UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

MSG 389 – Engineering Computation II
[Pengiraan Kejuruteraan II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of ELEVEN pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEBELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan**: Jawab **semua empat** [4] soalan.]*

1. (a) A tank initially contains 100 gallons of fresh water. Then salt water containing 2 lbs of salt per gallon is pumped in at a rate of 4 gallons per minute, and the well-mixed mixture is allowed to leave at the same rate. How many pounds of salt there will be in the tank after 30 min?

[30 marks]

(b) A cup of coffee initially at $55^{\circ}C$ is brought into a room of temperature $20^{\circ}C$. After 5 minutes, it cools down to $50^{\circ}C$. Assuming that Newton's Law of Cooling hold, determine the initial value problem that *T(t)* satisfies and solve it.

(Newton's Law of Cooling implies that $\frac{dT}{dt} = -k(T-20)$, *T* is temperature, *t* is time *k* is a constant)

[30 marks]

(c) Use the 2nd order Runge Kutta method with step size, *h=1* to estimate $y(2)$ for the initial value problem

$$y' = xy, y(0) = 4$$
$$y_{i+1} = y_i + hk_2$$
$$k_1 = f(x_i, y_i)$$
$$k_2 = f\left(x_i + \frac{h}{2}, \frac{1}{2}hk_1\right)$$

[40 marks]

1. (a) *Sebuah tangki mengandungi 100 gelen air. Air garam yang mengandungi 2 Ibs garam setiap gelan dipamkan ke dalam tangki dengan kadar 4 gelen setiap minit dan campuran sekata dibenarkan mengalir keluar dengan kadar yang sama. Berapakah banyakkah garam yang tinggal dalam tangki itu selepas 30 minit?*

[30 markah]

(b) *Secawan kopi dengan suhu awal* $55^{\circ}C$ *disejukkan pada suhu bilik* $20^{\circ}C$. *Selepas 5 minit, kopi itu menyejuk kepada* $50^{\circ}C$. *Dengan membuat andaian Hukum Penyejukan Newton dipatuhi, tentukan masalah nilai awal yang terlibat dan selesaikannya.*

(Hukum Penyejukan Newton menyatakan $\frac{dT}{dt} = -k(T-20)$, *T adalah suhu, t adalah masa dan k adalah pemalar)*

[30 markah]

(c) *Gunakan kaedah Runge Kutta tertib kedua dengan saiz langkah h=1, bagi mengangarkan nilai untuk y(2) bagi persamaan nilai awai*

$$y' = xy, y(0) = 4$$
$$y_{i+1} = y_i + hk_2$$
$$k_1 = f(x_i, y_i)$$
$$k_2 = f\left(x_i + \frac{h}{2}, \frac{1}{2}hk_1\right)$$

[40 markah]

2. Solve the boundary value problem (BVP)

$$x'' - x = t + 1 \qquad x(0) = -1, \qquad x(1) = -2$$

(a) Find the exact solution of the BVP.

[20 marks]

(b) By using linear shooting method

(i) Find the exact solution $u = x(t)$ for

$$x'' - x = 1 + t \qquad x(0) = -1, \qquad x'(0) = 0$$

[20 marks]

(ii) Find the exact solution $v = x(t)$ for

$$x'' - x = 0 \qquad x(0) = 0, \qquad x'(0) = 1$$

[10 marks]

(iii) Then combine u and v to get the exact solution.

[30 marks]

(c) Solve the finite difference equations for the BVP with step size, $h = 1/2$ to get an approximation of $x(1/2)$. Find the error.

[20 marks]

2. *Selesaikan masalah nilai batasan (BVP)*

$$x'' - x = t + 1 \qquad x(0) = -1, \qquad x(1) = -2$$

(a) *Cari penyelesaian tepat bagi BVP.*

[20 markah]

(b) *Dengan menggunakan kaedah Tembakan Linear.*

(i) *Cari penyelesaian tepat u=x(t) untuk*

$$x'' - x = 1 + t \qquad x(0) = -1, \qquad x'(0) = 0$$

[20 markah]

(ii) *. Cari penyelesaian tepat v=x(t) untuk*

$$x'' - x = 0 \qquad x(0) = 0, \qquad x'(0) = 1$$

[10 markah]

(iii) *Kemudian, cantumkan u dan v dengan bagi mendapat penyelesaian tepat.*

[30 markah]

(c) *Selesaikan persamaan pembezaan terhingga untuk BVP dengan saiz langkah,* $h = 1/2$ *bagi mendapat anggaran untuk* $x(1/2)$. *Cari ralat yang wujud.*

[20 markah]

3. Consider the initial boundary value problem

$$u_t - u_{xx} = 0, \quad 0 < x < 1, \quad 0 < t$$
$$u(0,t) = u(1,t) = 0, \quad t > 0$$
$$u(x,0) = 10\sin(\pi x), \quad 0 \le x \le 1$$

(a) Write the Forward Time Centered Space (FTCS) scheme for this problem. Using the scheme, compute the value of u at $x = 0.4$ at $t = 0.2$. Use $\Delta x = 0.2$, $\Delta t = 0.1$.

[50 marks]

(b) Write the Forward Time Backward Space scheme for the problem in 3(a). Investigate the stability of the scheme using the Fourier method.

[50 marks]

3. *Pertimbang masalah nilai awal sempadan*

$$u_t - u_{xx} = 0, \quad 0 < x < 1, \quad 0 < t$$
$$u(0,t) = u(1,t) = 0, \quad t > 0$$
$$u(x,0) = 10\sin(\pi x), \quad 0 \le x \le 1$$

(a) *Tulis skema FTCS untuk masalah ini. Dengan menggunakan skema ini, kira nilai u di* $x = 0.4$ *pada masa* $t = 0.2$. *Guna* $\Delta x = 0.2$, $\Delta t = 0.1$.

[50 markah]

(b) *Tulis kaedah masa ke depan, ruang ke belakang untuk masalah dalam 3 (a). Kaji kestabilan skema ini dengan menggunakan kaedah Fourier.*

[50 markah]

4. Consider the Laplace problem

$$\nabla^2 u = 0. \qquad (1)$$
$$u(0,y) = u(x,0) = 0, \qquad 0 \le x, y \le 1$$
$$u(x,1) = 50x, \qquad 0 < x \le 1$$
$$u(1,y) = 50y, \qquad 0 < y < 1$$

An important classical ordering of the grid points when applying the five-point difference formula on the partial differential equation (1) is the *red-black* ordering as illustrated in Figure 1.

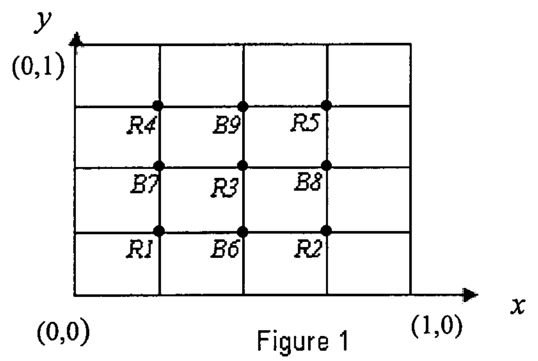


Figure 1

The grid points are divided into two types, *red* (*R*) and *black* (*B*). The ordering will be *R1, R2, R3, R4, R5*, followed by *B6, B7, B8* and *B9* for the case $n = 4$ as depicted in Figure 1.

(i) Generate the system $A\underline{u} = \underline{b}$ resulted from the discretisation according to this ordering for $n = 4$. Here, A is a 9x9 matrix.

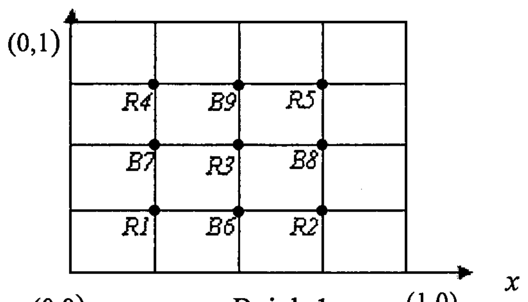
(ii) Show that the Gauss-Seidel iteration applied to this system will have the following form, i.e. it uncouples into two separate parts:

$$\underline{u}_R^{(k+1)} = D_R^{-1}(\underline{b}_1 - C\underline{u}_B^{(k)})$$
$$\underline{u}_B^{(k+1)} = D_B^{-1}(\underline{b}_2 - C^T\underline{u}_R^{(k+1)}).$$

4. *Pertimbangkan masalah Laplace*

$$\nabla^2 u = 0 \qquad (1)$$
$$u(0,y) = u(x,0) = 0, \qquad 0 \le x, y \le 1$$
$$u(x,1) = 50x, \qquad 0 < x \le 1$$
$$u(1,y) = 50y, \qquad 0 < y < 1$$

Satu tertib klasik bagi titik grid apabila diaplikasikan rumus beza lima-titik ke atas persamaan pembezaan separa (1) ialah tertib merah-hitam seperti yang diilustrasikan dalam Rajah 1.



Rajah 1

Titik grid dibahagikan kepada dua jenis, red (R) dan black (B). Tertibnya ialah R1, R2, R3, R4, R5, diikuti dengan B6, B7, B8 dan B9 bagi kes n = 4 seperti yang ditunjukkan dalam Rajah 1.

(i) *Janakan sistem* $A\underline{u} = \underline{b}$ *yang terhasil daripada pendiskretan mengikut tertib ini untuk* $n = 4$. *Di sini, A ialah matriks 9x9.*

(ii) *Tunjukkan bahawa lelaran Gauss-Seidel yang diaplikasikan pada sistem ini akan mempunyai bentuk yang berikut, yakni, ia ternyahpasangan kepada dua bahagian terpisah:*

$$\underline{u}_R^{(k+1)} = D_R^{-1}(\underline{b}_1 - C\underline{u}_B^{(k)})$$
$$\underline{u}_B^{(k+1)} = D_B^{-1}(\underline{b}_2 - C^T\underline{u}_R^{(k+1)}).$$

Here, $D_R = 4I_R$ and $D_B = 4I_B$, where I_R is the identity matrix of size 5x5 and I_B is the identity matrix of size 4x4. The matrix

$$C = \begin{bmatrix} -1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ -1 & -1 & -1 & -1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & -1 \end{bmatrix}, \ \underline{u}_R = \begin{bmatrix} u_{R1} \\ u_{R2} \\ u_{R3} \\ u_{R4} \\ u_{R5} \end{bmatrix}, \ \underline{u}_B = \begin{bmatrix} u_{B6} \\ u_{B7} \\ u_{B8} \\ u_{B9} \end{bmatrix},$$

$\underline{b}_1$ and $\underline{b}_2$ denote the right hand side values corresponding to the *red* and *black* points

respectively, specifically $\underline{b}_1 = \begin{bmatrix} 0 \\ 12.5 \\ 0 \\ 12.5 \\ 75 \end{bmatrix}$ and $\underline{b}_2 = \begin{bmatrix} 0 \\ 0 \\ 25 \\ 25 \end{bmatrix}$.

[100 marks]

Di sini, $D_R = 4I_R$ *dan* $D_B = 4I_B$ *, di mana* I_R *ialah matriks identiti bersaiz 5x5 dan* I_B *ialah matriks identiti bersaiz 4x4. Matriks*

$$C = \begin{bmatrix} -1 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ -1 & -1 & -1 & -1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & -1 \end{bmatrix}, \ \underline{u}_R = \begin{bmatrix} u_{R1} \\ u_{R2} \\ u_{R3} \\ u_{R4} \\ u_{R5} \end{bmatrix}, \ \underline{u}_B = \begin{bmatrix} u_{B6} \\ u_{B7} \\ u_{B8} \\ u_{B9} \end{bmatrix},$$

$\underline{b}_1$ *dan* $\underline{b}_2$ *ialah nilai-nilai di sebelah kanan yang bersepadan dengan titik-titik red dan black masing-masing, khususnya* $\underline{b}_1 = \begin{bmatrix} 0 \\ 12.5 \\ 0 \\ 12.5 \\ 75 \end{bmatrix}$ *dan* $\underline{b}_2 = \begin{bmatrix} 0 \\ 0 \\ 25 \\ 25 \end{bmatrix}$.

[100 markah]

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