UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2008/2009

Jun 2009

MAT 202 – Introduction to Analysis
[Pengantar Analisis]

Duration : 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all three** [3] questions.

*[**Arahan** : Jawab **semua tiga** [3] soalan.]*

1. (a) If $a \leq b + \varepsilon$ for each positive real number ε, show that $a \leq b$.

   (b) Define countability of a non-empty set S. Show that the set of all even integers is countable.

   (c) Prove that $\sqrt{3}$ is not rational.

   (d) Prove that between any two distinct real numbers there exists an irrational number.

   (e) State the Archimedean Principle. Use this principle to show that the infimum of the set $\left\{\frac{1}{n}; n \in Z^{+}\right\}$ is 0.

[100 marks]

2. (a) Prove that if $\{a_n\}$ is bounded and $\{b_n\} \to 0$, then $a_n b_n \to 0$.

   (b) Use the definition to determine whether the sequence $\left\{\frac{2^n - 1}{2^n}\right\}$ is a Cauchy sequence.

   (c) Let $A = (-1, 10] \cap Q^C$. Find the interior points, the limit points, the isolated points, and the boundary points of A.

   (d) Let $A \subset \Re$, If a is a limit point of A, show that there exists a sequence $\{a_n\} \subset A$ such that $a_n \to a$. Is the converse true?, if yes prove it, if not disprove using an example.

   (e) State Heine Borel Theorem of a compact set. Give an example of a compact set.

[100 marks]

1. (a) *Jika* $a \leq b + \varepsilon$ *untuk setiap nombor nyata* ε, *tunjukkan bahawa* $a \leq b$.

   (b) *Takrifkan keterbilangkan suatu set tak kosong S. Tunjukkan set semua nombor integer genap adalah terbilangkan.*

   (c) *Buktikan bahawa* $\sqrt{3}$ *adalah nombor tak nisbah*.

   (d) *Buktikan bahawa diantara dua nombor nyata yang berbeza wujud suatu nombor tak nisbah.*

   (e) *Nyatakan prinsip Archimedean. Dengan menggunakan prinsip ini tunjukkan bahawa infimum bagi set* $\left\{\frac{1}{n}; n \in Z^{+}\right\}$ *adalah* 0.

[100 markah]

2. (a) *Jika* $\{a_n\}$ *satu jujukan terbatas dan* $\{b_n\} \to 0$, *buktikan* $a_n b_n \to 0$.

   (b) *Dengan menggunakan takrif Cauchy tentukan sama ada jujukan* $\left\{\frac{2^n - 1}{2^n}\right\}$ *adalah Cauchy.*

   (c) *Biarkan* $A = (-1, 10] \cap Q^C$. *Dapatkan titik pedalaman, titik had, titik terpencil dan titik sempadan bagi set A.*

   (d) *Biarkan* $A \subset \Re$, *Jika a adalah titik had bagi set A, tunjukkan bahawa wujud jujukan* $\{a_n\} \subset A$ *supaya* $a_n \to a$. *Adakah akasnya benar? Jika ya, buktikan, jika tidak, sangkal dengan memberikan contoh.*

   (e) *Nyatakan Teorem Heine Borel bagi set padat. Berikan satu contoh set padat.*

[100 markah]

3. (a) Given a set $S = (-12, 2] \cup [8, 17)$. Using the definition of a disconnected set, determine whether or not the set S is disconnected.

(b) Given K_1 and K_2 are two compact sets, then show that $K_1 \cup K_2$ is compact.

(c) Let $f : \Re \to \Re$. If the function f is continuous, show that the pre-image of an open set U is open, that is $f^{-1}(U)$ open whenever U is open.

(d) Given a continuous function $f : \Re \to \Re$. For $a \in \Re$, determine whether the set $\{x : f(x) \geq a\}$ is closed or open.

(e) Let $\{f_n\}$ be a bounded sequence of functions on a set A and converge to a function f on A. Define $R_n = \sup_{x \in A} |f_n(x) - f(x)|$. Show that if $\lim_{n \to \infty} R_n = 0$, $\{f_n\}$ converge uniformly to f on A.

[100 marks]

3. (a) *Diberikan* $S = (-12,2] \cup [8,17)$. *Dengan menggunakan takrif set tak terkait, tentukan sama ada set S ini tak terkait.*

(b) *Diberikan* K_1 *dan* K_2 *dua set padat, seterusnya tunjukkan bahawa* $K_1 \cup K_2$ *adalah padat.*

(c) *Biarkan* $f : \Re \to \Re$. *Jika fungsi* f *adalah selanjar, tunjukkan bahawa pra-imej bagi set terbuka* U *adalah terbuka, iaitu* $f^{-1}(U)$ *adalah terbuka apabila* U *terbuka.*

(d) *Diberikan fungsi selanjar* $f : \Re \to \Re$. *Bagi* $a \in \Re$, *tentukan sama ada set* $\{x : f(x) \geq a\}$ *adalah tertutup atau terbuka.*

(e) *Biarkan* $\{f_n\}$ *suatu fungsi pada set* A *yang terbatas dan menumpu ke fungsi* f *pada* A. *Takrifkan* $R_n = \sup_{x \in A} |f_n(x) - f(x)|$. *Tunjukkan bahawa jika* $\operatorname{had}_{n \to \infty} R_n = 0$, *maka* $\{f_n\}$ *menumpu secara seragam ke* f *pada* A.

[100 markah]

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