UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

MSG 253 – Queueing Systems and Simulation
[Sistem Giliran dan Simulasi]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVENTEEN pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all three** [3] questions.

*[**Arahan:** Jawab **semua tiga** [3] soalan.]*

1. (a) Consider an M/M/1 queueing system with an arrival rate of λ and a service rate of μ.

i) Draw a rate-diagram to represent the queueing system.

ii) Using the birth and death process and under the assumption that the system is stable, show that the probability that the system is in state n is:

$$P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n \quad \text{for} \quad n = 0,1,2\cdots$$

iii) Next, show that the average state of the system is:

$$L = \frac{\lambda}{\mu - \lambda}$$

[40 marks]

(b) A bank has 5 tellers on duty, each with a separate waiting line. Customers arrive according to a Poisson process at rate of 100 per hour. Customers will select the shortest waiting line at the time of their arrival and will remain in the line until their transactions are completed. In effect, on average, the distribution of customers among the tellers are equal. Each teller transaction requires on average 2 minutes, distributed exponentially.

i) How long, on average, would each customer spend in the bank?

ii) How many customers, on average, are in the bank as a whole?

iii) What is the percentage busy time of a particular teller?

The bank is considering designating one teller as a quick-serve teller for customers with a deposit transaction only. A survey showed that 32 percent of all customers would be eligible for this category of services, with a uniform service time between 0.3 to 0.7 minutes. The service time for the remaining customers would remain exponential with an average of 2.7 minutes per customer. The non-quick-serve customers would select one of the four lines on the same basis as present.

iv) How long, on average, would each customer spend in the bank?

v) How many customers, on average, are in the bank as a whole?

vi) What is the percentage busy time of a particular normal teller? What about the percentage busy time of the quick-serve teller?

1. (a) *Pertimbangkan sistem giliran M/M/1 dengan kadar ketibaan λ dan kadar layanan μ.*

    i) *Lukiskan gambar rajah kadar bagi sistem giliran itu.*

    ii) *Dengan menggunakan proses lahir-mati dan di bawah andaian bahawa sistem berkeadaan mantap, tunjukkan bahawa kebarangkalian sisitem berkeadaan n adalah:*

$$P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n \quad untuk \quad n = 0,1,2\cdots$$

    iii) *Seterusnya, tunjukkan bahawa keadaan purata sistem adalah:*

$$L = \frac{\lambda}{\mu - \lambda}$$

[40 markah]

(b) *Sebuah bank menyediakan 5 teller yang setiap satunya mempunyai barisan menunggu yang tersendiri. Pelanggan tiba mengikut proses Poisson dengan kadar 100 sejam. Pelanggan akan memilih barisan menunggu yang terpendek semasa ketibaan mereka dan akan kekal menunggu di dalamnya sehingga selesai urusan mereka. Pada hitung panjangnya didapati bahawa pembahagian pelanggan di antara teller adalah sama rata. Masa layan setiap teller didapati mengikuti agihan eksponen dengan purata 2 minit.*

    i) *Pada puratanya, berapa lamakah setiap pelanggan akan berada di dalam bank?*

    ii) *Berapa ramaikah purata pelanggan yang akan berada di dalam bank pada sesuatu masa?*

    iii) *Apakah peratusan masa sibuk seseorang teller?*

Bank itu sedang membuat pertimbangan untuk menjadikan salah satu daripada teller sebagai teller-pantas yang akan hanya mengendalikan urusan penyimpanan wang sahaja. Data yang ada menunjukkan bahawa 32% daripada urusan dengan teller adalah urusan penyimpanan wang dan masa layan bagi urusan itu adalah seragam di antara 0.3 ke 0.7 minit. Masa layan bagi urusan lain masih lagi mengikuti agihan eksponen tetapi dengan purata 2.7 minit perpelanggan. Pembahagian pelanggan di antara teller sedia ada yang lain adalah sama rata.

    iv) *Pada puratanya, berapa lamakah setiap pelanggan akan berada di dalam bank?*

    v) *Berapa ramaikah purata pelanggan yang akan berada di dalam bank pada sesuatu masa?*

    vi) *Apakah peratusan masa sibuk seseorang teller biasa? Bagaimana pula dengan peratusan masa sibuk teller-pantas?*

Another option being considered by the bank is to have all customers, regardless of type, form a single waiting line. As soon as any teller is free, the first person in line would be dispatched to that teller. Arrival rates and service times are expected to be unchanged with this option.

vii) How long, on average, would each customer spend in the bank?
viii) How many customers, on average, are in the bank as a whole?
ix) What is the percentage busy time of a particular teller?
x) Which system do you think the bank should adopt?

[60 marks]

2. (a) A family with four children has a single bathroom. Each morning during a 1-hour period before school, there is a congestion problem related to its use. The parents minimize the problem by remaining in bed. While lying in bed, the mother, who is studying queueing theory, makes the following observations. The requirement to use the bathroom seems to be a random process. The time between the moment when one child leaves the bathroom and the moment when the same child returns averages 15 minutes, but actually the time is a random variable with an exponential distribution. All children are assumed to have the same bathroom needs and hence the same arrival rate. The time spent in the bathroom in any one instance is also exponentially distributed with a mean of 5 minutes.

A child arriving at the bathroom and finding it occupied will wait in a queue, all the while taunting the person inside in a loud voice and thus disturbing the parents. When there is no queue, the house is relatively calm. How many minutes in the 1-hour period should the parents expect to have peace and quiet?

[25 marks]

Satu lagi pilihan yang sedang dipertimbangan ialah dengan mengadakan satu sahaja barisan menunggu. Sebaik sahaja satu daripada teller itu bersenang, pelanggan yang menunggu di hadapan sekali akan berurusan dengan teller itu. Kadar ketibaan dan kadar layanan dijangkakan tidak berubah jika opsyen ini dilaksanakan.

vii) *Pada puratanya, berapa lamakah setiap pelanggan akan berada di dalam bank?*

viii) *Berapa ramaikah purata pelanggan yang akan berada di dalam bank pada sesuatu masa?*

ix) *Apakah peratusan masa sibuk seseorang teller?*

x) *Sistem manakah yang wajar dilaksanakan oleh pihak bank?*

[60 markah]

2. (a) *Satu keluarga yang mempunyai empat orang anak mendiami rumah yang mempunyai sebuah bilik air. Setiap pagi dalam jangkamasa 1-jam sebelum waktu persekolahan, timbul masalah kesibukan penggunaan bilik air itu. Untuk tidak merumitkan lagi keadaan, kedua ibu-bapa berada di katil pada waktu itu. Semasa berbaring di katil, siibu yang sedang belajar teori giliran, menyedari akan perlakuan berikut. Keperluan penggunaan bilik air adalah mengikut proses rawak. Masa di antara saat seorang kanak-kanak keluar daripada bilik air sehingga saat kanak-kanak yang sama kembali menggunakannya adalah pada puratanya 15 minit, dengan mengikuti agihan eksponen. Kesemua kanak-kanak dianggarkan mempunyai keperluan penggunaan bilik air yang sama, dengan itu, kadar ketibaan kesemuanya adalah sama. Masa penggunaan bilik air adalah juga mengikut agihan eksponen dengan min 5 minit.*

Seorang kanak-kanak yang hendak menggunakan bilik air dan mendapatinya sedang digunakan oleh kanak-kanak yang lain akan menunggu dalam satu barisan menunggu sambil memekik dan menggesa orang yang di dalam dan ini tentu sekali menganggu ibu-bapa mereka. Apabila tidak ada barisan menunggu, rumah itu kembali tenang. Berapa minitkah dalam masa 1-jam itu ibu-bapa boleh menjangkakan berada dalam keadaan aman dan tenang?

[25 markah]

(b) A finite queue has three servers and two additional spaces for waiting customers. There is an infinite input source, and the arrival rate is 50/hour. When arrivals find the queue full, they balk. The following steady-state probabilities have been determined for the number of customers in the systems:

$$P_0 = 0.05, P_1 = 0.12, P_2 = 0.23, P_3 = 0.25, P_4 = 0.20, P_5 = 0.15$$

Find each of the following values.

i) Expected number of customers in the queue.
ii) Expected time in the queue.
iii) Expected number of customers in the system.
iv) Expected time in the system.
v) Expected number of customers in service.
vi) Expected time in service.
vii) Proportion of customers who balk.
viii) Percentage utilization of each server.
ix) Probability that there will be fewer than two in the system.
x) Probability that there will be four or more in the system.
xi) Probability that an arriving customer will obtain immediate service (rather than having to wait in a queue).

[35 marks]

(b) *Suatu sistem giliran terhingga mempunyai tiga pelayan dan dua ruang tambahan untuk pelanggan menunggu. Sumber input adalah tak terhingga dan kadar ketibaan adalah 50/jam. Apabila pelanggan tiba dan mendapati tiada tempat untuk menunggu, mereka akan pergi ke tempat lain. Kebarangkalian dalam keadaan mantap berikut telah dikenalpasti bagi bilangan pelanggan di dalam sistem:*

$$P_0 = 0.05, P_1 = 0.12, P_2 = 0.23, P_3 = 0.25, P_4 = 0.20, P_5 = 0.15$$

Tentukan setiap nilai berikut.

i) *Bilangan purata pelanggan di dalam barisan menunggu.*
ii) *Masa purata dalam barisan menunggu.*
iii) *Bilangan purata pelanggan di dalam sistem.*
iv) *Masa purata di dalam sistem.*
v) *Bilangan purata pelanggan yang sedang dilayan.*
vi) *Masa purata layanan.*
vii) *Kadaran pelanggan yang pergi ke tempat lain.*
viii) *Peratusan masa manfaat setiap pelayan.*
ix) *Kebarangkalian bahawa terdapat kurang daripada dua pelanggan di dalam sistem.*
x) *Kebarangkalian bahawa terdapat empat atau lebih pelanggan di dalam sistem.*
xi) *Kebarangkalian bahawa pelanggan yang tiba akan terus dilayan (tidak perlu menunggu).*

[35 markah]

(c) The unloading platform of a warehouse needs to be renovated. The manager is considering three schemes (A, B and C) that have been suggested. Information about the schemes are as follows:

Scheme	Daily fixed cost (RM)	Daily variable operational cost (RM)	Hourly unloading rate (number of sacks)
A	60	100	1000
B	130	150	2000
C	250	200	6000

Fixed costs are expenses that have to be paid if a platform is open, regardless of whether it is being used or not. Variable operational costs are expenses that have to be paid only when the platform is used for unloading.

The arrival rate of trucks to the unloading platform is 15 per day and follows a Poisson distribution. The platform is open 10 hours per day and the capacity of each truck is 500 sacks. The unloading time of each truck follows an exponential distribution and only one truck can be unloaded at a time.

If loss due to waiting of each truck until unloading is completed is RM10 per hour, which scheme is the best?

[40 marks]

(c) *Platform memunggah barang di sebuah gudang perlu diubahsuai. Pengurus gudang itu sedang menimbangkan tiga skim (A, B dan C) yang dicadangkan. Maklumat mengenai skim itu adalah seperti berikut:*

Skim	*Kos tetap harian (RM)*	*Kos operasi boleh ubah harian (RM)*	*Kadar pemunggahan sejam (bilangan guni)*
A	*60*	*100*	*1000*
B	*130*	*150*	*2000*
C	*250*	*200*	*6000*

Kos tetap adalah kos yang perlu dibiayai jika platform dibuka, tidak kira sama ada ianya digunakan atau tidak. Kos operasi boleh ubah adalah kos yang perlu dibiayai hanya jika platform itu digunakan untuk memunggah.

Kadar ketibaan purata trak ke tempat memunggah adalah 15 sehari. Ketibaan didapati berlaku mengikut agihan Poisson. Platform dibuka 10 jam sehari dan muatan purata setiap trak adalah 500 guni. Masa memunggah setiap trak adalah mengikut agihan eksponen dan sebuah trak sahaja yang boleh dipunggah pada sesuatu masa.

Jika kerugian akibat daripada sesebuah trak terpaksa menunggu sehingga siap dipunggah adalah RM10 sejam, skim manakah yang paling baik?

[40 markah]

3. (a) There is a petrol pump and a car-wash machine at a petrol station. It is estimated that 40% of customers who fill-up petrol at the station will also have a car wash. Time taken for a car wash using the machine is a constant 2 minutes. The following data on customer arrivals have been gathered:

Interarrival time (minutes)	Number of occurrrences
1	136
2	34
3	102
4	51
5	17

The service time distribution at the gas pump is:

Service time (minutes)	Probability
1	0.20
2	0.40
3	0.30
4	0.10

Perform a hand simulation for the arrival and service of 10 cars to the petrol station. Determine the average waiting time at the petrol pump and also the average waiting time at the car-wash machine. Besides that, also determine the percentage idle time of the petrol pump and the percentage idle time of the car wash machine. What is the maximum number of cars waiting to be washed?

(Use the enclosed 2-digit random number table with the first column for *inter-arrival time*, second column for *determining whether to do a car wash or not* and the third column for *service time.*)

[50 marks]

(b) Referring to Question 3(a), write a GPSSPC program that can simulate the operation of the petrol station for a month, assuming that the station is open 12 hours per day. Assume that, besides the normal stream of customers who arrived for petrol and might need a car wash, there is another stream of customers who are interested just to have their car washed. The arrival pattern of this stream of customers is uniform 4 ± 2 minutes. This stream of customers will only enter the station if there is no cars waiting for the car-wash machine.

[50 marks]

3. (a) *Terdapat sebuah pam mengisi petrol dan sebuah mesin pencuci kereta di sebuah stesen minyak. Dianggarkan bahawa 40% daripada pelanggan yang mengisi petrol di stesen itu memerlukan perkhidmatan mencuci kereta. Masa yang diperlukan untuk mencuci kereta menggunakan mesin pencuci ialah 2 minit. Maklumat yang dikumpul tentang ketibaan pelanggan adalah seperti berikut:*

Lat ketibaan (minit)	*Bilangan kejadian*
1	136
2	34
3	102
4	51
5	17

Agihan masa layan di pam megisi petrol adalah:

Masa layan (minit)	*Kebarangkalian*
1	0.20
2	0.40
3	0.30
4	0.10

Lakukan simulasi dengan tangan untuk ketibaan dan layanan 10 buah kereta ke stesen minyak itu. Tentukan masa purata sesebuah kereta perlu menunggu untuk diisi petrol dan juga masa purata menunggu untuk dicuci. Selain daripada itu, tentukan juga peratusan masa pam petrol bersenang dan juga peratusan masa mesin pencuci bersenang. Berapa banyak keretakah paling maksimum yang menunggu untuk dicuci.

(Gunakan jadual nombor rawak 2-digit yang disertakan dengan lajur pertama untuk lat ketibaan, lajur kedua untuk menentukan sama ada hendak mencuci kereta ataupun tidak dan lajur ketiga untuk masa layanan.)

[50 markah]

(b) *Merujuk kepada Soalan 3(a), tulis satu aturcara GPSSPC yang dapat melakukan simulasi pengoperasian stesen minyak itu selama sebulan, dengan mengandaikan bahawa stesen itu beroperasi 12 jam sehari. Andaikan juga bahawa, selain daripada aliran pelanggan biasa yang tiba untuk mengisi petrol dan mungkin mengisi minyak kemudiannya, terdapat satu lagi aliran pelanggan yang tiba untuk mencuci kereta sahaja. Corak ketibaan aliran pelanggan ini adalah seragam 4 $\pm$ 2 minit. Aliran pelanggan ini akan memasuki stesen hanya apabila tidak ada kereta yang menunggu untuk dicuci.*

[50 markah]

APPENDIX 1 / *LAMPIRAN 1*

Formulas for Queueing Theory:

1. *M/M/1*:

$$\rho = \lambda / \mu$$

$$P_n = (1-\rho)\rho^n \quad \text{for } n = 0,1,2,\ldots$$

$$L = \frac{\lambda}{\mu - \lambda}$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$W = \frac{1}{\mu-\lambda} \quad , \quad W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$P[w > t] = e^{-t/w}$$

$$P\left[w_q > t\right] = \rho e^{-t/w}$$

2. *M/M/s*:

$$\rho = \frac{\lambda}{s\mu}$$

$$P_0 = \left[\frac{(\lambda/\mu)^s}{s!}\frac{1}{(1-\rho)} + \sum_{n=0}^{s-1}\frac{(\lambda/\mu)^n}{n!}\right]^{-1}$$

$$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!}P_0 & , \text{ if } 0 \le n \le s \\ \dfrac{(\lambda/\mu)^n}{s!s^{n-s}}P_0 & , \text{ if } n \le s \end{cases}$$

$$L_q = \frac{(\lambda/\mu)^s \rho}{s!(1-\rho)^2}P_0$$

$$W_q = \frac{L_q}{\lambda} \quad , \quad W = W_q + 1/\mu$$

$$L = L_q + \lambda/\mu$$

$$P\left[w_q > t\right] = e^{-\mu t}\left[1 + \frac{P_0\left(\dfrac{\lambda}{\mu}\right)^s}{s!(1-\rho)}\left(\frac{1-e^{\mu t(s-1-\lambda/\mu)}}{s-1-\lambda/\mu}\right)\right]$$

$$P\left[w_q > t\right] = \left[1 - P\left\{w_q = 0\right\}\right]e^{-s\mu(1-\rho)t}$$

where $P\left\{w_q = 0\right\} = \sum_{n=0}^{s-1} P_n$

APPENDIX 2 / *LAMPIRAN 2*

3. *M/M/s*: finite population of size *M*.

$$P_0=\left[\sum_{n=0}^{s-1}\binom{M}{n}\left(\frac{\lambda}{\mu}\right)^n+\sum_{n=s}^{M}\binom{M}{n}\frac{n!}{s^{n-s}s!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$P_n=\begin{cases}P_0\binom{M}{n}\left(\frac{\lambda}{\mu}\right)^n & ,\ \text{if } 0\le n\le s\\ P_0\binom{M}{n}\left(\frac{n!}{s^{n-s}s!}\right)\left(\frac{\lambda}{\mu}\right)^n & ,\ \text{if } s<n\le M\\ 0 & ,\ \text{if } n>M\end{cases}$$

$$L=P_0\left[\sum_{n=0}^{s-1}n\binom{M}{n}\left(\frac{\lambda}{\mu}\right)^n+\sum_{n=s}^{M}n\binom{M}{n}\frac{n!}{s^{n-s}s!}\left(\frac{\lambda}{\mu}\right)^n\right]$$

$$L_q=L-s+P_0\sum_{n=0}^{s-1}(s-n)\binom{M}{n}\left(\frac{\lambda}{\mu}\right)^n$$

$$W=\frac{L}{\lambda(M-L)}\quad,\quad W_q=\frac{L_q}{\lambda(M-L)}$$

4. *M/G/1*:

$$P_0=1-\rho$$

$$L_q=\frac{\lambda^2\sigma^2+\rho^2}{2(1-\rho)}$$

$$L=\rho+L_q$$

$$W_q=\frac{L_q}{\lambda}\quad,\quad W=w_q+\frac{1}{\mu}$$

5. $M/E_k/1$:

$$L_q=\frac{1+k}{2k}\frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$W_q=\frac{1+k}{2k}\frac{\lambda}{\mu(\mu-\lambda)}$$

$$W=W_q+1/\mu$$

$$L=\lambda W$$

APPENDIX 3 / *LAMPIRAN 3*

6. *M/M/1/k*:

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{k+1}} & (\rho \neq 1) \\ \dfrac{1}{k+1} & (\rho = 1) \end{cases}$$

For $\rho \neq 1$

$$L = \frac{\rho\left[1-(k+1)\rho^k + k\rho^{k+1}\right]}{\left(1-\rho^{k+1}\right)(1-\rho)}$$

$$L_q = L - (1 - P_0) = L - \frac{\rho\left(1-\rho^k\right)}{1-\rho^{k+1}}$$

$$W = L/\lambda' \quad , \quad \lambda' = \mu\left(L - L_q\right)$$

$$W_q = W - 1/\mu = L_q/\lambda'$$

For $\rho = 1$

$$L = \frac{k}{2}$$

7. *M/M/s/k*:

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (0 \leq n < s) \\ \dfrac{1}{s^{n-s}s!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (s \leq n \leq k) \end{cases}$$

$$P_0 = \begin{cases} \left[\displaystyle\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{(\lambda/\mu)^s}{s!}\,\frac{1-\left(\dfrac{\lambda}{s\mu}\right)^{k-s+1}}{1-\dfrac{\lambda}{s\mu}}\right]^{-1} & \text{for } \left(\dfrac{\lambda}{s\mu} \neq 1\right) \\ \left[\displaystyle\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{(\lambda/\mu)^s}{s!}(k-s+1)\right]^{-1} & \text{for } \left(\dfrac{\lambda}{s\mu} = 1\right) \end{cases}$$

$$L_q = \frac{P_0 (s\rho)^S \rho}{s!(1-\rho)^2}\left[1-\rho^{k-s+1} - (1-\rho)(k-s+1)\rho^{k-s}\right]$$

APPENDIX 4 / *LAMPIRAN 4*

$$L = L_q + s - P_0 \sum_{n=0}^{s-1} \frac{(s-n)(\rho s)^n}{n!}$$

$$W = \frac{L}{\lambda'} \quad , \quad \lambda' = \lambda(1-P_k)$$

$$W_q = W - \frac{1}{\mu}$$

$$W_q = \frac{L_q}{\lambda'}$$

8. *M/M/s/s*:

$$P_n = \frac{(\lambda/\mu)^n / n!}{\sum_{i=0}^{s} \left(\frac{\lambda}{\mu}\right)^i / i!} \quad \text{for } (0 \le n \le s)$$

$$P_s = \frac{(s\rho)^s / s!}{\sum_{i=0}^{s} (s\rho)^i / i!} \quad \text{where } \left(\rho = \frac{\lambda}{s\mu}\right).$$

$$L = \frac{\lambda}{\mu}(1-P_s) \quad , \quad W = \frac{L}{\lambda'} \text{ where } \lambda' = \lambda(1-P_s)$$

9. $M/M/\infty$:

$$P_n = \frac{(\lambda/\mu)^n e^{-\lambda/\mu}}{n!} \quad \text{for } n = 0, 1, 2, \ldots$$

$$L = \lambda/\mu$$

$$W = \frac{1}{\mu}$$

APPENDIX 5 / *LAMPIRAN 5*

10. *M/M/1*: state-dependent service

$$\mu_n = \begin{cases} \mu_1 & (1 \le n \le k) \\ \mu & (n \ge k) \end{cases}$$

$$P_0 = \left[\frac{1-\rho_1^k}{1-\rho_1} + \frac{\rho\rho_1^{k-1}}{1-\rho} \right]^{-1} \quad (\rho_1 = \lambda/\mu_1, \rho = \lambda/\mu < 1)$$

$$L = P_0 \left[\frac{\rho_1\left[1+(k-1)\rho_1^k - k\rho_1^{k-1}\right]}{(1-\rho_1)^2} + \frac{\rho\rho_1^{k-1}\left[k-(k-1)\rho\right]}{(1-\rho)^2} \right]$$

$$L_q = L - (1-P_0)$$

$$W = \frac{L}{\lambda} \qquad W_q = \frac{L_q}{\lambda}$$

$$W = W_q + \frac{1-P_0}{\lambda}$$

$$P_n = \begin{cases} \left(\dfrac{\lambda}{\mu_1}\right)^n P_0 & (0 \le n < k) \\ \dfrac{\lambda^n}{\mu_1^{k-1}\mu^{n-k+1}} P_0 & (n \ge k) \end{cases}$$

11. *M/M/1*: finite population of size *M*.

$$P_0 = \left[\sum_{n=0}^{M} \frac{M!}{(M-n)!} \left(\frac{\lambda}{\mu}\right)^n \right]^{-1}$$

$$P_n = \frac{M!}{(M-n)!} \left(\frac{\lambda}{\mu}\right)^n P_0 \quad \text{for } n = 1, 2, \ldots, M$$

$$L = M - \frac{\mu}{\lambda}[1-P_0]$$

$$L_q = M - \frac{\lambda+\mu}{\lambda}(1-P_0)$$

$$W = \frac{L}{\lambda'} \;,\quad W_q = \frac{L_q}{\lambda'} \quad \text{where } \lambda' = \lambda(M-L)$$

APPENDIX 6 / *LAMPIRAN 6*

TWO-DIGIT RANDOM NUMBER TABLE

03	26	48	92	38	96	41	04	35	84
71	44	81	46	44	47	07	20	58	04
33	75	06	41	87	72	63	88	59	54
53	71	27	13	37	45	89	61	30	26
41	15	43	91	46	81	57	39	34	86
16	18	75	11	26	80	93	97	29	33
88	50	00	56	70	19	90	00	93	95
13	10	08	15	29	33	75	70	43	05
15	72	73	69	27	75	72	95	99	56
64	10	99	02	18	26	78	69	19	12
98	66	53	86	34	71	09	88	56	08
43	05	06	19	91	78	03	65	08	16
69	82	02	61	98	50	74	84	60	41
06	40	10	24	68	42	39	97	25	55
34	86	83	41	33	83	85	92	32	29
46	05	92	36	82	04	67	05	18	69
28	73	59	56	43	88	61	17	07	48
35	53	49	39	98	14	16	76	69	10
90	90	18	27	75	08	75	17	55	68
62	32	97	16	33	66	02	34	62	26

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