
UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2008/2009

Jun 2009

MAT 122 – Differential Equations I
[Persamaan Pembezaan I]

Duration : 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all three** [3] questions.

Arahan : Jawab **semua tiga** [3] soalan.]

1. (a) (i) Show that the parabola $y = x^2$ and a line $y = 2x - 1$ are both solutions for the differential equation

$$y' = 2x - 2\sqrt{x^2 - y}, \quad y(1) = 1.$$

- (ii) Does the above fact contradict with the Existence and Uniqueness Theorem? Explain your answer.
- (b) Given a differential equation

$$(3x^2 + 2y^2)dx + (4xy + 6y^2)dy = 0.$$

Verify that this differential equation is exact and find its general solution.

- (c) Using the method of undetermined coefficients, find the general solution of the differential equation

$$y'' + 2y' - 3y = 1 + xe^x.$$

[50 marks]

2. (a) Use the Euler method to approximate the value of the solution ϕ of the initial value problem $y' = 2x + y, y(0) = 1$. Find approximations to $\phi(x)$ at $x = 0.2, 0.4, 0.6, 0.8$ and 1.0 by using $h = 0.2$.
- (b) Solve the equation $2xy'' + (1+x)y' + y = 0$ by the method of Frobenius to obtain two linearly independent series solutions about $x = 0$.
- (c) Determine the singular points of $(x^2 - 9)^2 y'' + (x+3)y' + 2y = 0$ and classify them as regular or irregular.

[50 marks]

1. (a) (i) Tunjukkan bahawa parabola $y = x^2$ dan garis lurus $y = 2x - 1$ adalah penyelesaian bagi persamaan pembezaan

$$y' = 2x - 2\sqrt{x^2 - y}, \quad y(1) = 1.$$

- (ii) Adakah fakta di atas bercanggah dengan Teorem Kewujudan dan Keunikan? Jelaskan jawapan anda.

- (b) Diberikan persamaan pembezaan

$$(3x^2 + 2y^2)dx + (4xy + 6y^2)dy = 0.$$

Tentukan bahawa persamaan pembezaan ini adalah tepat dan cari penyelesaian amnya.

- (c) Menggunakan kaedah pekali tak tentu, cari penyelesaian am bagi persamaan pembezaan

$$y'' + 2y' - 3y = 1 + xe^x.$$

[50 markah]

2. (a) Gunakan Kaedah Euler untuk menganggar nilai penyelesaian ϕ bagi masalah nilai awal $y' = 2x + y, y(0) = 1$. Cari anggaran $\phi(x)$ bila $x = 0.2, 0.4, 0.6, 0.8$ dan 1.0 dengan menggunakan $h = 0.2$.

- (b) Selesaikan persamaan $2xy'' + (1+x)y' + y = 0$ dengan kaedah Frobenius untuk memperoleh dua penyelesaian siri tak bersandar di sekitar $x = 0$.

- (c) Tentukan titik singular bagi $(x^2 - 9)^2 y'' + (x + 3)y' + 2y = 0$ dan kelaskan mereka samada nalar atau tak nalar.

[50 markah]

3. (a) Solve the non-homogeneous system $\bar{x}' = \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix} \bar{x} + \begin{pmatrix} 3t \\ e^{-t} \end{pmatrix}$ on $(-\infty, \infty)$.
- (b) Radioactive isotope thorium 234 disintegrates at a rate proportional to the amount remaining. If 100 milligram of the substance is reduced to 82.04 milligram in 1 week, find an expression for the amount remaining at any time t . Also, find the time taken for the substance to disintegrate to half of its original amount (half-life).
- (c) Given that $\bar{\phi}_1 = \begin{pmatrix} e^{2t} \\ -e^{2t} \\ -e^{2t} \end{pmatrix}$, $\bar{\phi}_2 = \begin{pmatrix} e^{3t} \\ -2e^{3t} \\ -e^{3t} \end{pmatrix}$, $\bar{\phi}_3 = \begin{pmatrix} 3e^{5t} \\ -6e^{5t} \\ -e^{5t} \end{pmatrix}$ are the solutions of
- $$\bar{x}' = \begin{pmatrix} 7 & -1 & 6 \\ -10 & 4 & -12 \\ -2 & 1 & -1 \end{pmatrix} \bar{x}.$$

Prove that the set is a fundamental set of solutions. Write the fundamental matrix and the general solution of the given system of differential equations.

[50 marks]

3. (a) Selesaikan sistem tak homogen $\vec{x}' = \begin{pmatrix} -3 & 1 \\ 2 & -4 \end{pmatrix} \vec{x} + \begin{pmatrix} 3t \\ e^{-t} \end{pmatrix}$ pada $(-\infty, \infty)$.
- (b) Radioaktif isotop Thorium 234 mereput pada kadar seimbang dengan kuantiti bahan yang tinggal. Jika 100 miligram bahan ini berkurang kepada 82.04 miligram dalam 1 minggu, cari suatu pernyataan bagi kuantiti bahan yang tinggal pada masa t . Cari juga masa diambil bagi bahan untuk mereput kepada setengah kuantiti asalnya (setengah hayat).

(c) Diberi bahawa $\bar{\phi}_1 = \begin{pmatrix} e^{2t} \\ -e^{2t} \\ -e^{2t} \end{pmatrix}$, $\bar{\phi}_2 = \begin{pmatrix} e^{3t} \\ -2e^{3t} \\ -e^{3t} \end{pmatrix}$, $\bar{\phi}_3 = \begin{pmatrix} 3e^{5t} \\ -6e^{5t} \\ -e^{5t} \end{pmatrix}$ adalah penyelesaian bagi $\vec{x}' = \begin{pmatrix} 7 & -1 & 6 \\ -10 & 4 & -12 \\ -2 & 1 & -1 \end{pmatrix} \vec{x}$.

Buktikan bahawa set ini adalah set penyelesaian asasi. Tuliskan matrik asasi dan penyelesaian am bagi sistem persamaan pembezaan yang diberi.

[50 markah]