UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2008/2009

Jun 2009

MAT 111 – Linear Algebra
[Aljabar Linear]

Duration : 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all five** [5] questions.

[Arahan : *Jawab* ***semua lima*** *[5] soalan.]*

1. Given $S=\{v_1,v_2,v_3\}$ where $v_1=(1,-1,1)$, $v_2=(1,0,2)$ and $v_3=(0,-2,-2)$.

(a) (i) Show that $\{v_1,v_2\}$ is linearly independent.

(ii) Show that v_3 is a linear combination of v_1 and v_2.

(b) Find <u>one</u> vector e_i from the standard basis $\{e_1,e_2,e_3\}$ of $\mathbb{R}^3$ such that $S'=\{v_1,v_2,e_i\}$ is linearly independent. Justify your answer.

(c) Show that $S''=\{v_1,v_1-v_2,e_i\}$ is also linearly independent.
(e_i is the vector you have chosen in (b))

(d) Can $S\cup\{e_i\}$ generate $\mathbb{R}^3$? Explain your answer.

[100 marks]

2. Given the system of linear equation:

$$\begin{aligned} x-y&=1\\ 2x-3y&=3\\ x+3y&=2 \end{aligned}$$

(a) Write the system into augmented matrix $[A|b]$.

(b) Use the Gauss-Jordan method to obtain the row-reduced echelon form (RREF) of the matrix in (a). (Show all elementary operations used)

(c) Explain clearly why there are no solutions to the system.

(d) Find the best approximate solution to the system. (Explain in detail including writing out A, v and b for the system $vA=b$, the normal equation and the calculations done to get the answer).

[100 marks]

1. *Diberi* $S=\{v_1,v_2,v_3\}$ *dengan* $v_1=(1,-1,1)$, $v_2=(1,0,2)$ *dan* $v_3=(0,-2,-2)$.

(a) (i) *Tunjukkan bahawa* $\{v_1,v_2\}$ *adalah tidak bersandar linear.*

(ii) *Tunjukkan bahawa* v_3 *adalah gabungan linear* v_1 *dan* v_2.

(b) *Cari satu vektor* e_i *daripada asas piawai* $\{e_1,e_2,e_3\}$ *untuk* $\mathbb{R}^3$ *sedemikian hingga* $S'=\{v_1,v_2,e_i\}$ *adalah tidak bersandar linear. Justifikasikan jawapan anda.*

(c) *Tunjukkan bahawa* $S''=\{v_1,v_1-v_2,e_i\}$ *adalah tidak bersandar linear juga.* (e_i *ialah vektor yang anda pilih dalam (b))*

(d) *Bolehkah* $S\cup\{e_i\}$ *menghasilkan* $\mathbb{R}^3$ *? Terangkan jawapan anda.*

[100 markah]

2. *Diberi sistem persamaan linear:*

$$\begin{aligned} x-y&=1\\ 2x-3y&=3\\ x+3y&=2 \end{aligned}$$

(a) *Tulis sistem di atas dalam bentuk matriks imbuhan* $[A|b]$.

(b) *Gunakan kaedah Gauss-Jordan untuk mendapatkan bentuk eselon baris terturun (BEBT) bagi matriks dalam (a). (Tunjukkan semua operasi baris permulaan yang digunakan)*

(c) *Terangkan dengan jelas mengapa sistem di atas tidak mempunyai penyelesaian.*

(d) *Cari penyelesaian penghampiran terbaik kepada sistem tersebut. (Terangkan secara terperinci termasuk memaparkan* A, v *dan* b *untuk sistem* $vA=b$ *, persamaan normal dan pengiraan yang dibuat untuk mendapatkan jawapan).*

[100 markah]

3. Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be a transformation defined by

$$(x,y,z)T = (x-2y+z, 3x-2y, 2x-z)$$

(a) Show that T is a linear transformation.

(b) Find the basis and dimension of the kernel of T, Ker T. (Show all steps)

(c) (i) Show that the set that defines the image of T is Im $T = \{\alpha(1,3,2)+\beta(-2,2,0)+\gamma(1,0,-1) | \alpha,\beta,\gamma \in \mathbb{R}\}$ and then deduce that the basis of ImT is $\{(1,3,2),(-2,2,0)\}$

(ii) Verify the dimension theorem based on your answer in (b) and (c)(i).

(d) Use the Gram-Schmidt process to find the **orthogonal** basis of ImT.

[100 marks]

4. Given that $S = \{(1,2,0),(1,0,1)\}$.

(a) Write out the set that defines the linear span of S, $\mathscr{L}(S)$.

(b) Show that $\mathscr{L}(S)$ is a subspace of $\mathbb{R}^3$.

(c) Let $W = \mathscr{L}(S)$. Give the set that defines the orthogonal complement of W, $W^{\perp}$ and find its basis.

(d) (i) Define the set $W + W^{\perp}$.

(ii) Show that $W \oplus W^{\perp} = \mathbb{R}^3$

[Hint : You just need to show that dim $(W + W^{\perp}) = 3$ and $W \cap W^{\perp} = \{\underline{0}\}$]

[100 marks]

3. *Biar* $T:\mathbb{R}^3 \to \mathbb{R}^3$ *suatu transformasi yang ditakrifkan dengan*

$$(x,y,z)T = (x-2y+z, 3x-2y, 2x-z)$$

(a) *Tunjukkan bahawa* T *adalah suatu transformasi linear.*

(b) *Cari asas dan dimensi kernel bagi* T, *Ker* T. *(Tunjuk semua langkah)*

(c) (i) *Tunjukkan bahawa set yang mentakrifkan imej bagi* T *adalah* $\text{Im } T = \{\alpha(1,3,2)+\beta(-2,2,0)+\gamma(1,0,-1) | \alpha,\beta,\gamma \in \mathbb{R}\}$ *dan kemudian buat kesimpulan bahawa asas bagi* $\text{Im}T$ *ialah* $\{(1,3,2),(-2,2,0)\}$

(ii) *Tentusahkan teorem dimensi berdasarkan jawapan anda dalam (b) dan (c)(i).*

(d) *Guna proses Gram-Schmidt untuk mencari asas* ***berortogon*** *bagi* $\text{Im}T$.

[100 markah]

4. *Diberi* $S = \{(1,2,0),(1,0,1)\}$.

(a) *Tulis set yang mentakrifkan rentangan linear bagi* S, $\mathscr{L}(S)$.

(b) *Tunjukkan bahawa* $\mathscr{L}(S)$ *ialah subruang dari* $\mathbb{R}^3$.

(c) *Biar* $W = \mathscr{L}(S)$. *Beri set yang mentakrifkan pelengkap berortogon bagi* W, $W^{\perp}$ *dan cari asasnya.*

(d) (i) *Takrifkan set* $W + W^{\perp}$.

(ii) *Tunjukkan bahawa* $W \oplus W^{\perp} = \mathbb{R}^3$

[Petunjuk : Anda hanya perlu tunjukkan $\dim(W+W^{\perp}) = 3$ *dan* $W \cap W^{\perp} = \{\underline{0}\}$ *]*

[100 markah]

5. Given

$$A=\begin{bmatrix} 3 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(a) Find the polynomial $f(x)=(x-\lambda_1)(x-\lambda_2)(x-\lambda_3)$, $\lambda_i \in \mathbb{R}$ such that $f(A)=\underline{0}$.
[Hint : A is a lower triangular matrix]

(b) Find $\beta=\{v_1,v_2,v_3\}$ where $v_i \in Ker(A-\lambda_i I)$, $i=1,2,3$.

(c) Find the change of basis matrices $I_{\alpha,\beta}$ and $I_{\beta,\alpha}$ where $\alpha=\{(1,0,0),(0,1,0),(0,0,1)\}$.

(d) From your results above, is A diagonalizable? Justify your answer.

[100 marks]

5. *Diberi*

$$A=\begin{bmatrix} 3 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(a) *Cari polinomial* $f(x)=(x-\lambda_1)(x-\lambda_2)(x-\lambda_3)$, $\lambda_i \in \mathbb{R}$ *sedemikian hingga* $f(A)=\underline{0}$.
[Petunjuk : A ialah matriks segitiga bawah]

(b) *Cari* $\beta=\{v_1,v_2,v_3\}$ *yang mana* $v_i \in Ker(A-\lambda_i I)$, $i=1,2,3$.

(c) *Cari matriks pertukaran asas* $I_{\alpha,\beta}$ *dan* $I_{\beta,\alpha}$ *yang mana* $\alpha=\{(1,0,0),(0,1,0),(0,0,1)\}$.

(d) *Daripada keputusan anda di atas, adakah A terpepenjurukan? Justifikasikan jawapan anda.*

[100 markah]

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