UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

MSG 284 – Introduction to Geometric Modelling
[Pengenalan kepada Pemodelan Geometri]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all three** [3] questions.

*[**Arahan:** Jawab **semua tiga** [3] soalan.]*

1. (a) Given three points $A(1, 1)$, $B(-3, 4)$ and $C(4, 13)$. Find the barycentric coordinates of the point $(2, 5)$ with respect to the points A, B and C.

(b) Let $\boldsymbol{T}(s)$ and $\boldsymbol{N}(s)$ be the unit tangent vector and the principal normal vector to a parametric curve $\boldsymbol{C}(s)$ at arc length parameter s. Prove that the vectors $\boldsymbol{T}(s)$ and $\boldsymbol{N}(s)$ are mutually orthogonal.

(c) Use monomial basis to derive a polynomial that interpolates the points $(1, 1)$, $(2, 5)$ and $(4, 5)$.

(d) Given a tensor product Bézier surface of degree $(2, 2)$

$$\boldsymbol{S}(u, v) = \sum_{i=0}^{2} \sum_{j=0}^{2} \boldsymbol{C}_{i,j}\, B_j^2(v)\, B_i^2(u), \qquad 0 \le u, v \le 1,$$

where

$$B_i^2(t) = \frac{2!}{i!\,(2-i)!}\, t^i (1-t)^{2-i}, \qquad 0 \le t \le 1, \qquad i = 0, 1, 2,$$

and

$$\boldsymbol{C}_{0,0} = (-1, -1, 0), \qquad \boldsymbol{C}_{0,1} = (0, -1, 2), \qquad \boldsymbol{C}_{0,2} = (1, -1, 1),$$
$$\boldsymbol{C}_{1,0} = (-1, 0, 2), \qquad \boldsymbol{C}_{1,1} = (0, 0, 3), \qquad \boldsymbol{C}_{1,2} = (1, 0, 2),$$
$$\boldsymbol{C}_{2,0} = (-1, 1, 1), \qquad \boldsymbol{C}_{2,1} = (0, 1, 2), \qquad \boldsymbol{C}_{2,2} = (1, 1, 1).$$

(i) Find the surface point at $(u, v) = (0, 0.5)$.

(ii) Find the cross boundary derivative to surface $\boldsymbol{S}$ at $(u, v) = (0, 0.5)$.

(iii) Find the unit normal vector to surface $\boldsymbol{S}$ at $(u, v) = (0, 0.5)$.

[100 marks]

1. (a) *Diberi tiga titik* $\boldsymbol{A}(1, 1)$, $\boldsymbol{B}(-3, 4)$ *dan* $\boldsymbol{C}(4, 13)$. *Cari koordinat baripusat bagi titik* $(2, 5)$ *terhadap titik-titik* $\boldsymbol{A}$, $\boldsymbol{B}$ *dan* $\boldsymbol{C}$.

(b) *Katakan* $\boldsymbol{T}(s)$ *dan* $\boldsymbol{N}(s)$ *ialah vektor tangen unit dan vektor normal prinsipal kepada suatu lengkung berparameter* $\boldsymbol{C}(s)$ *pada parameter panjang lengkok* s. *Buktikan bahawa vektor-vektor* $\boldsymbol{T}(s)$ *dan* $\boldsymbol{N}(s)$ *adalah saling berserenjang.*

(c) *Gunakan asas monomial untuk membina satu polinomial yang menginterpolasi titik-titik* $(1, 1)$, $(2, 5)$ *dan* $(4, 5)$.

(d) *Diberi suatu permukaan hasil darab tensor Bézier berdarjah* $(2, 2)$

$$\boldsymbol{S}(u, v) = \sum_{i=0}^{2} \sum_{j=0}^{2} \boldsymbol{C}_{i,j} B_j^2(v) B_i^2(u), \qquad 0 \le u, v \le 1,$$

di mana

$$B_i^2(t) = \frac{2!}{i!\,(2-i)!} t^i (1-t)^{2-i}, \qquad 0 \le t \le 1, \qquad i = 0, 1, 2,$$

dan

$$\boldsymbol{C}_{0,0} = (-1, -1, 0), \quad \boldsymbol{C}_{0,1} = (0, -1, 2), \quad \boldsymbol{C}_{0,2} = (1, -1, 1),$$
$$\boldsymbol{C}_{1,0} = (-1, 0, 2), \quad \boldsymbol{C}_{1,1} = (0, 0, 3), \quad \boldsymbol{C}_{1,2} = (1, 0, 2),$$
$$\boldsymbol{C}_{2,0} = (-1, 1, 1), \quad \boldsymbol{C}_{2,1} = (0, 1, 2), \quad \boldsymbol{C}_{2,2} = (1, 1, 1).$$

(i) *Cari titik permukaan pada* $(u, v) = (0, 0.5)$.

(ii) *Cari terbitan silang sempadan kepada permukaan* $\boldsymbol{S}$ *pada* $(u, v) = (0, 0.5)$.

(iii) *Cari vektor normal unit kepada permukaan* $\boldsymbol{S}$ *pada* $(u, v) = (0, 0.5)$.

[100 markah]

2. (a) Suppose a Bézier curve of degree n is defined by

$$\boldsymbol{P}(t) = \sum_{i=0}^{n} \boldsymbol{C}_i B_i^n(t), \qquad t \in [0, 1],$$

where

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!} t^i (1-t)^{n-i}$$

are the Bernstein polynomials of degree n and $\boldsymbol{C}_i$ are the Bézier control points of the curve.

(i) Prove that $B_i^n(t) = B_{n-i}^n(1-t)$.

Then show that $\boldsymbol{P}(t) = \sum_{i=0}^{n} \boldsymbol{C}_{n-i} B_i^n(1-t)$.

(ii) Show that $\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{P}(t) = n\sum_{i=0}^{n-1} (\boldsymbol{C}_{i+1} - \boldsymbol{C}_i) B_i^{n-1}(t)$.

(iii) Let $n = 2$, $\boldsymbol{C}_0 = (2, 1)$, $\boldsymbol{C}_1 = (0, 3)$ and $\boldsymbol{C}_2 = (4, 3)$. The curve $\boldsymbol{P}(t)$ is subdivided at $t = 0.25$ into two quadratic Bézier curve segments, $\boldsymbol{P}_1(u)$ and $\boldsymbol{P}_2(u)$, where $u \in [0, 1]$ is the local parameter of each curve segment. Evaluate the respective Bézier control points of the curves $\boldsymbol{P}_1(u)$ and $\boldsymbol{P}_2(u)$.

(b) Let $\boldsymbol{R}(u)$ be composed of two adjacent curve segments as

$$\boldsymbol{R}(u) = \begin{cases} \boldsymbol{P}(u), & u \in [0, 1], \\ \boldsymbol{Q}(u), & u \in [1, 4]. \end{cases}$$

$\boldsymbol{P}(u)$ is a quadratic Bézier curve with its Bézier control points $\boldsymbol{C}_0 = (1, 0)$, $\boldsymbol{C}_1 = (2, 2)$ and $\boldsymbol{C}_2 = (4, 2)$. $\boldsymbol{Q}(u)$ is a polynomial which can be represented locally as a cubic Bézier curve

$$\boldsymbol{Q}(t) = \sum_{i=0}^{3} \boldsymbol{D}_i B_i^3(t), \qquad t \in [0, 1],$$

where $B_i^3(t)$ are the Bernstein polynomials of degree 3 and $\boldsymbol{D}_i$ are the associated Bézier control points. Suppose $\boldsymbol{D}_1 = (5, 2)$, evaluate the Bézier points $\boldsymbol{D}_0$, $\boldsymbol{D}_2$ and $\boldsymbol{D}_3$ such that $\boldsymbol{R}(u)$, $u \in [0, 4]$, is a G^2 continuous curve which passes through the point (6, 1) at $u = 4$, and it derivative vectors $\frac{\mathrm{d}}{\mathrm{d}u}\boldsymbol{R}(u)$ at $u = 1$ and $u = 4$ are perpendicular.

[100 marks]

2. (a) *Andaikan lengkung Bézier berdarjah n ditakrif sebagai*

$$\mathbf{P}(t) = \sum_{i=0}^{n} \mathbf{C}_i B_i^n(t), \qquad t \in [0, 1],$$

di mana

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!} t^i (1-t)^{n-i}$$

ialah polinomial Bernstein berdarjah n *dan* $\mathbf{C}_i$ *adalah titik-titik kawalan Bézier.*

(i) *Buktikan bahawa* $B_i^n(t) = B_{n-i}^n(1-t)$.

Seterusnya, tunjukkan bahawa $\mathbf{P}(t) = \sum_{i=0}^{n} \mathbf{C}_{n-i} B_i^n(1-t)$.

(ii) *Tunjukkan bahawa* $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{P}(t) = n\sum_{i=0}^{n-1} (\mathbf{C}_{i+1} - \mathbf{C}_i) B_i^{n-1}(t)$.

(iii) *Katakan* $n = 2$, $\mathbf{C}_0 = (2, 1)$, $\mathbf{C}_1 = (0, 3)$ *dan* $\mathbf{C}_2 = (4, 3)$. *Lengkung* $\mathbf{P}(t)$ *dibahagikan pada* $t = 0.25$ *kepada dua segmen lengkung Bézier kuadratik,* $\mathbf{P}_1(u)$ *dan* $\mathbf{P}_2(u)$, *di mana* $u \in [0, 1]$ *ialah parameter setempat bagi setiap segmen lengkung. Nilaikan titik-titik kawalan Bézier bagi lengkung* $\mathbf{P}_1(u)$ *dan* $\mathbf{P}_2(u)$ *masing-masing.*

(b) *Katakan* $\mathbf{R}(u)$ *digubah dengan dua segmen lengkung yang bersebelahan sebagai*

$$\mathbf{R}(u) = \begin{cases} \mathbf{P}(u), & u \in [0, 1], \\ \mathbf{Q}(u), & u \in [1, 4]. \end{cases}$$

$\mathbf{P}(u)$ *ialah satu lengkung Bézier kuadratik dengan titik-titik kawalan Bézier* $\mathbf{C}_0 = (1, 0)$, $\mathbf{C}_1 = (2, 2)$ *dan* $\mathbf{C}_2 = (4, 2)$. $\mathbf{Q}(u)$ *ialah suatu polinomial yang dapat diwakili secara setempat sebagai satu lengkung Bézier kubik*

$$\mathbf{Q}(t) = \sum_{i=0}^{3} \mathbf{D}_i B_i^3(t), \qquad t \in [0, 1],$$

di mana $B_i^3(t)$ *ialah polinomial Bernstein berdarjah* 3 *dan* $\mathbf{D}_i$ *adalah titik-titik kawalan Bézier. Andaikan* $\mathbf{D}_1 = (5, 2)$, *nilaikan titik-titik Bézier* $\mathbf{D}_0$, $\mathbf{D}_2$ *dan* $\mathbf{D}_3$ *supaya* $\mathbf{R}(u)$, $u \in [0, 4]$, *adalah satu lengkung berkeselanjaran* G^2 *yang melalui titik* $(6, 1)$ *pada* $u = 4$, *dan vektor-vektor terbitan* $\frac{\mathrm{d}}{\mathrm{d}u}\mathbf{R}(u)$ *pada* $u = 1$ *dan* $u = 4$ *adalah saling berserenjang.*

[100 markah]

3. (a) Let $\boldsymbol{u} = (u_0, u_1, \ldots, u_{n+k})$ be a knot vector where n and k are positive integers. The normalized B-splines of order k are defined recursively by

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

and

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad k > 1.$$

(i) Suppose the knots $u_i = i$. Show that $N_0^3(u)$ has parametric continuity C^1 at each of the knots u_i.

(ii) Given $k = 3$ and $\boldsymbol{u} = (0, 0, 0, 1, 1, 1)$. Show that

$$N_i^3(u) = B_i^2(u), \qquad u \in [0, 1], \qquad i = 0, 1, 2,$$

where $B_i^2(u)$ are the Bernstein polynomials of degree 2.

(b) Given a knot vector $\boldsymbol{u} = (0, 1, 2, 3, 4, 5)$. A uniform B-spline curve of order 3 is defined by

$$\boldsymbol{P}(u) = \boldsymbol{D}_0 N_0^3(u) + \begin{pmatrix} 2 \\ 1 \end{pmatrix} N_1^3(u) + \boldsymbol{D}_1 N_2^3(u), \qquad u \in [2, 3],$$

where $\boldsymbol{D}_0$, $\boldsymbol{D}_1 \in \mathbb{R}^2$. If $\boldsymbol{P}(2) = (1, 2)$ and $\boldsymbol{P}(3) = (4, 3)$, evaluate the points $\boldsymbol{D}_0$ and $\boldsymbol{D}_1$.

(c) Given a bilinearly blended Coons patch $\boldsymbol{F}(u, v)$, $0 \le u, v \le 1$, that interpolates four boundary curves as

$$\boldsymbol{F}(u, 0) = \left(\frac{5}{5 - 4u},\ 1 - u,\ 0\right), \qquad u \in [0, 1],$$

$$\boldsymbol{F}(u, 1) = (2 + 2u,\ 3,\ 1 + u - u^2), \qquad u \in [0, 1],$$

$$\boldsymbol{F}(0, v) = \left(1 + v^2,\ 2v + 1,\ \sin\left(\tfrac{\pi}{2} v\right)\right), \qquad v \in [0, 1],$$

$$\boldsymbol{F}(1, v) = \left(5 - v,\ 3v^2,\ 1 - \cos\left(\tfrac{\pi}{2} v\right)\right), \qquad v \in [0, 1].$$

Evaluate the patch point $\boldsymbol{F}(0.5, 0.5)$.

[100 marks]

3. (a) *Katakan* $\boldsymbol{u} = (u_0, u_1, \ldots, u_{n+k})$ *ialah suatu vektor knot di mana* n *dan* k *adalah nombor integer positif. Splin-B ternormal berperingkat* k *ditakrif secara rekursi sebagai*

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & \text{di tempat lain} \end{cases}$$

and

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad k > 1.$$

(i) *Andaikan nilai-nilai knot* $u_i = i$. *Tunjukkan bahawa* $N_0^3(u)$ *mempunyai keselanjaran berparameter* C^1 *pada setiap knot* u_i.

(ii) *Diberi* $k = 3$ *dan* $\boldsymbol{u} = (0, 0, 0, 1, 1, 1)$. *Tunjukkan bahawa*

$$N_i^3(u) = B_i^2(u), \qquad u \in [0, 1], \qquad i = 0, 1, 2,$$

di mana $B_i^2(u)$ *ialah polinomial Bernstein berdarjah* 2.

(b) *Diberi vektor knot* $\boldsymbol{u} = (0, 1, 2, 3, 4, 5)$. *Suatu lengkung splin-B seragam berperingkat* 3 *ditakrif sebagai*

$$\boldsymbol{P}(u) = \boldsymbol{D}_0 N_0^3(u) + \begin{pmatrix} 2 \\ 1 \end{pmatrix} N_1^3(u) + \boldsymbol{D}_1 N_2^3(u), \qquad u \in [2, 3],$$

di mana $\boldsymbol{D}_0$, $\boldsymbol{D}_1 \in \mathbb{R}^2$. *Jika* $\boldsymbol{P}(2) = (1, 2)$ *dan* $\boldsymbol{P}(3) = (4, 3)$, *nilaikan titik-titik* $\boldsymbol{D}_0$ *dan* $\boldsymbol{D}_1$.

(c) *Diberi satu tampalan Coons teraduan bilinear* $\boldsymbol{F}(u, v)$, $0 \le u, v \le 1$, *yang menginterpolasi empat lengkung sempadan sebagai*

$$\boldsymbol{F}(u, 0) = \left(\frac{5}{5 - 4u}, \; 1 - u, \; 0\right), \qquad u \in [0, 1],$$

$$\boldsymbol{F}(u, 1) = (2 + 2u, \; 3, \; 1 + u - u^2), \qquad u \in [0, 1],$$

$$\boldsymbol{F}(0, v) = (1 + v^2, \; 2v + 1, \; \sin(\tfrac{\pi}{2} v)), \qquad v \in [0, 1],$$

$$\boldsymbol{F}(1, v) = (5 - v, \; 3v^2, \; 1 - \cos(\tfrac{\pi}{2} v)), \qquad v \in [0, 1].$$

Nilaikan titik tampalan $\boldsymbol{F}(0.5, 0.5)$.

[100 markah]

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