
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

MSG 162 – Applied Statistical Methods
[Kaedah Statistik Gunaan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIXTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

Arahan: Jawab **semua empat** [4] soalan.]

1. (a) A study was designed to evaluate the effectiveness of new treatments to reduce the systolic blood pressure of patients determined to have high blood pressure. Three drugs (A, B and C) and three nondrug treatments (controlled diet, exercise programs and biofeedback) were considered for evaluation in the study. A group of three patients was then randomly assigned to a combination of one of the three drugs and one of the three nondrug treatments. After participating in the program for 2 months, the reduction in systolic blood pressure from the blood pressure readings at the beginning of the program was recorded for each patient.

Drug	Diet	Nondrug		Total
		Exercise	Biofeedback	
A	32	48	49	362
	26	47	42	
	39	38	41	
	97	133	132	
B	35	47	48	384
	32	48	49	
	39	44	42	
	106	139	139	
C	38	48	40	382
	45	47	41	
	33	41	49	
	116	136	130	
Total	319	408	401	1128

$$\sum_i \sum_j \sum_k Y_{ijk}^2 = 48,146$$

- (i) State a model and its assumptions.
(ii) Perform an appropriate analysis at $\alpha = 0.01$.
(iii) Perform a follow-up analysis.
- (b) Two anticoagulant drugs are studied for their effectiveness in dissolving blood clots. Each of six subjects receives the drugs at equally spaced time intervals and in random order. Time periods between drug applications permit a drug to be passed out of a subject's body before the subject receives the next drug. After each drug is in the bloodstream, the length of time required for a cut of specified size to stop bleeding is recorded.

Drug	Individual						Total
	1	2	3	4	5	6	
Drug A in pill form	127.5	130.6	118.3	122.5	180.7	128.6	808.2
Drug B in pill form	129.0	129.1	111.7	144.3	174.4	113.2	801.7
Drug A in liquid form	135.5	138.0	110.1	162.3	181.8	191.6	919.3
Drug B in liquid form	139.0	131.1	115.7	184.3	175.9	183.2	929.2
Total	531.0	528.8	455.8	613.4	712.8	616.6	3,458.4

$$\sum_i \sum_j Y_{ij}^2 = 515,335$$

- (i) State a model and its assumptions.
(ii) Perform an appropriate analysis at $\alpha = 0.10$.
(iii) Perform a follow-up analysis.

[100 marks]

1. (a) Suatu kajian direkabentuk untuk menilai keberkesanan rawatan baru bagi mengurangkan tekanan darah sistolik pesakit yang dikenalpasti mengalami tekanan darah tinggi. Tiga ubat (A, B, dan C) dan tiga rawatan tanpa ubat (diet yang dikawal, program senaman dan bio-timbalbalas) dipertimbangkan untuk penilaian di dalam kajian. Sekumpulan tiga pesakit diumpukkan secara rawak kepada suatu gabungan daripada satu daripada tiga ubat dan satu daripada tiga rawatan tanpa ubat. Selepas menyertai program tersebut untuk 2 bulan, pengurangan di dalam tekanan darah sistolik daripada bacaan-bacaan tekanan darah pada awal program dicatat bagi setiap pesakit.

Ubat	Diet	Tanpa ubat		Jumlah
		Senaman	Bio-timbalbalas	
A	32	48	49	362
	26	47	42	
	39	38	41	
	97	133	132	
B	35	47	48	384
	32	48	49	
	39	44	42	
	106	139	139	
C	38	48	40	382
	45	47	41	
	33	41	49	
	116	136	130	
Jumlah	319	408	401	1128

$$\sum_i \sum_j \sum_k Y_{ijk}^2 = 48,146$$

- (i) Nyatakan suatu model serta anggapannya.
(ii) Lakukan suatu analisis yang sesuai pada $\alpha = 0.01$.
(iii) Lakukan suatu analisis lanjutan.
- (b) Dua ubat anti pembekuan dikaji untuk keberkesanan dalam mencairkan darah beku. Setiap enam subjek menerima ubat-ubat tersebut pada selang masa yang sama dan secara rawak. Tempoh masa antara penggunaan ubat membolehkan satu ubat dikeluarkan dari tubuh subjek sebelum subjek menerima ubat seterusnya. Selepas setiap ubat berada di dalam aliran darah, panjang masa yang diperlukan bagi suatu luka yang bersaiz tertentu berhenti berdarah dicatat.

Ubat	Individu						Jumlah
	1	2	3	4	5	6	
Ubat A di dalam bentuk pil	127.5	130.6	118.3	122.5	180.7	128.6	808.2
Ubat B di dalam bentuk pil	129.0	129.1	111.7	144.3	174.4	113.2	801.7
Ubat A di dalam bentuk cecair	135.5	138.0	110.1	162.3	181.8	191.6	919.3
Ubat B di dalam bentuk cecair	139.0	131.1	115.7	184.3	175.9	183.2	929.2
Jumlah	531.0	528.8	455.8	613.4	712.8	616.6	3,458.4

$$\sum_i \sum_j Y_{ij}^2 = 515,335$$

- (i) Nyatakan suatu model serta anggapannya.
(ii) Lakukan suatu analisis yang sesuai pada $\alpha = 0.10$.
(iii) Lakukan suatu analisis lanjutan.

[100 markah]

2. (a) In a study to examine the quality of fish after 7 days in ice storage, ten raw fish of the same kind and approximately the same size were caught and prepared for ice storage. Two of the fish were placed in storage immediately after being caught, two were placed in storage 3 hours after being caught, and two each were placed in storage at 6, 9, and 12 hours after being caught. A measurement of fish quality (on a 100-point scale) after the 7 days of storage were recorded.

Quality score	Storage time (in hours)
27	0
28	0
70	3
73	3
76	6
78	6
81	9
79	9
84	12
85	12

- (i) State a model and its assumptions.
(ii) Perform an appropriate analysis at $\alpha = 0.05$.
- (b) A clinician studied the effects of two drugs used either alone or together on the blood flow in human subjects. Six healthy middle-aged males were randomly assigned to the various drug treatments. The increase in blood flow from before to shortly after the administration of the drugs were measured.

Drug	Individual						Total
	1	2	3	4	5	6	
Both drugs A and B	25	21	24	28	25	23	146
Drug B alone	10	8	11	10	8	8	55
Drug A alone	9	6	8	10	10	6	49
Placebo (neither drug)	2	-1	0	-2	2	-1	0
Total	46	34	43	46	45	36	250

$$\sum \sum Y_{ij}^2 = 4,524$$

- (i) State a model and its assumptions.
(ii) Perform an appropriate analysis at $\alpha = 0.01$.
(iii) Perform a follow-up analysis using Duncan multiple range test.
(iv) Determine the following effects on blood flow:
a. drug effect.
b. drug combination effect.

[100 marks]

2. (a) Di dalam suatu kajian pemeriksaan kualiti ikan selepas 7 hari di dalam simpanan berair, sepuluh ikan mentah sama jenis yang bersaiz hampir sama ditangkap dan disediakan untuk simpanan. Dua daripada ikan tersebut diletakkan di dalam simpanan sejurus selepas ditangkap, dua diletakkan di dalam simpanan 3 jam selepas ditangkap dan setiap dua diletakkan di dalam simpanan pada 6, 9 dan 12 jam selepas ditangkap. Suatu ukuran kualiti ikan (berdasarkan suatu skala 100) selepas 7 hari disimpan dicatat.

Skor kualiti	Masa simpanan (dalam jam)
27	0
28	0
70	3
73	3
76	6
78	6
81	9
79	9
84	12
85	12

- (i) Nyatakan suatu model serta anggapannya.
(ii) Lakukan suatu analisis yang sesuai pada $\alpha = 0.05$.
- (b) Seorang ahli perubatan mengkaji keberkesanan dua ubat yang digunakan sama ada bersendirian atau bergabung ke atas aliran darah dalam manusia. Enam lelaki sihat pertengahan umur diumpukkan secara rawak kepada rawatan ubat yang berlainan. Peningkatan di dalam aliran darah daripada sebelum ke sejurus selepas penggunaan ubat-ubat tersebut dicatat.

Ubat	Individu						Jumlah
	1	2	3	4	5	6	
Kedua-dua ubat A dan B	25	21	24	28	25	23	146
Ubat B sahaja	10	8	11	10	8	8	55
Ubat A sahaja	9	6	8	10	10	6	49
Placebo (tanpa ubat)	2	-1	0	-2	2	-1	0
Jumlah	46	34	43	46	45	36	250

$$\sum \sum Y_{ij}^2 = 4,524$$

- (i) Nyatakan suatu model serta anggapannya.
(ii) Lakukan suatu analisis yang sesuai pada $\alpha = 0.01$.
(iii) Lakukan suatu analisis lanjutan menggunakan ujian julat berganda Duncan.
(iv) Tentukan kesan berikut ke atas aliran darah:
a. kesan ubat.
b. kesan gabungan ubat.

[100 markah]

3. (a) An advertising executive is interested in the effect of colour on advertisement appeal. Six individuals are selected randomly and asked to rate each of four magazine layouts in order of eye appeal, with a rank of 1 being given to the most appealing advertisement. The layouts are identical except for the colour scheme involved.

Advertisement	Individual						Total
	1	2	3	4	5	6	
No color	1	1	3	4	1	2	12
Red predominant	2	2	4	5	2	1	16
Yellow predominant	4	5	5	3	5	4	26
Mixed colors	5	4	2	5	3	3	22
Total	12	12	14	17	11	10	76

$$\sum \sum Y_{ij}^2 = 290$$

- (i) State a model and its assumptions.
 - (ii) Perform an appropriate analysis at $\alpha = 0.05$.
 - (iii) Perform a follow-up analysis using Tukey HSD test.
- (b) It was known that a toxic material was dumped in a river leading into a large salt-water commercial fishing area. Civil engineers studied the way the water carried the toxic material by measuring the amount of the material (in parts per million) found in oysters harvested at three randomly selected locations, ranging from the estuary out into the bay where the majority of commercial fishing were carried out.

Oyster	Location			Total
	1	2	3	
1	30	19	22	71
2	26	15	26	67
3	20	10	24	54
4	20	16	26	62
5	29	11	15	55
6	28	20	17	65
Total	153	91	130	374

$$\sum \sum Y_{ij}^2 = 8,390$$

- (i) State a model and its assumptions.
- (ii) Perform an appropriate analysis at $\alpha = 0.01$.
- (iii) Perform a follow-up analysis.

[100 marks]

3. (a) Seorang pegawai pengiklanan berminat dalam kesan warna ke atas tarikan iklan. Enam individu dipilih secara rawak dan dikehendaki untuk menilai setiap daripada empat lakaran majalah yang menarik hati mengikut peringkat, dengan pangkat 1 diberikan kepada iklan yang paling menarik. Lakaran-lakaran tersebut adalah sama melainkan pada skema warna yang digunakan.

Iklan	Individu						Jumlah
	1	2	3	4	5	6	
Tiada warna	1	1	3	4	1	2	12
Warna utama merah	2	2	4	5	2	1	16
Warna utama kuning	4	5	5	3	5	4	26
Warna bercampur	5	4	2	5	3	3	22
Jumlah	12	12	14	17	11	10	76

$$\sum \sum Y_{ij}^2 = 290$$

- (i) Nyatakan suatu model serta anggapannya.
(ii) Lakukan suatu analisis yang sesuai pada $\alpha = 0.05$.
(iii) Lakukan suatu analisis lanjutan menggunakan ujian Tukey HSD.
- (b) Diketahui bahawa suatu bahan toksik dibuang ke dalam sebuah sungai yang mengalir ke dalam suatu kawasan perniagaan penangkapan ikan air masin yang luas. Jurutera-jurutera sivil mengkaji cara air membawa bahan toksik dengan mengukur amaun bahan (dalam bahagian per juta) yang terdapat di dalam kerang di tiga lokasi yang dipilih secara rawak, daripada muara hingga keluar ke kawasan teluk, tempat kebanyakan penangkapan ikan komersial dilakukan.

Kerang	Lokasi			Jumlah
	1	2	3	
1	30	19	22	71
2	26	15	26	67
3	20	10	24	54
4	20	16	26	62
5	29	11	15	55
6	28	20	17	65
Jumlah	153	91	130	374

$$\sum \sum Y_{ij}^2 = 8,390$$

- (i) Nyatakan suatu model serta anggapannya.
(ii) Lakukan suatu analisis yang sesuai pada $\alpha = 0.01$.
(iii) Lakukan suatu analisis lanjutan.

[100 markah]

4. For many years, educators have suggested that home environment contribute heavily to the educational achievement of students. In a study, averages on a reading test were compared with the percentage of students with three or more types of reading material at home (books, encyclopedias, magazines, newspapers) and the percentage of students who watch television (TV) for six hours or more per day. The following data were collected for second year students in 32 states:

Reading score	Reading material(%)	TV watch (%)
52	78	18
59	73	12
56	77	20
56	68	11
67	85	9
70	86	12
61	83	18
31	76	33
55	73	19
58	80	17
31	64	20
51	69	23
72	84	7
67	84	11
78	88	8
56	78	14
46	76	19
60	83	19
64	84	14
76	88	7
80	88	6
76	88	9
69	84	13
61	79	17
50	78	21
81	90	6
66	86	10
58	70	15
18	76	27
64	82	16
56	80	16
74	86	8

Refer to the appendix output for question 4.

- (i) Perform an appropriate analysis to obtain the model and its assumptions. Use $\alpha = 0.025$.
- (ii) Obtain an estimation interval and prediction interval for a state with 75 percent of students with three or more types of reading material at home and 10 percent of students who watch television for six hours or more per day.

[100 marks]

4. Untuk beberapa tahun, para pendidik telah menyarankan bahawa suasana rumah paling banyak menyumbang kepada pencapaian pendidikan para pelajar. Dalam suatu kajian, purata bagi suatu ujian bacaan dibandingkan dengan peratusan pelajar yang mempunyai tiga atau lebih jenis bahan bacaan di rumah (buku, encyclopedia, suratkhobar) dan peratusan pelajar yang menonton televisyen (TV) selama enam jam atau lebih sehari. Data berikut dikumpul bagi pelajar tahun dua di 32 buah negeri.

Skor Bacaan	Bahan Bacaan (%)	Menonton TV (%)
52	78	18
59	73	12
56	77	20
56	68	11
67	85	9
70	86	12
61	83	18
31	76	33
55	73	19
58	80	17
31	64	20
51	69	23
72	84	7
67	84	11
78	88	8
56	78	14
46	76	19
60	83	19
64	84	14
76	88	7
80	88	6
76	88	9
69	84	13
61	79	17
50	78	21
81	90	6
66	86	10
58	70	15
18	76	27
64	82	16
56	80	16
74	86	8

Rujuk kepada output appendiks untuk soalan 4.

- (i) Lakukan suatu analisis yang sesuai untuk mendapatkan model dan anggapananya. Guna $\alpha = 0.025$.
- (ii) Dapatkan suatu selang anggaran dan selang ramalan bagi sesebuah negeri dengan 75 peratus pelajar yang mempunyai tiga atau lebih jenis bahan bacaan di rumah dan 10 peratus pelajar yang menonton televisyen untuk enam jam atau lebih sehari.

[100 markah]

APPENDIX: OUTPUT

Question 4

Predictor	Coef	SE Coef	t
Constant	24.58	17.81	1.38
Material	0.7263	0.1967	3.69
TV	-1.5361	0.2068	-7.43

S = 5.79411 R-Sq = 84.7% R-Sq(adj) = 83.6%

Analysis of Variance

Source	DF	SS	MS
Regression	2	5377.4	2688.7
Error	29	973.6	33.6
Total	31	6351.0	

$$(X'X)^{-1} = \begin{bmatrix} 9.45008 & -0.10320 & -0.07745 \\ -0.10320 & 0.001153 & 0.000731 \\ -0.07745 & 0.000731 & 0.0012734 \end{bmatrix}$$

APPENDIX: FORMULAS

1. Completely Randomized Design

$$SST = \sum_i \sum_j Y_{ij}^2 - \frac{Y_{..}^2}{N}$$

$$SSA = \sum_i \frac{Y_{i.}^2}{n_i} - \frac{Y_{..}^2}{N}$$

For any contrast : $L = \sum_i c_i \bar{Y}_{i.}$

$$SSL = \frac{\left(\sum_i c_i \bar{Y}_{i.} \right)^2}{\sum_i \frac{c_i^2}{n_i}}$$

2. Completely Randomized Block Design

$$SST = \sum_i \sum_j Y_{ij}^2 - \frac{Y_{..}^2}{N}$$

$$SSA = \sum_i \frac{Y_{i.}^2}{b} - \frac{Y_{..}^2}{N}$$

$$SSB = \sum_j \frac{Y_{.j}^2}{a} - \frac{Y_{..}^2}{N}$$

3. Latin Square Design

$$SST = \sum_i \sum_j \sum_k Y_{ijk}^2 - \frac{Y_{...}^2}{N}$$

$$SSR = \sum_i \frac{Y_{i..}^2}{a} - \frac{Y_{...}^2}{N}$$

$$SSC = \sum_j \frac{Y_{.j.}^2}{a} - \frac{Y_{...}^2}{N}$$

$$SSA = \sum_k \frac{Y_{..k}^2}{a} - \frac{Y_{...}^2}{N}$$

4. Two-way Factorial Design

$$SST = \sum_i \sum_j \sum_k Y_{ijk}^2 - \frac{Y_{\dots}^2}{N}$$

$$SS_{\text{Cell}} = \sum_i \sum_j \frac{Y_{ij.}^2}{n} - \frac{Y_{\dots}^2}{N}$$

$$SSA = \sum_i \frac{Y_{i..}^2}{bn} - \frac{Y_{\dots}^2}{N}$$

$$SSB = \sum_j \frac{Y_{.j.}^2}{an} - \frac{Y_{\dots}^2}{N}$$

$$SSE = \sum_i \sum_j \sum_k Y_{ijk}^2 - \frac{Y_{ij.}^2}{n}$$

5. Distributions

Duncan multiple range: $r_{\alpha, p, df}$, $p = \text{range}$ $df = \text{degrees of freedom}$

Tukey HSD: $q_{\alpha, a, df}$, $a = \text{number of treatments}$ $df = \text{degrees of freedom}$

Scheffe': $\sqrt{(a-1)F_{\alpha, a-1, df}}$, $a = \text{number of treatments}$ $df = \text{degrees of freedom}$

6. Correlation

$$r = \frac{SS_{xy}}{\sqrt{SS_x SS_y}}$$

$$t = r \sqrt{\frac{n-2}{1-r^2}}$$

7. Regression

$$b_1 = \frac{SS_{xy}}{SS_x}, \quad b_0 = \bar{Y} - b_1 \bar{X}$$

$$SS_{xy} = \sum X_i Y_i - \frac{(\sum X_i)(\sum Y_i)}{n}$$

$$SS_x = \sum X_i^2 - \frac{(\sum X_i)^2}{n}$$

$$SS_y = \sum Y_i^2 - \frac{(\sum Y_i)^2}{n}$$

$$\text{Var}(b_1) = \frac{\sigma^2}{SS_x}$$

$$\text{Var}(\hat{Y}_h) = \sigma^2 \left[\frac{1}{n} + \frac{(X_h - \bar{X})^2}{SS_x} \right], \quad \text{Var}(\hat{Y}_h) = \text{MSE}(\mathbf{X}_h'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}_h)$$

8. Kruskal-Wallis Test

$$T = \frac{12}{N(N+1)} \sum_i \frac{R_i^2}{n_i} - 3(N+1)$$

9. Friedman Test

$$T = \frac{12}{ba(a+1)} \sum_i R_i^2 - 3b(a+1)$$

10. Cochran Test

$$T = \frac{a(a-1) \sum_i A_i^2 - (a-1)N^2}{aN - \sum_j B_j^2}$$

11. Spearman Test

$$r_s = 1 - \frac{6 \sum_i d_i^2}{n(n^2 - 1)}$$

APPENDIX: TABLES

Duncan Multiple Range Table

		$r_{.01}(p, f)$											
		p											
f		2	3	4	5	6	7	8	9	10	20	50	100
1		90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
2		14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0
3		8.26	8.5	8.6	8.7	8.8	8.9	8.9	9.0	9.0	9.3	9.3	9.3
4		6.51	6.8	6.9	7.0	7.1	7.1	7.2	7.2	7.3	7.5	7.5	7.5
5		5.70	5.96	6.11	6.18	6.26	6.33	6.40	6.44	6.5	6.8	6.8	6.8
6		5.24	5.51	5.65	5.73	5.81	5.88	5.95	6.00	6.0	6.3	6.3	6.3
7		4.95	5.22	5.37	5.45	5.53	5.61	5.69	5.73	5.8	6.0	6.0	6.0
8		4.74	5.00	5.14	5.23	5.32	5.40	5.47	5.51	5.5	5.8	5.8	5.8
9		4.60	4.86	4.99	5.08	5.17	5.25	5.32	5.36	5.4	5.7	5.7	5.7
10		4.48	4.73	4.88	4.96	5.06	5.13	5.20	5.24	5.28	5.55	5.55	5.55
11		4.39	4.63	4.77	4.86	4.94	5.01	5.06	5.12	5.15	5.39	5.39	5.39
12		4.32	4.55	4.68	4.76	4.84	4.92	4.96	5.02	5.07	5.26	5.26	5.26
13		4.26	4.48	4.62	4.69	4.74	4.84	4.88	4.94	4.98	5.15	5.15	5.15
14		4.21	4.42	4.55	4.63	4.70	4.78	4.83	4.87	4.91	5.07	5.07	5.07
15		4.17	4.37	4.50	4.58	4.64	4.72	4.77	4.81	4.84	5.00	5.00	5.00
16		4.13	4.34	4.45	4.54	4.60	4.67	4.72	4.76	4.79	4.94	4.94	4.94
17		4.10	4.30	4.41	4.50	4.56	4.63	4.68	4.73	4.75	4.89	4.89	4.89
18		4.07	4.27	4.38	4.46	4.53	4.59	4.64	4.68	4.71	4.85	4.85	4.85
19		4.05	4.24	4.35	4.43	4.50	4.56	4.61	4.64	4.67	4.82	4.82	4.82
20		4.02	4.22	4.33	4.40	4.47	4.53	4.58	4.61	4.65	4.79	4.79	4.79
30		3.89	4.06	4.16	4.22	4.32	4.36	4.41	4.45	4.48	4.65	4.71	4.71
40		3.82	3.99	4.10	4.17	4.24	4.30	4.34	4.37	4.41	4.59	4.69	4.69
60		3.76	3.92	4.03	4.12	4.17	4.23	4.27	4.31	4.34	4.53	4.66	4.66
100		3.71	3.86	3.98	4.06	4.11	4.17	4.21	4.25	4.29	4.48	4.64	4.65
∞		3.64	3.80	3.90	3.98	4.04	4.09	4.14	4.17	4.20	4.41	4.60	4.68

 f = degrees of freedom.

Tukey HSD Table

$$q_{\alpha} = \frac{\bar{x}_{max} - \bar{x}_{min}}{s\sqrt{p}}$$

Upper 5% Points

Error df	p = number of treatment means																			
	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1	18.0	27.0	32.8	37.1	40.4	43.1	45.4	47.4	49.1	50.6	52.0	53.2	54.3	55.4	56.3	57.2	58.0	58.8	59.6	
2	6.09	8.3	9.8	10.9	11.7	12.4	13.0	13.5	14.0	14.4	14.7	15.1	15.4	15.7	15.9	16.1	16.4	16.6	16.8	
3	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46	9.72	9.95	10.15	10.35	10.52	10.69	10.84	10.98	11.11	11.24	
4	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83	8.03	8.21	8.37	8.52	8.66	8.79	8.91	9.03	9.13	9.23	
5	3.63	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17	7.32	7.47	7.60	7.72	7.83	7.93	8.03	8.12	8.19	
6	3.46	4.34	4.90	5.31	5.63	5.89	6.12	6.32	6.49	6.65	6.79	6.92	7.03	7.14	7.24	7.34	7.43	7.51	7.59	
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30	6.43	6.55	6.66	6.76	6.85	6.94	7.02	7.09	7.17	
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05	6.18	6.29	6.39	6.48	6.57	6.65	6.73	6.80	6.87	
9	3.20	3.95	4.42	4.76	5.02	5.24	5.43	5.60	5.74	5.87	5.98	6.09	6.19	6.28	6.36	6.44	6.51	6.58	6.64	
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72	5.83	5.93	6.03	6.11	6.20	6.27	6.34	6.40	6.47	
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61	5.71	5.81	5.90	5.99	6.06	6.14	6.20	6.26	6.33	
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.40	5.51	5.62	5.71	5.80	5.88	5.95	6.03	6.09	6.15	6.21	
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43	5.53	5.63	5.71	5.79	5.86	5.93	6.00	6.05	6.11	
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36	5.46	5.55	5.64	5.72	5.79	5.85	5.92	5.97	6.03	
15	3.01	3.67	4.08	4.37	4.60	4.78	4.94	5.08	5.20	5.31	5.40	5.49	5.58	5.65	5.72	5.79	5.85	5.90	5.96	
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26	5.35	5.44	5.52	5.59	5.66	5.72	5.79	5.84	5.90	
17	2.98	3.63	4.02	4.30	4.52	4.71	4.86	4.99	5.11	5.21	5.31	5.39	5.47	5.55	5.61	5.68	5.74	5.79	5.84	
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.17	5.27	5.35	5.43	5.50	5.57	5.63	5.69	5.74	5.79	
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04	5.14	5.23	5.32	5.39	5.46	5.53	5.59	5.65	5.70	5.75	
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.11	5.20	5.28	5.36	5.43	5.49	5.55	5.61	5.66	5.71	
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01	5.10	5.18	5.25	5.32	5.38	5.44	5.50	5.54	5.59	
30	2.89	3.49	3.84	4.10	4.30	4.46	4.60	4.72	4.83	4.92	5.00	5.08	5.15	5.21	5.27	5.33	5.38	5.43	5.48	
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.74	4.82	4.91	4.98	5.05	5.11	5.16	5.22	5.27	5.31	5.36	
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73	4.81	4.88	4.94	5.00	5.06	5.11	5.16	5.20	5.24	
120	2.80	3.36	3.69	3.92	4.10	4.24	4.36	4.48	4.56	4.64	4.72	4.78	4.84	4.90	4.95	5.00	5.05	5.09	5.13	
∞	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55	4.62	4.68	4.74	4.80	4.85	4.89	4.93	4.97	5.01	

Spearman Table

<i>n</i>	.001	.005	.010	.025	.050	.100
4	—	—	—	—	.8000	.8000
5	—	—	.9000	.9000	.8000	.7000
6	—	.9429	.8857	.8286	.7714	.6000
7	.9643	.8929	.8571	.7450	.6786	.5357
8	.9286	.8571	.8095	.7143	.6190	.5000
9	.9000	.8167	.7667	.6833	.5833	.4667
10	.8667	.7818	.7333	.6364	.5515	.4424
11	.8364	.7545	.7000	.6091	.5273	.4182
12	.8182	.7273	.6713	.5804	.4965	.3986
13	.7912	.6978	.6429	.5549	.4780	.3791
14	.7670	.6747	.6220	.5341	.4593	.3626
15	.7464	.6536	.6000	.5179	.4429	.3500
16	.7265	.6324	.5824	.5000	.4265	.3382
17	.7083	.6152	.5637	.4853	.4118	.3260
18	.6904	.5975	.5480	.4716	.3994	.3148
19	.6737	.5825	.5333	.4579	.3895	.3070
20	.6586	.5684	.5203	.4451	.3789	.2977
21	.6455	.5545	.5078	.4351	.3688	.2909
22	.6318	.5426	.4963	.4241	.3597	.2829
23	.6186	.5306	.4852	.4150	.3518	.2767
24	.6070	.5200	.4748	.4061	.3435	.2704
25	.5962	.5100	.4654	.3977	.3362	.2646
26	.5856	.5002	.4564	.3894	.3299	.2588
27	.5757	.4915	.4481	.3822	.3236	.2540
28	.5660	.4828	.4401	.3749	.3175	.2490
29	.5567	.4744	.4320	.3685	.3113	.2443
30	.5479	.4665	.4251	.3620	.3059	.2400