UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2008/2009

Jun 2009

MSS 212 – Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all eight** [8] questions.

[Arahan : *Jawab **semua lapan** [8] soalan.]*

1. Solve the simultaneous equations using Cramer's Rule.

$$2x_1 + x_2 + 5x_3 + x_4 = 5$$
$$x_1 + x_2 - 3x_3 - 4x_4 = -1$$
$$3x_1 + 6x_2 - 2x_3 + x_4 = 8$$
$$2x_1 + 2x_2 + 2x_3 - 3x_4 = 2$$

[20 marks]

2. Let

$$A = \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{pmatrix}.$$

(i) Find adjoint of A.

[10 marks]

(ii) Find A^{-1}.

[5 marks]

3. Suppose $F: V \to U$ is a linear transformation. Suppose V has finite dimension and $\dim(V) = \dim(U)$. Show that F is an isomorphism if and only if $Ker\ (F) = \{0\}$.

[10 marks]

4. Let V be a vector space of 2×2 matrices. Let $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $S: V \to V$ be the linear transformation defined by $S(A) = AM$. Find the matrix representation of S relative to the standard basis of V.

[10 marks]

1. *Selesaikan persamaan serentak dengan menggunakan Petua Cramer.*

$$2x_1 + x_2 + 5x_3 + x_4 = 5$$
$$x_1 + x_2 - 3x_3 - 4x_4 = -1$$
$$3x_1 + 6x_2 - 2x_3 + x_4 = 8$$
$$2x_1 + 2x_2 + 2x_3 - 3x_4 = 2$$

[20 markah]

2. *Biar*

$$A = \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 1 \end{pmatrix}.$$

(i) Cari adjoin bagi A.

[10 markah]

(ii) Cari A^{-1}.

[5 markah]

3. *Andai* $F : V \to U$ *suatu transformasi linear. Andai V mempunyai dimensi terhingga dan* $\dim(V) = \dim(U)$. *Tunjukkan bahawa F adalah suatu isomorfisma jika dan hanya jika Inti* $(F) = \{\underline{0}\}$.

[10 markah]

4. *Biar V suatu ruang vektor bagi matriks* 2×2. *Biar* $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *dan* $S : V \to V$ *suatu transformasi linear yang ditakrifkan sebagai* $S(A) = AM$. *Cari matriks perwakilan bagi S relatif kepada asas piawai bagi V.*

[10 markah]

5. Let

$$A=\begin{pmatrix}1 & -3 & 3\\ 3 & -5 & 3\\ 6 & -6 & 4\end{pmatrix}.$$

(a) Determine whether A can be diagonalised.

[16 marks]

(b) Find the Jordan Canonical Form of A.

[4 marks]

6. Let T be the linear operator on $\mathbb{C}^3$ defined by

$$T(x,y,x)=(2x+iy, y-5iz, x+(1-i)y+3z).$$

Find $T^*(x,y,z)$.

[10 marks]

7. Suppose $f:V\to U$ is a linear transformation. Show that the kernel of f is a subspace of V.

[10 marks]

8. Define a vector space isomorphism.

[5 marks]

5. *Biar*

$$A=\begin{pmatrix}1 & -3 & 3\\ 3 & -5 & 3\\ 6 & -6 & 4\end{pmatrix}.$$

(a) Tentukan samada A boleh dipepenjurukan.

[16 markah]

(b) Cari Bentuk Berkanun Jordan bagi A.

[4 markah]

6. *Biar T suatu operator linear pada* $\mathbb{C}^3$ *ditakrifkan sebagai*

$$T(x,y,x)=(2x+iy,y-5iz,x+(1-i)y+3z).$$

Cari $T^*(x,y,z)$.

[10 markah]

7. *Andai* $f:V\to U$ *suatu transformasi linear. Tunjukkan bahawa inti bagi f adalah suatu subruang bagi V.*

[10 markah]

8. *Takrifkan isomorfisma bagi suatu ruang vektor.*

[5 markah]

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