UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

MSG 252 – Linear and Integer Programming
[Pengaturcaraan Linear dan Integer]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer all **nine [9]** questions.

[Arahan: *Jawab semua **sembilan** [9] soalan.]*

1. Write the dual of the following LP:

$$\begin{aligned} \text{Maximize} \quad & z = 4x_1 + x_2 \\ \text{Subject to} \quad & 3x_1 + x_2 \geq 6 \\ & 2x_1 + x_2 \geq 4 \\ & x_1 + x_2 = 3 \\ & x_1,\ x_2 \geq 0 \end{aligned}$$

Give a lower bound and an upper bound for the objective value of the problem.

[10 marks]

2. Consider the following LP and its partial optimal tableau.

$$\begin{aligned} \text{Maximize} \quad & z = 3x_1 + 4x_2 + x_3 \\ \text{Subject to} \quad & x_1 + x_2 + x_3 \leq 50 \\ & 2x_1 - x_2 + x_3 \geq 15 \\ & x_1 + x_2 = 10 \\ & x_1,\ x_2,\ x_3 \geq 0 \end{aligned}$$

Basic	x_1	x_2	x_3	s_1	s_2	A_2	A_3	Solution
z	1	0	0	1	0			
s_2	−3	0	0	1	1			
x_3	0	0	1	1	0			
x_2	1	1	0	0	0			

The slack and surplus variables of the first and second constraints are s_1 and s_2 respectively while A_2 and A_3 are the artificial variables of the second and third constraints respectively.

From the tableau determine
(a) the missing values.
(b) the optimal dual solution.

[10 marks]

3. Construct a pair of primal and dual problems, each with two decision variables and two functional constraints such that the primal problem has no feasible solution and the dual has an unbounded objective function.

[10 marks]

1. *Tuliskan dual bagi masalah PL berikut:*

$$\begin{aligned} &\textit{Maksimumkan} && z = 4x_1 + x_2 \\ &\textit{Terhadap} && 3x_1 + x_2 \geq 6 \\ & && 2x_1 + x_2 \geq 4 \\ & && x_1 + x_2 = 3 \\ & && x_1,\ x_2 \geq 0 \end{aligned}$$

Berikan suatu batas bawah dan batas atas bagi nilai objektif masalah ini.

[10 markah]

2. *Pertimbangkan masalah PL berikut dan sebahagian daripada tablo optimumnya.*

$$\begin{aligned} &\textit{Maksimumkan} && z = 3x_1 + 4x_2 + x_3 \\ &\textit{terhadap} && x_1 + x_2 + x_3 \leq 50 \\ & && 2x_1 - x_2 + x_3 \geq 15 \\ & && x_1 + x_2 = 10 \\ & && x_1,\ x_2,\ x_3 \geq 0 \end{aligned}$$

Asas	x_1	x_2	x_3	s_1	s_2	A_2	A_3	*Penyelesaian*
z	1	0	0	1	0			
s_2	−3	0	0	1	1			
x_3	0	0	1	1	0			
x_2	1	1	0	0	0			

Pembolehubah lalai dan lebihan bagi kekangan pertama dan kedua masing-masing adalah s_1 dan s_2 manakala A_2 dan A_3 masing-masing adalah pembolehubah buatan dari kekangan kedua dan ketiga.

Daripada tablo tentukan
(a) nilai-nilai yang tertinggal.
(b) penyelesaian optimum masalah dual.

[10 markah]

3. *Bentukkan sepasang masalah primal dan dual, setiap satunya dengan dua pembolehubah dan dua kekangan berfungsi dengan masalah primal tidak mempunyai penyelesaian tersaur manakala fungsi objektif dual tak terbatas.*

[10 markah]

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4. Consider the following LP problem and its optimal tableau.

Maximize $z = 2x_1 + 7x_2 - 3x_3$

Subject to $x_1 + 3x_2 + 4x_3 \leq 30$

$x_1 + 4x_2 - x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$

Basic	x_1	x_2	x_3	x_4	x_5	solution
z	0	1	1	0	2	20
x_4	0	−1	5	1	−1	20
x_1	1	4	−1	0	1	10

The slack variables are represented by x_4 and x_5. Conduct sensitivity analysis by independently investigating each of the following changes in the original model. Obtain a new optimal solution if the current solution changes.

(a) Change the right-hand sides to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$

(b) Change the coefficients of x_3 to $\begin{bmatrix} c_3 \\ a_{13} \\ a_{23} \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$

(c) Introduce a new variable x_6 with coefficients $\begin{bmatrix} c_6 \\ c_{16} \\ a_{26} \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$

(d) Introduce a new constraint $3x_1 + 2x_2 + 3x_3 \leq 25$

[20 marks]

5. Solve the following IP by using the branch and bound method.

Maximize $z = 3x_1 + x_2$

Subject to $5x_1 + x_2 \leq 12$

$2x_1 + x_2 \leq 8$

$x_1, x_2 \geq 0$ and integer

[10 marks]

4. *Pertimbangkan masalah PL berikut dan tablo optimumnya.*

Maksimumkan $z = 2x_1 + 7x_2 - 3x_3$

Terhadap $x_1 + 3x_2 + 4x_3 \leq 30$

$x_1 + 4x_2 - x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$

Asas	x_1	x_2	x_3	x_4	x_5	*Penyelesaian*
z	0	1	1	0	2	20
x_4	0	−1	5	1	−1	20
x_1	1	4	−1	0	1	10

Pembolehubah lalai diwakili oleh x_4 *dan* x_5. *Lakukan analisis kepekaan secara berasingan dengan memeriksa perubahan-perubahan berikut di dalam masalah asal. Dapatkan penyelesaian optimum yang baru jika penyelesaian semasa berubah.*

(a) Tukar nilai-nilai sebelah kanan kepada $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$

(b) Tukar pekali-pekali x_3 *kepada* $\begin{bmatrix} c_3 \\ a_{13} \\ a_{23} \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}$

(c) Tambahkan satu pembolehubah baru x_6 *dengan pekali-pekali* $\begin{bmatrix} c_6 \\ c_{16} \\ a_{26} \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$

(d) Masukkan kekangan baru $3x_1 + 2x_2 + 3x_3 \leq 25$

[20 markah]

5. *Selesaikan masalah PI berikut dengan menggunakan kaedah cabang dan batas.*

Maksimumkan $z = 3x_1 + x_2$

Terhadap $5x_1 + x_2 \leq 12$

$2x_1 + x_2 \leq 8$

$x_1, x_2 \geq 0$ *dan integer*

[10 markah]

6. Consider the following LP and its optimal tableau.

Maximize $z = 40x_1 + 50x_2$

Subject to $x_1 + 2x_2 \leq 40$

$x_1 + x_2 \leq 30$

$2x_1 + x_2 \leq 40$

$x_1, x_2 \geq 0$

Basic	x_1	x_2	x_3	x_4	x_5	Solution
z	0	0	20	0	10	1200
x_2	0	1	$\frac{2}{3}$	0	$-\frac{1}{3}$	$\frac{40}{3}$
x_4	0	0	$-\frac{1}{3}$	1	$-\frac{1}{3}$	$\frac{10}{3}$
x_1	1	0	$-\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{40}{3}$

The slack variables are x_4 , x_4 and x_5. If x_1 is required to be an integer, find a new optimal solution.

[10 marks]

7. Solve the following goal programming problem graphically.

Minimize $z = P_1 d_1^- + P_2 d_2^- + P_3 d_3^+ + P_4 d_1^+$

Subject to $x_1 + 2x_2 + d_1^- - d_1^+ = 40$

$40x_1 + 50x_2 + d_2^- - d_2^+ = 1600$

$4x_1 + 3x_2 + d_3^- - d_3^+ = 120$

$x_1, x_2 , d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ \geq 0$

[10 marks]

6. *Pertimbangkan masalah PL berikut dan tablo optimumnya.*

$$\begin{aligned} \textit{Maksimumkan} \quad & z = 40x_1 + 50x_2 \\ \textit{Terhadap} \quad & x_1 + 2x_2 \le 40 \\ & x_1 + x_2 \le 30 \\ & 2x_1 + x_2 \le 40 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Asas	x_1	x_2	x_3	x_4	x_5	*Penyelesaian*
z	0	0	20	0	10	1200
x_2	0	1	$\frac{2}{3}$	0	$-\frac{1}{3}$	$\frac{40}{3}$
x_4	0	0	$-\frac{1}{3}$	1	$-\frac{1}{3}$	$\frac{10}{3}$
x_1	1	0	$-\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{40}{3}$

Pembolehubah lalai diwakili oleh x_4 , x_4 *dan* x_5. *Jika* x_1 *diperlukan sebagai integer, cari penyelesaian optimum yang baru.*

[10 markah]

7. *Selesaikan masalah gol berikut secara bergraf.*

$$\begin{aligned} \textit{Minimumkan} \quad & z = P_1 d_1^- + P_2 d_2^- + P_3 d_3^+ + P_4 d_1^+ \\ \textit{Terhadap} \quad & x_1 + 2x_2 + d_1^- - d_1^+ = 40 \\ & 40x_1 + 50x_2 + d_2^- - d_2^+ = 1600 \\ & 4x_1 + 3x_2 + d_3^- - d_3^+ = 120 \\ & x_1, x_2 \ , \ d_1^-, d_1^+, \ d_2^-, d_2^+, \ d_3^-, d_3^+ \ge 0 \end{aligned}$$

[10 markah]

8. Meatmix Corporation produces sausages by blending beef, chicken, mutton and water. The cost per kg, percentage of fat and protein per kg for these ingredients are given below.

	Beef	Chicken	Mutton
Fat per kg	0.05	0.24	0.11
Protein per kg	0.20	0.26	0.08
Cost per kg (RM)	12	9	8

Meatmix needs to produce 100 kg of sausages and has the following goals in order of priority.
Goal 1 Sausage should consist of at least 20% protein.
Goal 2 Sausage should consist of at most 7% fat.
Goal 3 Cost per kg of sausage should not exceed RM10.

Formulate a preemptive goal programming model for Meatmix.

[10 marks]

9. Solve the following 0-1 problem.

$$\text{Maximize } z = 300x_1 + 90x_2 + 400x_3 + 150x_4$$
$$\text{Subject to } 35x_1 + 10x_2 + 25x_3 + 90x_4 \leq 120$$
$$4x_1 + 2x_2 + 7x_3 + 3x_4 \leq 12$$
$$x_1 + x_2 \leq 1$$
$$x_1, x_2, x_3, x_4 = 0, 1$$

[10 marks]

8. *Meatmix Corporation mengeluarkan sosej-sosej dengan mencampurkan daging lembu, ayam, kambing dan air. Kos per kg, lemak per kg dan protein per kg bagi bahan-bahan ini diberikan seperti berikut.*

	Lembu	*Ayam*	*Kambing*
Lemak per kg	0.05	0.24	0.11
Protein per kg	0.20	0.26	0.08
Kos per kg (RM)	12	9	8

Meatmix perlu menghasilkan 100kg sosej dan mempunyai gol-gol berikut mengikut keutamaan.

Gol 1 Sosej harus mengandungi sekurang-kurangnya 20% protein.
Gol 2 Sosej harus mengandungi paling tinggi 7% lemak.
Gol 3 Kos per kg bagi sosej mesti tidak melebihi RM10.

Rumuskan suatu model pengaturcaraan gol pra-imtif untuk Meatmix.

[10 markah]

9. *Selesaikan masalah 0-1 berikut.*

$$\text{Maksimumkan } z = 300x_1 + 90x_2 + 400x_3 + 150x_4$$

$$\text{Terhadap } 35x_1 + 10x_2 + 25x_3 + 90x_4 \le 120$$

$$4x_1 + 2x_2 + 7x_3 + 3x_4 \le 12$$

$$x_1 + x_2 \le 1$$

$$x_1, x_2, x_3, x_4 = 0, 1$$

[10 markah]

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