbb{C})$ suatu ruang vektor bagi matriks 3×2 di atas $\mathbb{C}$ dan biar $P_5(\mathbb{C})$ suatu ruang vektor bagi polinomial yang darjahnya tidak melebihi 5 di atas $\mathbb{C}$. Bina suatu isomorfisma untuk menunjukkan $M_{3\times2}(\mathbb{C})$ adalah isomorfik terhadap $P_5(\mathbb{C})$.*

[14 markah]

6. Let V be a vector space of 2×2 matrices. Let $M=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $S:V\to V$ be the linear transformation defined by $S(A)=AM$. Find the matrix representation of S relative to the standard basis of V.

[10 marks]

7. Let

$$A=\begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$$

(a) Determine whether A can be diagonalised.

[16 marks]

(b) Find the Jordan Canonical Form of A.

[4 marks]

8. Let T be the linear operator on $\mathbb{C}^3$ defined by

$$T(x,y,z)=\left(2x+(1-i)y,(3+2i)x-4iz,2ix+(4-3i)y-3z\right).$$

Find $T^*(x,y,z)$.

[10 marks]

9. Suppose T is a self adjoint linear operator of V and $T^2=0$. Show that $T=0$.

[12 marks]

6. *Biar V suatu ruang vektor bagi matriks 2×2. Biar $M=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ dan $S:V\to V$ suatu transformasi linear yang ditakrifkan sebagai $S(A)=AM$. Cari matriks perwakilan bagi S relatif kepada asas piawai bagi V.*

[10 markah]

7. *Biar*

$$A=\begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$$

(a) Tentukan samada A boleh dipepenjurukan.

[16 markah]

(b) Cari Bentuk Berkanun Jordan bagi A.

[4 markah]

8. *Biar T suatu operator linear pada $\mathbb{C}^3$ ditakrifkan sebagai*

$$T(x,y,z)=\left(2x+(1-i)y,(3+2i)x-4iz,2ix+(4-3i)y-3z\right).$$

Cari $T^*(x,y,z)$.

[10 markah]

9. *Andai T suatu linear operator swadampingan bagi V dan $T^2=0$. Tunjukkan bahawa $T=0$.*

[12 markah]

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