UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

MSG 265 – Design and Analysis of Experiments
[Rekabentuk dan Analisis Uji Kaji]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINETEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

[Arahan: *Jawab* ***semua empat*** *[4] soalan.]*

1. (a) A two-factor factorial experiment was conducted and the following ANOVA table was obtained.

Source	DF	SS	MS	F	p-value
A	1	_____	0.0002	_____	_____
B	_____	180.378	_____	_____	_____
Interaction	3	8.479	_____	_____	0.932
Error	8	158.797			
Total	15	347.653			

(i) Fill in the blanks in the ANOVA table.
(ii) How many levels were used for factor *B*?
(iii) How many replicates of the experiment were performed?
(iv) What conclusions would you draw about this experiment?

[40 marks]

(b) An experiment was conducted to determine whether either firing temperature or furnace position affects the baked density of carbon anode. Suppose that the levels of the firing temperature and the furnace position were randomly selected.The data are shown below:

Position	Temperature			Total
	800	825	850	
1	70 65 (218) 83	563 580 (1686) 543	65 10 (165) 90	2069
2	28 47 (96) 21	488 526 (1518) 504	26 38 (96) 32	1710
Total	314	3204	261	3779

Given $SST = 958690.9444$

(i) Is there a significant interaction effect? Does firing temperature or furnace position affect the baked density? State your conclusions. Use $\alpha = 0.05$.
(ii) Estimate the variance components.

[40 marks]

(c) Consider the experiment in part (b). Demonstrate how the assignment of the treatment combinations to the experimental units would be determined if this experiment was run as
(i) a factorial design in a randomized block
(ii) a completely randomized factorial design

[20 marks]

1. (a) *Satu ujikaji dua-faktor telah dijalankan dan jadual ANOVA berikut diperoleh.*

Punca Ubahan	DF	SS	MS	F	Nilai-p
A	1		0.0002		
B		180.378			
Saling tindak	3	8.479			0.932
Ralat	8	158.797			
Jumlah	15	347.653			

(i) Isikan tempat kosong jadual ANOVA ini.
(ii) Berapakah bilangan aras yang diguna untuk faktor B?
(iii) Berapa replika uji kaji yang telah dijalankan?
(iv) Apakah kesimpulan yang anda boleh buat tentang uji kaji ini?

[40 markah]

(b) *Satu uji kaji telah dijalankan untuk menentukan sama ada suhu pembakaran atau kedudukan peleburan memberi kesan kepada ketumpatan anode karbon yang dikeraskan. Andaikan bahawa aras suhu pembakaran dan aras kedudukan peleburan dipilih secara rawak. Data diperoleh diberikan di bawah:*

Kedudukan	Suhu			Jumlah
	800	825	850	
1	70 65 (218) 83	563 580 (1686) 543	65 10 (165) 90	2069
2	28 47 (96) 21	488 526 (1518) 504	26 38 (96) 32	1710
Jumlah	314	3204	261	3779

Diberi SST = 958690.9444

(i) Adakah terdapat kesan saling tindak yang bererti? Adakah suhu pembakaran atau kedudukan peleburan memberi kesan terhadap ketumpatan bahan yang dikeraskan? Nyatakan kesimpulan anda. Guna $\alpha = 0.05$.
(ii) Anggarkan komponen variansnya.

[40 markah]

(c) *Pertimbangkan uji kaji di bahagian (b). Jelaskan bagaimana pembahagian gabungan rawatan kepada unit uji kaji boleh ditentukan jika uji kaji ini dijalankan sebagai*

(i) satu rekabentuk faktoran dalam satu blok rawakan
(ii) satu rekabentuk faktoran rawak lengkap

[20 markah]

2.. An experiment was conducted to study the life (in hours) of two different brands of batteries in three different devices (radio, camera and portable DVD player). A completely randomized two-factor factorial experiment was conducted and the following data obtained:

Brand of Battery	Device		
	Radio	Camera	DVD Player
A	8.6	7.9	5.4
	8.2	8.4	5.7
B	9.4	8.5	5.8
	8.8	8.9	5.9

(i) Is there a significant interaction effect? Does brand of battery or devices affect the life? State your conclusions. Refer to the ANOVA table below and use $\alpha = 0.05$.

Dependent Variable: life

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	23.327(a)	5	4.665	54.355	.000
Intercept	697.687	1	697.687	8128.398	.000
battery	.801	1	.801	9.330	.022
device	22.445	2	11.223	130.748	.000
battery * device	.082	2	.041	.476	.643
Error	.515	6	.086		
Total	721.530	12			
Corrected Total	23.842	11			

a R Squared = .978 (Adjusted R Squared = .960)

(ii) Obtain point estimates of the life of each device. Use Tukey's test to determine which devices are significantly different for the given data. Given that critical value for the differences is $T_{0.05} = 2.324$.

(iii) Which brand of batteries would you recommend?

(iv) State the statistical model without the interaction term. Conduct the analysis of variance and test the hypotheses on the main effects. What conclusions can be drawn? Use $\alpha = 0.05$.

(v) Assume that the replicates are used as blocking variables. Present the ANOVA table for the full model of this two-factor factorial design.

(vi) Compare the plots for parts (i) and (v) and comment on the adequacy of the models considered. Refer to the attached residual plots in the Appendix.

[100 marks]

2. *Satu uji kaji telah dijalankan untuk mengkaji hayat (dalam unit jam) dua jenama bateri yang berlainan di dalam tiga alat yang berlainan (radio, kamera dan pemain DVD mudahalih). Satu uji kaji faktorial dua-faktor rawak lengkap telah dijalankan dan data berikut diperoleh:*

Jenama bateri	Alat		
	Radio	Kamera	Pemain DVD
A	8.6	7.9	5.4
	8.2	8.4	5.7
B	9.4	8.5	5.8
	8.8	8.9	5.9

(i) *Adakah terdapat kesan saling tindak yang bererti? Adakah jenama bateri atau alat memberi kesan terhadap hayat? Nyatakan kesimpulan anda. Rujuk kepada jadual ANOVA di bawah dan guna* $\alpha = 0.05$

Dependent Variable: life

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	23.327(a)	5	4.665	54.355	.000
Intercept	697.687	1	697.687	8128.398	.000
battery	.801	1	.801	9.330	.022
device	22.445	2	11.223	130.748	.000
battery * device	.082	2	.041	.476	.643
Error	.515	6	.086		
Total	721.530	12			
Corrected Total	23.842	11			

a R Squared = .978 (Adjusted R Squared = .960)

(ii) *Dapatkan penganggar titik hayat bagi setiap alat. Guna ujian Tukey untuk tentukan alat yang berbeza secara bererti bagi data yang diberi. Diberikan bahawa nilai kritikal bagi beza ialah* $T_{0.05} = 2.324$.

(iii) *Jenama manakah yang anda akan syorkan?*

(iv) *Tuliskan model statistik tanpa sebutan saling tindak. Jalankan analisis varians dan uji hipotesis-hipotesis kesan utama. Apakah kesimpulan yang boleh diperoleh? Guna* $\alpha = 0.05$.

(v) *Andaikan bahawa replika telah diguna sebagai pembolehubah pemblok. Berikan jadual ANOVA bagi model penuh rekabentuk faktoran dua-faktor ini.*

(vi) *Bandingkan plot bagi bahagian (i) dan bahagaian (v) dan berikan komen tentang kecukupan model yang dipertimbang. Rujuk kepada plot reja yang diberi dalam Lampiran.*

[100 markah]

3. (a) An experiment was conducted to compare the effect of poultry manure, M with sulphate of ammonia, N and superphosphate, P on the yield of a plant. This experiment was being run in two blocks with NPM confounded and three replicates made.

The data below shows the yield obtained from the experiment.

Replicate I		Replicate II		Replicate III	
Block 1	Block 2	Block 1	Block 2	Block 1	Block 2
$np = 48.81$	$m = 40.49$	$nm = 50.43$	$m = 51.94$	$nm = 49.93$	$m = 46.94$
$nm = 58.88$	$p = 32.75$	$(1) = 40.26$	$p = 32.36$	$(1) = 39.23$	$p = 37.25$
$pm = 46.11$	$n = 55.07$	$pm = 52.31$	$n = 53.86$	$pm = 39.30$	$n = 47.37$
$(1) = 38.62$	$npm = 61.55$	$np = 49.62$	$npm = 48.49$	$np = 51.43$	$npm = 46.87$

(i) Based on the given ANOVA table below, analyze the data and state your conclusions.

Source of Variation	Sum of Squares	df	Mean Square	F	p-value
Replicate	42.585	2	21.293	0.750	0.493
Block(NPM)	4.158	1	4.158	0.146	0.709
Rep x NPM	1.382	2	0.691	0.024	0.976
N	648.440	1	648.440	22.832	0.000
P	28.536	1	28.536	1.005	0.336
M	184.871	1	184.871	6.509	0.025
NP	3.103	1	3.103	0.109	0.747
NM	90.598	1	90.598	3.190	0.099
PM	13.817	1	13.817	0.486	0.499
Error	340.810	12	28.401		
Total	1358.301	23			

(ii) Suppose that NPM is confounded in replicate I, NP is confounded in replicate II and PM is confounded in replicate III. Construct the design and the ANOVA table. List only the source of variation and degrees of freedom.

[50 marks]

3. (a) *Satu uji kaji telah dijalankan untuk membanding kesan baja asli, M dengan ammonia sulfat, N dan superfosfat, P terhadap hasil sejenis tanaman. Uji kaji ini dijalankan dalam dua blok dengan NPM dibaurkan dan tiga replika dibuat.*

Data berikut menunjukkan hasil yang diperoleh dari uji kaji ini.

Replika I		*Replika II*		*Replika III*	
Blok 1	*Blok 2*	*Blok 1*	*Blok 2*	*Blok 1*	*Blok 2*
$np = 48.81$	$m = 40.49$	$nm = 50.43$	$m = 51.94$	$nm = 49.93$	$m = 46.94$
$nm = 58.88$	$p = 32.75$	$(1) = 40.26$	$p = 32.36$	$(1) = 39.23$	$p = 37.25$
$pm = 46.11$	$n = 55.07$	$pm = 52.31$	$n = 53.86$	$pm = 39.30$	$n = 47.37$
$(1) = 38.62$	$npm = 61.55$	$np = 49.62$	$npm = 48.49$	$np = 51.43$	$npm = 46.87$

(i) *Berdasarkan jadual ANOVA yang diberi di bawah, jalankan analisis data dan nyatakan kesimpulan anda.*

Punca Ubahan	*Hasiltambah Kuasadua*	*dk*	*Min Kuasadua*	*F*	*Nilai-p*
Replicate	*42.585*	*2*	*21.293*	*0.750*	*0.493*
Block(NPM)	*4.158*	*1*	*4.158*	*0.146*	*0.709*
Rep x NPM	*1.382*	*2*	*0.691*	*0.024*	*0.976*
N	*648.440*	*1*	*648.440*	*22.832*	*0.000*
P	*28.536*	*1*	*28.536*	*1.005*	*0.336*
M	*184.871*	*1*	*184.871*	*6.509*	*0.025*
NP	*3.103*	*1*	*3.103*	*0.109*	*0.747*
NM	*90.598*	*1*	*90.598*	*3.190*	*0.099*
PM	*13.817*	*1*	*13.817*	*0.486*	*0.499*
Error	*340.810*	*12*	*28.401*		
Total	*1358.301*	*23*			

(ii) *Andaikan bahawa NPM dibaurkan dalam replika I, NP dibaurkan dalam replika II dan PM dibaurkan dalam replika III. Bina reka bentuk dan jadual ANOVA yang berkaitan. Hanya punca ubahan dan darjah kebebasan yang perlu disenaraikan.*

[50 markah]

(b) In an experiment on the preparation of chocolate cakes, three recipes for preparing the batter were compared. In addition six different baking temperatures were tested: 175° , 185° , 195° , 205° , 215° and 225°. Each time that a mix was made by any recipe, enough batter was prepared for six cakes, each of which was baked at a different temperature. Thus the recipes are the whole plot treatments, while the baking temperatures are the subplot treatments. There were 15 replications, treated as blocks. The data consists of measurements made on the texture of the cakes produced. The sum of squares of the effects and the total are given in the table below.

Effects	Sum of Squares
Blocks	10,204
Recipes	135
Temperatures	2100
Blocks x Recipes	1,199
Recipes x Temperatures	206
Total	18,143

(i) State the experimental design used. Construct the ANOVA table.

(ii) Which effects are significant? State your conclusions.

[50 marks]

(b) *Dalam satu uji kaji terhadap penyediaan kek coklat, tiga resipi penyediaan adunan telah dibanding. Sebagai tambahan enam suhu membakar kek telah diuji: 175°, 185°, 195°, 205°, 215° and 225°. Setiap kali campuran dibuat mengikut sebarang resipi, adunan dibuat secukupnya untuk enam biji kek, setiap satu dibakar pada suhu yang berlainan. Oleh itu, resipi adalah rawatan plot keseluruhan, sementara suhu adalah rawatan subplot. Terdapat 15 replika yang dijalankan dan dianggap sebagai blok. Data terdiri daripada ukuran yang dibuat terhadap tekstur kek yang dimasak. Hasiltambah kuasa dua kesan dan hasitambah kuasa dua jumlah diberi dalam jadual di bawah:*

Kesan	*Hasiltambah kuasa dua*
Blok	*10,204*
Resipi	*135*
Suhu	*2100*
Blok x Resipi	*1,199*
Resipi x Suhu	*206*
Jumlah	*18,143*

(i) *Namakan reka bentuk uji kaji yang diguna. Bina jadual ANOVA yang berkaitan.*

(ii) *Kesan manakah yang bererti? Nyatakan kesimpulan anda.*

[50 markah]

4. An experiment was conducted to determine the effect of CO_2 pressure (A), CO_2 temperature (B), peanut moisture (C), CO_2 flow rate (D), and peanut particle size (E) on the total yield of oil per batch of peanuts (y). The levels used for these factors are as follows:

Coded Level	*A* Pressure (bar)	*B* Temp (C)	*C* Moisture (% by weight)	*D* Flow (liters/min)	*E* Particle Size (mm)
-1	415	25	5	40	1.28
+1	550	95	15	60	4.05

The 16-run fractional factorial experiment conducted is as shown below:

Run	*A*	*B*	*C*	*D*	*E*	y
1	415	25	5	40	1.28	63
2	550	25	5	40	4.05	21
3	415	95	5	40	4.05	36
4	550	95	5	40	1.28	99
5	415	25	15	40	4.05	24
6	550	25	15	40	1.28	66
7	415	95	15	40	1.28	71
8	550	95	15	40	4.05	54
9	415	25	5	60	4.05	23
10	550	25	5	60	1.28	74
11	415	95	5	60	1.28	80
12	550	95	5	60	4.05	33
13	415	25	15	60	1.28	63
14	550	25	15	60	4.05	21
15	415	95	15	60	4.05	44
16	550	95	15	60	1.28	96

(i) What type of design has been used? Identify the defining relation and the alias relationships.

(ii) The table below shows the estimates of the factor effects and its normal probability plot.

A = 7.5	*AB = 5.25*	*BD = -1.75*
B = 19.75	*AC = 1.25*	*BE = 0.25*
C = 1.25	*AD = -4.0*	*CD = 2.25*
D = 0.0	*AE = 7.0*	*CE = -6.25*
E = 44.5	*BC = 3.0*	*DE = 3.5*

4. *Satu uji kaji dijalankan untuk menentukan kesan tekanan CO_2 (A), suhu CO_2 (B), kelembapan kacang tanah (C), kadar flow CO_2 (D) dan saiz particle kacang tanah (E) terhadap jumlah hasil minyak setiap batch kacang tanah (y). Aras-aras yang diguna untuk faktor-faktor ini adalah seperti berikut:*

Kod Aras	A Tekanan (bar)	B Suhu (C)	C Kelembapan (% berat)	D Arus (liters/min)	E Saiz Particle (mm)
-1	415	25	5	40	1.28
+1	550	95	15	60	4.05

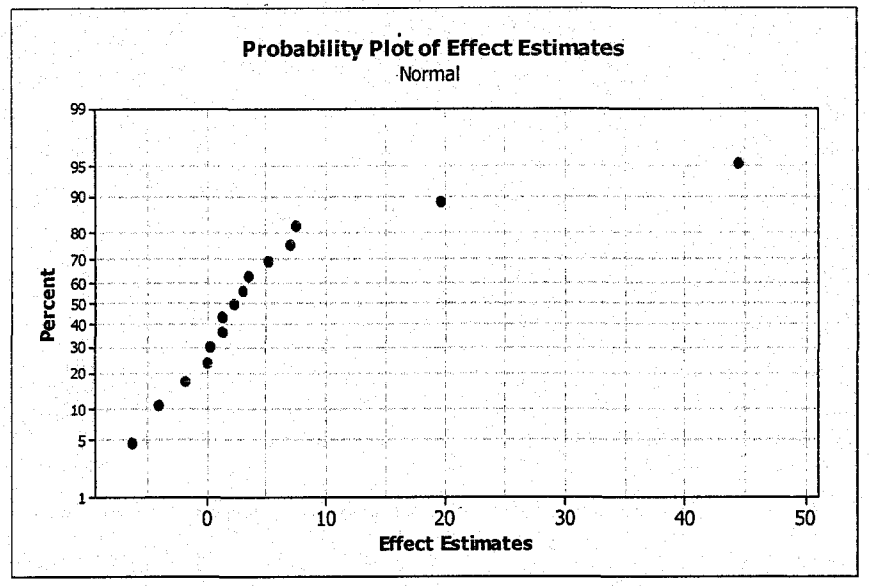
Uji kaji faktoran pecahan 16-larian yang dijalankan adalah seperti berikut:

Larian	A	B	C	D	E	y
1	415	25	5	40	1.28	63
2	550	25	5	40	4.05	21
3	415	95	5	40	4.05	36
4	550	95	5	40	1.28	99
5	415	25	15	40	4.05	24
6	550	25	15	40	1.28	66
7	415	95	15	40	1.28	71
8	550	95	15	40	4.05	54
9	415	25	5	60	4.05	23
10	550	25	5	60	1.28	74
11	415	95	5	60	1.28	80
12	550	95	5	60	4.05	33
13	415	25	15	60	1.28	63
14	550	25	15	60	4.05	21
15	415	95	15	60	4.05	44
16	550	95	15	60	1.28	96

(i) *Apakah jenis reka bentuk yang diguna? Camkan hubungan pentakrif dan hubungan aliasnya.*

(ii) *Jadual berikut menunjukkan anggaran kesan faktor dan plot kebarangkalian normal anggaran-anggaran tersebut.*

$A = 7.5$	$AB = 5.25$	$BD = -1.75$
$B = 19.75$	$AC = 1.25$	$BE = 0.25$
$C = 1.25$	$AD = -4.0$	$CD = 2.25$
$D = 0.0$	$AE = 7.0$	$CE = -6.25$
$E = 44.5$	$BC = 3.0$	$DE = 3.5$



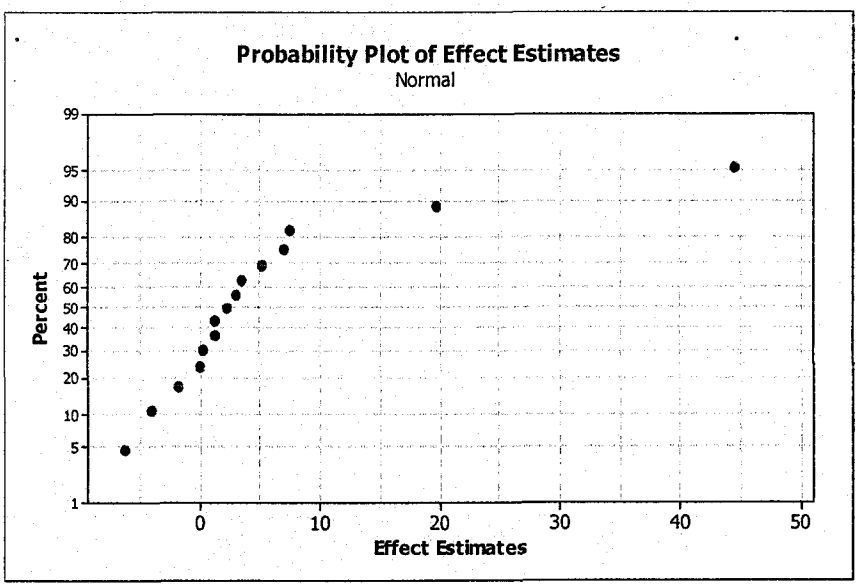
Based on the above normal probability plot, which factors can be identified as important. Perform an appropriate statistical analysis to test the hypothesis that the factors identified have a significant effect on the yield of peanut oil.

(iii) Fit a model that could be used to predict peanut oil yield in terms of the factors that you have identified as important in part (ii).

(iv) Find the predicted peanut oil yield at run number 2 and find its residual. Use the coded levels.

(v) Show how the runs of the alternate fraction can be constructed.

[100 marks]



Berdasarkan plot kebarangkalian normal di atas, faktor manakah yang boleh dikenalpasti sebagai penting. Jalankan analisis statistik yang sesuai untuk menguji bahawa faktor-faktor yang dikenalpasti mempunyai kesan bererti terhadap hasil minyak kacang tanah.

(iii) *Suaikan satu model regresi yang boleh diguna untuk meramal hasil minyak kacang tanah dalam sebutan faktor-faktor yang telah anda kenalpasti sebagai penting di dalam bahagian (ii).*

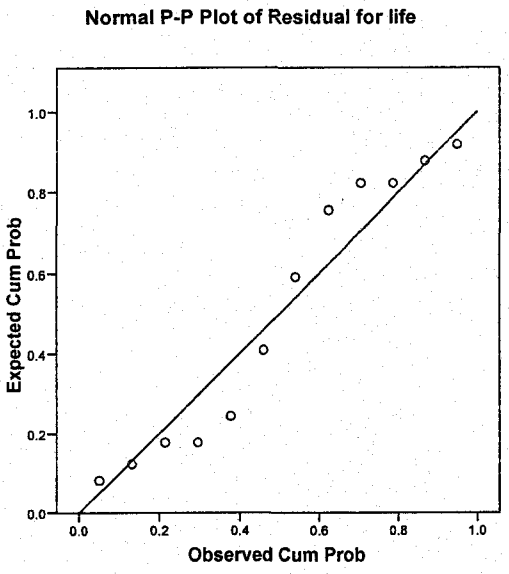
(iv) *Dapatkan ramalan hasil minyak kacang tanah pada larian bernombor 2 dan dapatkan rejanya. Guna aras yang telah dikod.*

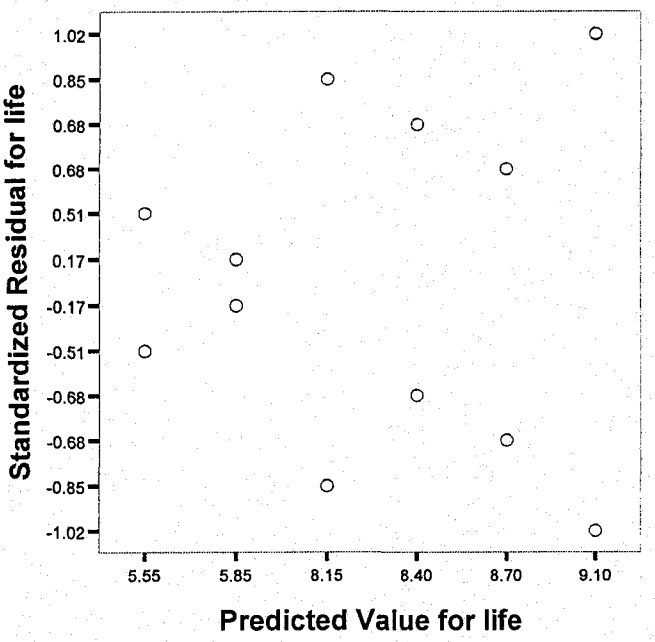
(v) *Tunjukkan bagaimana larian bagi pecahan kedua reka bentuk ini boleh dibina.*

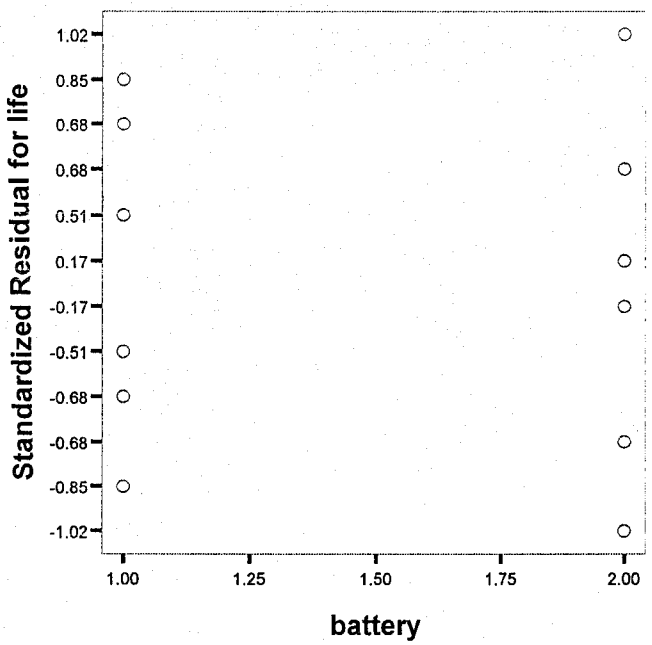
[100 markah]

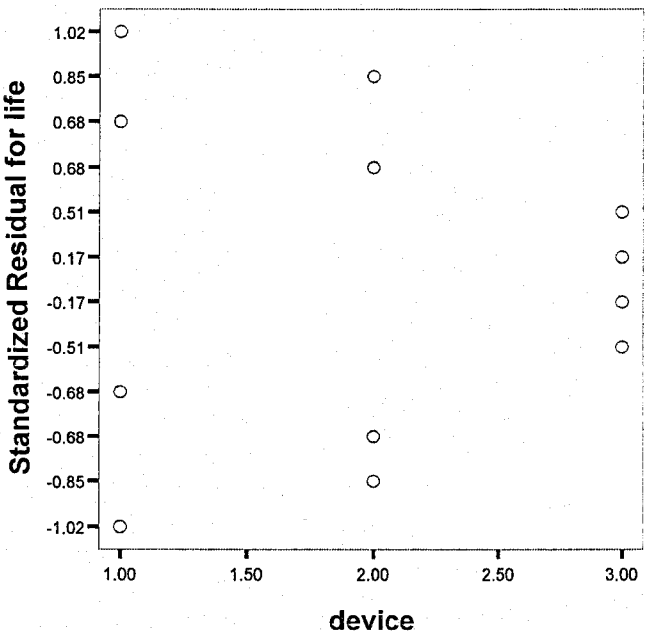
Appendix/Lampiran

Residual Plots for Question 2 (i)/ Plot Reja bagi Soalan 2 (i)



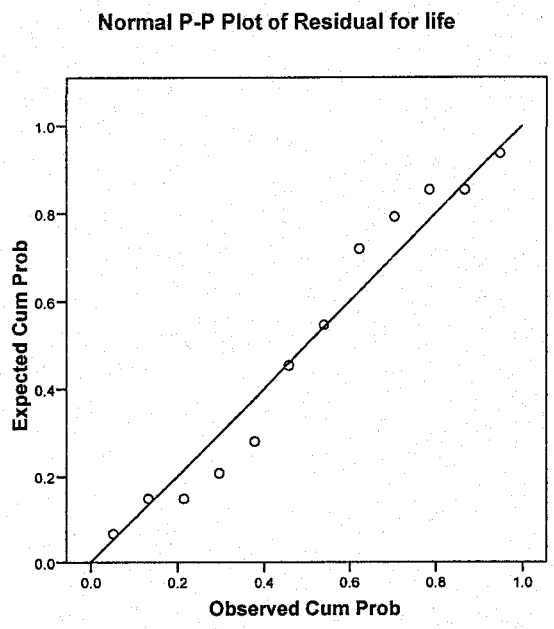


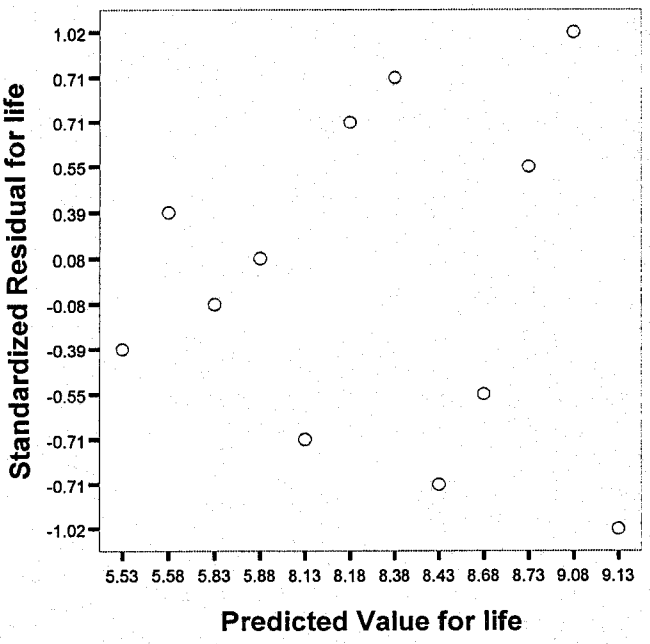

Standardized Residual for life
1.02
0.85
0.68
0.68
0.51
0.17
-0.17
-0.51
-0.68
-0.68
-0.85
-1.02
1.00
1.25
1.50
1.75
2.00
battery


Standardized Residual for life
1.02
0.85
0.68
0.68
0.51
0.17
-0.17
-0.51
-0.68
-0.68
-0.85
-1.02
1.00
1.50
2.00
2.50
3.00
device

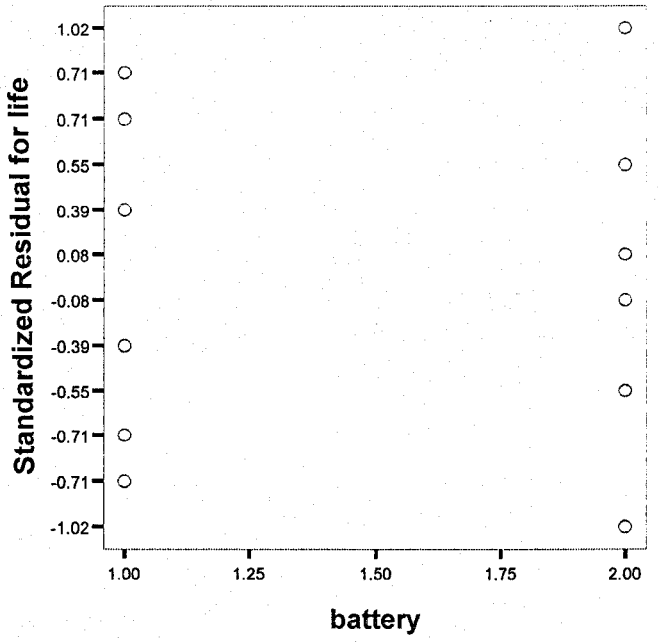
...16/-

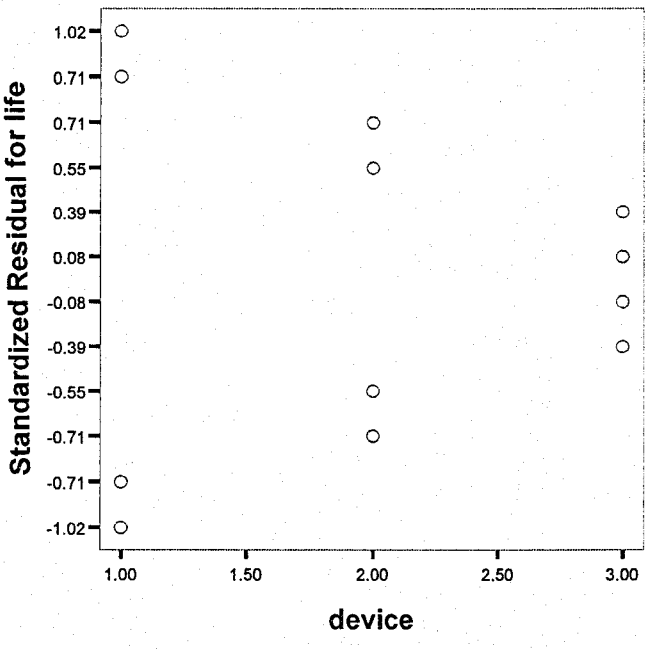
Residual Plots for Question 2 (v)/ Plot Reja bagi Soalan 2 (v)





...17/-


Standardized Residual for life
1.02
0.71
0.71
0.55
0.39
0.08
-0.08
-0.39
-0.55
-0.71
-0.71
-1.02
1.00
1.25
1.50
1.75
2.00
battery


Standardized Residual for life
1.02
0.71
0.71
0.55
0.39
0.08
-0.08
-0.39
-0.55
-0.71
-0.71
-1.02
1.00
1.50
2.00
2.50
3.00
device

...18/-

MSG265/4 – DESIGN AND ANALYSIS OF EXPERIMENTS
Formula/ Rumus

1. Two-factor Factorial Design

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_{i=1}^{a} \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_B = \sum_{j=1}^{b} \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn}$$

$$SS_{Subtotal} = \sum_{i=1}^{a}\sum_{j=1}^{b} \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{abn}$$

$$SS_{AB} = SS_{Subtotal} - SS_A - SS_B$$

$$SS_E = SS_T - SS_{Subtotal} \quad \text{atau} \quad SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

Expected Mean Square
Random Effects Model:

$$E[MS_A] = \sigma^2 + n\sigma_{\tau\beta}^2 + bn\sigma_\tau^2$$

$$E[MS_B] = \sigma^2 + n\sigma_{\tau\beta}^2 + an\sigma_\beta^2$$

$$E[MS_{AB}] = \sigma^2 + n\sigma_{\tau\beta}^2$$

$$E[MS_E] = \sigma^2$$

Mixed Model :
(*A*: fixed, *B*: random)

$$E[MS_A] = \sigma^2 + n\sigma_{\tau\beta}^2 + \frac{bn\sum_{i=1}^{a}\tau_i^2}{a-1}$$

$$E[MS_B] = \sigma^2 + an\sigma_\beta^2$$

$$E[MS_{AB}] = \sigma^2 + n\sigma_{\tau\beta}^2$$

$$E[MS_E] = \sigma^2$$

...19/-

2. Three-factor Factorial Design

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n} y_{ijkl}^2 - \frac{y_{....}^2}{abcn}$$

$$SS_A = \sum_{i=1}^{a}\frac{y_{i...}^2}{bcn} - \frac{y_{....}^2}{abcn}$$

$$SS_B = \sum_{j=1}^{b}\frac{y_{.j..}^2}{acn} - \frac{y_{....}^2}{abcn}$$

$$SS_C = \sum_{k=1}^{c}\frac{y_{..k.}^2}{abn} - \frac{y_{....}^2}{abcn}$$

$$SS_{Subtotal(AB)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\frac{y_{ij..}^2}{cn} - \frac{y_{....}^2}{abcn}$$

$$SS_{AB} = SS_{Subtotal(AB)} - SS_A - SS_B$$

$$SS_{Subtotal(AC)} = \sum_{i=1}^{a}\sum_{k=1}^{c}\frac{y_{i.k.}^2}{bn} - \frac{y_{....}^2}{abcn}$$

$$SS_{AC} = SS_{Subtotal(AC)} - SS_A - SS_C$$

$$SS_{Subtotal(BC)} = \sum_{j=1}^{b}\sum_{k=1}^{c}\frac{y_{.jk.}^2}{an} - \frac{y_{....}^2}{abcn}$$

$$SS_{BC} = SS_{Subtotal(BC)} - SS_B - SS_C$$

$$SS_{Subtotal(ABC)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\frac{y_{i.k.}^2}{bn} - \frac{y_{....}^2}{abcn}$$

$SS_E = SS_T - SS_{Subtotal}$ atau $SS_E = SS_T - SS_A - SS_B - SS_{AB}$

3. Two-stage Nested Design

$$SS_T = \sum_i\sum_j\sum_k y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_{i=1}^{a}\frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_{B(A)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\frac{y_{ij.}^2}{n} - \sum_{i=1}^{a}\frac{y_{i..}^2}{bn}$$

$$SS_E = SS_T - SS_A - SS_{B(A)}$$