UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

MSG 367 – Time Series Analysis
[Analisis Siri Masa]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of TEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEPULUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

*[**Arahan:** Jawab **semua sepuluh** [10] soalan.]*

1. Find the mean and covariance function for each of the following processes. In each case, determine whether or not the process is stationary.

(a) $Y_t = \theta_0 + tZ_t$

(b) $W_t = \nabla Y_t$, where Y_t is as given in part (a)

(c) $Y_t = Z_t Z_{t-1}$ (assume that $\{Z_t\}$ is normal white noise)

[8 marks]

2. Show that the variance function of ARMA(1,1) model

$$Y_t = \phi Y_{t-1} + Z_t + \theta Z_{t-1}$$

where $Z_t \sim N(0,1)$ is given by $Var(Y_t) = \dfrac{1 + 2\phi\theta + \theta^2}{1 - \phi^2}$.

[10 marks]

3. Describe the important characteristics of the autocorrelation and partial autocorrelation functions for the following models:

(a) ARIMA(0,0,1)

(b) ARIMA(0,0,2)

(c) ARIMA(1,0,0)

(d) ARIMA(2,0,0)

(e) ARIMA(1,0,1)

[15 marks]

4. Consider a process that satisfies the zero-mean, "stationary" AR(1) equation $Y_t = \phi Y_{t-1} + Z_t$ with $-1 < \phi < 1$. Let c be any nonzero constant, and define $W_t = Y_t + c\phi^t$.

(a) Show that $E[W_t] = c\phi^t$.

(b) Show that $\{W_t\}$ satisfies the "stationary" AR(1) equation $W_t = \phi W_{t-1} + Z_t$.

(c) Is $\{W_t\}$ stationary?

[8 marks]

1. *Dapatkan fungsi min dan kovarians bagi setiap proses berikut. Bagi setiap proses tentukan sama ada setiap proses adalah pegun.*

   (a) $Y_t = \theta_0 + tZ_t$

   (b) $W_t = \nabla Y_t$, *yang mana* Y_t *adalah dari bahagian (a)*

   (c) $Y_t = Z_t Z_{t-1}$ *(Anggapkan* $\{Z_t\}$ *adalah proses hingar putih yang normal)*

[8 markah]

2. *Tunjukkan fungsi varians bagi model ARMA(1,1)*

$$Y_t = \phi Y_{t-1} + Z_t + \theta Z_{t-1}$$

yang mana $Z_t \sim N(0,1)$ *dinyatakan sebagai* $Var(Y_t) = \dfrac{1 + 2\phi\theta + \theta^2}{1 - \phi^2}$.

[10 markah]

3. *Berikan ciri-ciri penting fungsi autokolerasi dan fungsi autokolerasi separa bagi model-model berikut:*

   (a) *ARIMA(0,0,1)*

   (b) *ARIMA(0,0,2)*

   (c) *ARIMA(1,0,0)*

   (d) *ARIMA(2,0,0)*

   (e) *ARIMA(1,0,1)*

[15 markah]

4. *Pertimbangkan proses AR(1) pegun dengan min sifar yang memenuhi persamaan* $Y_t = \phi Y_{t-1} + Z_t$ *dengan* $-1 < \phi < 1$. *Biarkan c nilai malar tak sifar dan takrifkan sebagai* $W_t = Y_t + c\phi^t$.

   (a) *Tunjukkan* $E[W_t] = c\phi^t$.

   (b) *Tunjukkan bahawa* $\{W_t\}$ *memenuhi persamaan proses AR(1) pegun* $W_t = \phi W_{t-1} + Z_t$.

   (c) *Adakah* $\{W_t\}$ *pegun?*

[8 markah]

5. Consider the ARIMA (0,1,1) model

$$(1-B)Y_t = (1-\theta B)Z_t$$

(a) Write down the forecast equation that generates the forecasts, $\hat{Y}_n(l)$.

(b) Find the 95% forecast limits for $\hat{Y}_n(l)$.

(c) Express the forecasts as a weighted average of previous observations.

(d) Discuss the connection of this forecast $\hat{Y}_n(l)$ with the one generated by the simple exponential smoothing method.

[12 marks]

6. Identify the values of $p,d,q,\ P,D,Q,\ s$ for the following multiplicative seasonal ARIMA models:

(a) $Y_t = 0.5Y_{t-1} + Y_{t-4} - 0.5Y_{t-5} + Z_t - 0.3Z_{t-1}$

(b) $Y_t = Y_{t-1} + Y_{t-12} + Y_{t-13} + Z_t - 0.5Z_{t-1} - Z_{t-12} + 0.25Z_{t-13}$

(c) $Y_t = Y_{t-12} + 0.7(Y_{t-1} - Y_{t-13}) + Z_t - 0.3Z_{t-12}$

(d) $(1-0.27B-0.21B^2)(1-B^{12})Y_t = (1-0.49B^{12})Z_t$

[8 marks]

7. (a) For a series of length 64, the sample partial autocorrelations are given as:

Lag	1	2	3	4	5
PACF	0.47	-0.34	0.20	0.02	-0.06

Find a suitable model for the series above.

(b) The sample ACF for a series and its first difference are given in the following table. Here $n = 100$.

Lag	1	2	3	4	5	6
ACF for Y_t	0.97	0.97	0.93	0.85	0.80	0.71
ACF for ∇Y_t	-0.42	0.18	-0.02	0.07	-0.10	-0.09

Based on this information alone, find a suitable model for the series above.

[7 marks]

5. *Pertimbangkan model ARIMA (0,1,1)*

$$(1-B)Y_t = (1-\theta B)Z_t$$

(a) *Tuliskan persamaan telahan yang menjana telahan,* $\hat{Y}_n(l)$.

(b) *Dapatkan had telahan 95% untuk* $\hat{Y}_n(l)$.

(c) *Nyatakan telahan sebagai purata pemberat cerapan terdahulu.*

(d) *Bincangkan hubungan antara telahan* $\hat{Y}_n(l)$ *dengan telahan yang dijana oleh kaedah pelicinan exponen ringkas.*

[12 markah]

6. *Kenalpasti nilai-nilai p,d,q, P,D,Q, s bagi setiap model berdaraban ARIMA bermusim berikut:*

(a) $Y_t = 0.5Y_{t-1} + Y_{t-4} - 0.5Y_{t-5} + Z_t - 0.3Z_{t-1}$

(b) $Y_t = Y_{t-1} + Y_{t-12} + Y_{t-13} + Z_t - 0.5Z_{t-1} - Z_{t-12} + 0.25Z_{t-13}$

(c) $Y_t = Y_{t-12} + 0.7(Y_{t-1} - Y_{t-13}) + Z_t - 0.3Z_{t-12}$

(d) $(1-0.27B-0.21B^2)(1-B^{12})Y_t = (1-0.49B^{12})Z_t$

[8 markah]

7. (a) *Nilai-nilai autokolerasi separa sampel bagi suatu siri dengan 64 nilai diberikan seperti berikut:*

Susulan	*1*	*2*	*3*	*4*	*5*
PACF	*0.47*	*-0.34*	*0.20*	*0.02*	*-0.06*

Dapatkan model yang sesuai bagi siri di atas.

(b) *Jadual menunjukkan nilai-nilai autokolerasi sampel untuk suatu siri dan perbezan pertama siri tersebut dengan n = 100.*

Susulan	*1*	*2*	*3*	*4*	*5*	*6*
ACF untuk Y_t	*0.97*	*0.97*	*0.93*	*0.85*	*0.80*	*0.71*
ACF untuk ∇Y_t	*-0.42*	*0.18*	*-0.02*	*0.07*	*-0.10*	*-0.09*

Berdasarkan maklumat ini, dapatkan model yang sesuai bagi siri di atas.

[7 markah]

8. An AR model has AR characteristic polynomial $\left(1-1.6Y+0.7Y^2\right)\left(1-0.8Y^{12}\right)$

   (a) Is the model stationary?

   (b) Identify the model as a certain seasonal ARIMA model.

[6 marks]

9. Consider the ARCH(l) model $r_t = \sigma_{t|t-1}\varepsilon_t$ where the ε_t are independently identically distributed (i.i.d) random variable with mean 0 and variance 1, and $\sigma_{t|t-1}^2 = \varpi + \alpha r_{t-1}^2$ where α and ϖ are unknown parameters. Show that the unconditional variance of r_t is $Var\left(r_t\right) = \varpi/1-\alpha$

[6 marks]

10. (a) Summarize the Box-Jenkins three-stage modeling procedure.

    (b) The series analyzed here is the monthly price of Tahir Limited common shares from January 2003 to October 2008 with a total of 70 observations. Based on the output given in Appendix discuss the best fitted model for the series using the Box-Jenkins procedures.

[20 marks]

8. *Suatu model AR mempunyai polinomial cirian* $\left(1-1.6Y+0.7Y^2\right)\left(1-0.8Y^{12}\right)$

(a) *Adakah model ini pegun?*

(b) *Kenalpasti model ini sebagai suatu model ARIMA bermusim.*

[6 markah]

9. *Pertimbangkan model ARCH(l)* $r_t=\sigma_{t|t-1}\varepsilon_t$ *di mana* ε_t *adalah pemboleh ubah rawak tertabur secara secaman dan tak bersandar (i.i.d) dengan min 0 and variance 1, dan* $\sigma^2_{t|t-1}=\varpi+\alpha r^2_{t-1}$ *yang mana* α *dan* ϖ *adalah parameter yang tidak diketahui. Tunjukkan bahawa varians bersyarat* r_t *adalah* $Var\left(r_t\right)=\varpi/1-\alpha$

[6 markah]

10. (a) *Terangkan secara ringkas kaedah pemodelan tiga peringkat Box-Jenkins.*

(b) *Siri yang dianalisis adalah harga bulanan saham umum bagi Tahir Limited dari Januari 2003 hingga Oktober 2008 dengan jumlah cerapan sebanyak 70. Berdasarkan output yang diberikan dalam Lampiran bincangkan penyuaian model yang terbaik dengan menggunakan kaedah pemodelan Box-Jenkins..*

[20 markah]

APPENDIX

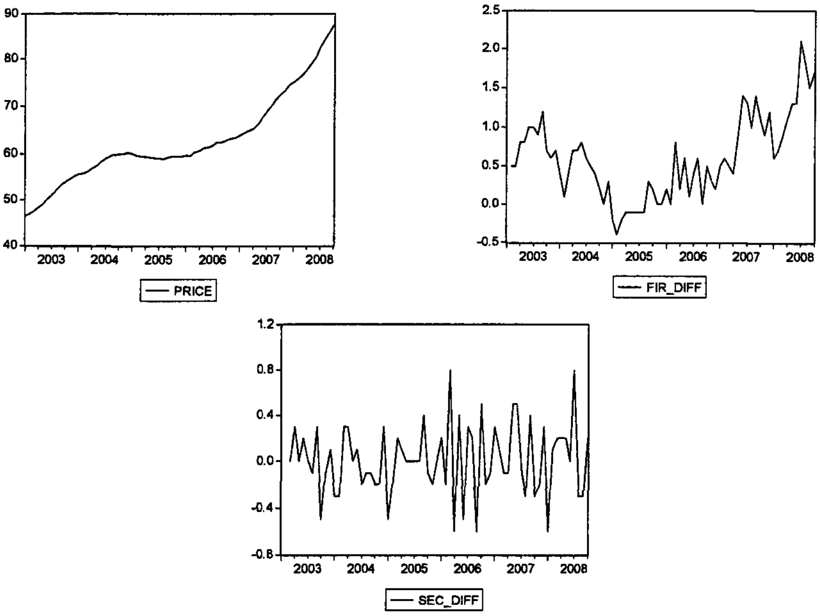

Figure 1 Time Series Plots for Price, First Differences and Second Differences of Price

Correlogram of PRICE

Date: 02/15/09 Time: 11:08
Sample: 2003:01 2008:10
Included observations: 70

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.926	0.926	62.577	0.000
		2	0.853	-0.026	116.51	0.000
		3	0.782	-0.029	162.50	0.000
		4	0.715	-0.013	201.50	0.000
		5	0.653	0.001	234.56	0.000
		6	0.595	-0.012	262.43	0.000
		7	0.540	-0.008	285.80	0.000
		8	0.489	-0.013	305.24	0.000
		9	0.441	-0.010	321.29	0.000
		10	0.394	-0.023	334.30	0.000
		11	0.347	-0.029	344.57	0.000
		12	0.303	-0.009	352.56	0.000
		13	0.261	-0.021	358.58	0.000
		14	0.220	-0.021	362.95	0.000
		15	0.184	-0.004	366.05	0.000
		16	0.151	-0.005	368.17	0.000
		17	0.122	0.007	369.59	0.000
		18	0.100	0.018	370.57	0.000
		19	0.082	0.007	371.24	0.000
		20	0.066	-0.005	371.68	0.000
		21	0.051	-0.001	371.95	0.000
		22	0.039	0.000	372.10	0.000
		23	0.027	-0.004	372.18	0.000
		24	0.016	-0.008	372.21	0.000
		25	0.005	-0.014	372.22	0.000
		26	-0.006	-0.012	372.22	0.000
		27	-0.018	-0.024	372.26	0.000
		28	-0.029	-0.006	372.36	0.000
		29	-0.040	-0.011	372.56	0.000
		30	-0.051	-0.020	372.89	0.000
		31	-0.062	-0.006	373.38	0.000
		32	-0.072	-0.016	374.07	0.000

Figure 2 Estimated ACF and PACF for Price

Correlogram of FIR_DIFF

Date: 02/15/09 Time: 11:17
Sample: 2003:01 2008:10
Included observations: 69

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.801	0.801	46.189	0.000
		2	0.732	0.254	85.413	0.000
		3	0.649	0.025	116.71	0.000
		4	0.501	-0.232	135.64	0.000
		5	0.435	0.038	150.16	0.000
		6	0.353	0.010	159.87	0.000
		7	0.337	0.179	168.82	0.000
		8	0.307	0.007	176.39	0.000
		9	0.320	0.114	184.76	0.000
		10	0.368	0.138	196.02	0.000
		11	0.332	-0.111	205.35	0.000
		12	0.294	-0.192	212.77	0.000
		13	0.264	-0.022	218.88	0.000
		14	0.181	-0.061	221.81	0.000
		15	0.115	-0.029	223.01	0.000
		16	0.045	-0.057	223.20	0.000
		17	-0.049	-0.151	223.42	0.000
		18	-0.118	-0.097	224.76	0.000
		19	-0.149	0.030	226.93	0.000
		20	-0.171	-0.017	229.84	0.000
		21	-0.169	0.036	232.77	0.000
		22	-0.229	-0.230	238.24	0.000
		23	-0.217	0.021	243.24	0.000
		24	-0.223	0.049	248.65	0.000
		25	-0.272	-0.073	256.90	0.000
		26	-0.247	0.046	263.88	0.000
		27	-0.295	-0.027	274.02	0.000
		28	-0.314	0.005	285.77	0.000

Figure 3 Estimated ACF and PACF for First Differences of Price

Correlogram of SEC_DIFF

Date: 02/15/09 Time: 11:19
Sample: 2003:01 2008:10
Included observations: 68

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.361	-0.361	9.2782	0.002
		2	-0.006	-0.157	9.2808	0.010
		3	0.139	0.094	10.703	0.013
		4	-0.106	-0.020	11.545	0.021
		5	0.048	0.018	11.717	0.039
		6	-0.163	-0.194	13.752	0.033
		7	0.078	-0.044	14.221	0.047
		8	-0.101	-0.132	15.031	0.059
		9	-0.090	-0.157	15.686	0.074
		10	0.148	0.024	17.472	0.065
		11	0.055	0.175	17.722	0.088
		12	-0.071	0.017	18.150	0.111
		13	0.110	0.074	19.194	0.117
		14	0.024	0.038	19.243	0.156
		15	-0.046	-0.028	19.437	0.195
		16	0.057	0.039	19.737	0.232
		17	0.005	0.074	19.739	0.288
		18	-0.027	0.037	19.810	0.344
		19	-0.050	0.020	20.053	0.391
		20	-0.090	-0.134	20.659	0.405
		21	0.194	0.120	24.677	0.261
		22	-0.160	-0.018	27.314	0.200
		23	0.027	-0.005	27.393	0.240
		24	0.089	0.018	28.252	0.250
		25	-0.135	-0.071	30.283	0.214
		26	0.112	-0.017	31.694	0.203
		27	-0.055	-0.042	32.040	0.231
		28	0.025	-0.041	32.114	0.270

Figure 4 Estimated ACF and PACF for Second Differences Price

Dependent Variable: PRICE
Method: Least Squares
Date: 02/15/09 Time: 11:22
Sample(adjusted): 2003:02 2008:10
Included observations: 69 after adjusting endpoints
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	1.009988	0.000935	1080.486	0.0000

R-squared	0.997269	Mean dependent var	62.93478
Adjusted R-squared	0.997269	S.D. dependent var	9.358699
S.E. of regression	0.489063	Akaike info criterion	1.421736
Sum squared resid	16.26442	Schwarz criterion	1.454114
Log likelihood	-48.04988	Durbin-Watson stat	0.394814

Inverted AR Roots 1.01
Estimated AR process is nonstationary

Dependent Variable: FIR_DIFF
Method: Least Squares
Date: 02/15/09 Time: 11:27
Sample(adjusted): 2003:04 2008:10
Included observations: 67 after adjusting endpoints
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.637015	0.117256	5.432695	0.0000
AR(2)	0.353158	0.120268	2.936416	0.0046

R-squared	0.708628	Mean dependent var	0.598507
Adjusted R-squared	0.704145	S.D. dependent var	0.537529
S.E. of regression	0.292375	Akaike info criterion	0.407841
Sum squared resid	5.556423	Schwarz criterion	0.473652
Log likelihood	-11.66266	Durbin-Watson stat	2.080882

Inverted AR Roots .99 -.36

Dependent Variable: SEC_DIFF
Method: Least Squares
Date: 02/15/09 Time: 11:30
Sample(adjusted): 2003:03 2008:10
Included observations: 68 after adjusting endpoints
Convergence achieved after 6 iterations
Backcast: 2003:02

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.378958	0.112843	-3.358293	0.0013

R-squared	0.134790	Mean dependent var	0.017647
Adjusted R-squared	0.134790	S.D. dependent var	0.308071
S.E. of regression	0.286558	Akaike info criterion	0.352842
Sum squared resid	5.501719	Schwarz criterion	0.385482
Log likelihood	-10.99662	Durbin-Watson stat	2.041494

Inverted MA Roots .38

Figure 5 Three estimated Model for Price Series

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