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UNIVERSITI SAINS MALAYSIA

Second Semester Examination  
Academic Session 2008/2009

April/May 2009

**MAT 122 – Differential Equations I**  
**[Persamaan Pembezaan I]**

Duration : 3 hours  
[Masa : 3 jam]

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Please check that this examination paper consists of FIVE pages of printed materials before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions:** Answer **all four** [4] questions.

**Arahan:** Jawab **semua empat** [4] soalan.]

1. (a) (i) Show that the function  $y = \phi(x) = \left(\frac{2}{3}x\right)^{\frac{3}{2}}$ ,  $x \geq 0$  and  $y = \psi(x) = 0$  are both solutions for the differential equation  $y' = y^{\frac{1}{3}}$ ,  $y(0) = 0$  for  $x \geq 0$ .
- (ii) Does the above fact contradict with the Existence and Uniqueness Theorem? Explain your answer.
- (b) Find the general solution of the following differential equations:
- (i)  $(x - y^3 + y^2 \sin x)dx = (3xy^2 + 2y \cos x)dy$
- (ii)  $2xy' - 3y = 9x^3$
- (iii)  $x^2 y' + 2xy = 5y^3$ .

[50 marks]

2. (a) Verify that  $y_1 = x$  and  $y_2 = x^2$  are linearly independent solutions on the entire real line of the differential equation

$$x^2 y'' - 2xy' + 2y = 0,$$

but that the Wronskian function of  $y_1$  and  $y_2$ , vanishes at  $x = 0$ . Explain your answer.

- (b) Find the general solution of the differential equation  $2y'' - 7y' + 3y = 0$ .
- (c) Find the general solution for the differential equation

$$y'' + 3y' + 2y = \frac{1}{1 + e^x}.$$

- (d) Determine the particular solution  $y_p(x)$  and hence, find the general solution for the differential equation

$$y'' + 9y = 2x^2 e^{3x} + 5.$$

[50 marks]

1. (a) (i) Tunjukkan bahawa fungsi  $y = \phi(x) = \left(\frac{2}{3}x\right)^{\frac{3}{2}}$ ,  $x \geq 0$  dan  $y = \psi(x) = 0$  adalah penyelesaian bagi persamaan pembezaan  $y' = y^{\frac{1}{3}}$ ,  $y(0) = 0$  pada  $x \geq 0$ .
- (ii) Adakah fakta di atas bercanggah dengan Teorem Kewujudan dan Keunikan. Terangkan jawapan anda.
- (b) Cari penyelesaian am bagi persamaan pembezaan berikut :
- (i)  $(x - y^3 + y^2 \sin x)dx = (3xy^2 + 2y \cos x)dy$
- (ii)  $2xy' - 3y = 9x^3$
- (iii)  $x^2y' + 2xy = 5y^3$ .

[50 markah]

2. (a) Tentusahkan bahawa  $y_1 = x$  dan  $y_2 = x^2$  adalah penyelesaian tak bersandar secara linear pada seluruh garis nyata bagi persamaan pembezaan

$$x^2y'' - 2xy' + 2y = 0,$$

tetapi fungsi Wronskian bagi  $y_1$  dan  $y_2$ , lenyap pada  $x = 0$ . Terangkan jawapan anda.

- (b) Cari penyelesaian am bagi persamaan pembezaan  $2y'' - 7y' + 3y = 0$ .
- (c) Cari penyelesaian am bagi persamaan pembezaan

$$y'' + 3y' + 2y = \frac{1}{1 + e^x}.$$

- (d) Tentukan penyelesaian khusus  $y_p(x)$  dan seterusnya, cari penyelesaian am bagi persamaan pembezaan

$$y'' + 9y = 2x^2e^{3x} + 5.$$

[50 markah]

3. (a) Use the Heun formula to obtain the approximate value of  $y(1.5)$  for the solution of the initial value problem  $y' = 2xy, y(1) = 1$ . Compare the results for  $h = 0.1$  and  $h = 0.05$ .
- (b) Find a power series solution for  $(x^2 + 1)y'' + xy' - y = 0$ .
- (c) Determine the singular points of  $(x^3 + 4x)y'' - 2xy' + 6y = 0$  and classify them as regular or irregular.
- (d) What is the radius of convergence and interval of convergence for the power series  $\sum_{k=1}^{\infty} \frac{(-1)^k}{10^k} (x-5)^k$ .

[50 marks]

4. (a) Solve the non-homogeneous system  $\bar{x}' = \begin{pmatrix} 6 & 1 \\ 4 & 3 \end{pmatrix} \bar{x} + \begin{pmatrix} 6t \\ -10t + 4 \end{pmatrix}$  on  $(-\infty, \infty)$ .
- (b) Let the vector functions  $\bar{\varphi}_1(t), \bar{\varphi}_2(t), \dots, \bar{\varphi}_n(t)$  defined by  $\bar{\varphi}_i(t) = [\varphi_{i1}(t), \varphi_{i2}(t), \dots, \varphi_{in}(t)]^T$  be  $n$  solutions of the homogeneous linear vector differential equation  $\bar{x}' = A(t)\bar{x}$  where  $\bar{x}' = [x_1'(t), x_2'(t), \dots, x_n'(t)]^T$ ,  $\bar{x} = [x_1(t), x_2(t), \dots, x_n(t)]^T$ ,  $A(t) = [a_{ij}(t)]$  of order  $n \times n$ . If the  $n$  vectors  $\bar{\varphi}_i(t)$  are linearly dependent on  $a \leq t \leq b$ , then prove that the Wronskian of these vectors,  $W(\bar{\varphi}_1, \bar{\varphi}_2, \dots, \bar{\varphi}_n)(t) \neq 0$  for  $t \in [a, b]$ .
- (c) A 4-lb roast, initially at  $50^\circ F$ , is placed in a  $375^\circ F$  oven at 5 pm. After 75 minutes it is found that the temperature  $T(t)$  of the roast is  $125^\circ F$ . When will the roast be at the temperature  $150^\circ F$  (medium rare)?

[50 marks]

3. (a) Guna formula Heun untuk memperoleh nilai penghampiran bagi  $y(1.5)$  untuk penyelesaian masalah nilai awal  $y' = 2xy, y(1) = 1$ . Bandingkan keputusan untuk  $h = 0.1$  dan  $h = 0.05$ .
- (b) Cari penyelesaian siri kuasa bagi  $(x^2 + 1)y'' + xy' - y = 0$ .
- (c) Tentukan titik singular bagi  $(x^3 + 4x)y'' - 2xy' + 6y = 0$  dan kelaskan mereka sebagai regular atau tak regular.
- (d) Apakah jejari penumpuan dan selang penumpuan untuk siri kuasa  $\sum_{k=1}^{\infty} \frac{(-1)^k}{10^k} (x-5)^k$ .

[50 markah]

4. (a) Penyelesaian sistem tak homogen  $\bar{x}' = \begin{pmatrix} 6 & 1 \\ 4 & 3 \end{pmatrix} \bar{x} + \begin{pmatrix} 6t \\ -10t + 4 \end{pmatrix}$  pada  $(-\infty, \infty)$ .
- (b) Andaikan fungsi vektor  $\bar{\varphi}_1(t), \bar{\varphi}_2(t), \dots, \bar{\varphi}_n(t)$  ditakrifkan oleh  $\bar{\varphi}_i(t) = [\varphi_{1i}(t), \varphi_{2i}(t), \dots, \varphi_{mi}(t)]^T$  sebagai  $n$  penyelesaian bagi persamaan pembezaan vektor linear yang homogen  $\bar{x}' = A(t)\bar{x}$  dimana  $\bar{x}' = [x_1'(t), x_2'(t), \dots, x_n'(t)]^T$ ,  $\bar{x} = [x_1(t), x_2(t), \dots, x_n(t)]^T$ ,  $A(t) = [a_{ij}(t)]$  berperingkat  $n \times n$ . Jika  $n$  vektor  $\bar{\varphi}_i(t)$  adalah bersandar linear pada  $a \leq t \leq b$ , buktikan bahawa fungsi Wronskian bagi vektor,  $W(\bar{\varphi}_1, \bar{\varphi}_2, \dots, \bar{\varphi}_n)(t) \neq 0$  untuk  $t \in [a, b]$ .
- (c) Satu daging panggang 4-lb pada suhu awal  $50^\circ F$ , ditempatkan ke dalam oven bersuhu  $375^\circ F$  pada jam 5 petang. Selepas 75 minit di dapati suhu  $T(t)$  bagi daging panggang adalah  $125^\circ F$ . Bilakah daging panggang tersebut akan berada pada suhu  $150^\circ F$  (suhu pertengahan)?

[50 markah]

