
UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2008/2009

Jun 2009

MAT 102 – Advanced Calculus
[Kalkulus Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all nine** [9] questions.

Arahan: Jawab **semua sembilan** [9] soalan.]

1. Find the following limits. Use L'Hospital's rule where appropriate.

$$(a) \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x}\right)^x \qquad (b) \lim_{x \rightarrow 0} \frac{x - \tan x}{x - \sin x}$$

[16 marks]

2. (a) (i) What does it mean for a sequence $\{a_n\}_{n=1}^{\infty}$ to be convergent?

(ii) What does it mean for a series $\sum_{n=1}^{\infty} a_n$ to be convergent?

(b) Consider $a_n = \frac{(-1)^n}{n3^n}$.

(i) Determine whether the sequence $\{a_n\}_{n=1}^{\infty}$ is bounded, monotonic and convergent.

(ii) Determine whether the series $\sum_{n=1}^{\infty} a_n$ converges or diverges.

[26 marks]

3. (a) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n3^n}.$$

(b) By using a suitable power series representation, find a power series for

$$\frac{3}{1-x^4}.$$

[20 marks]

4. (a) Evaluate the following integrals.

$$(i) \int_1^2 \frac{1}{2x-1} dx \qquad (ii) \int_0^1 \frac{1}{2x-1} dx$$

(b) Determine whether $\int_{10}^{\pi/2} \frac{1}{x \sin x} dx$ is convergent or divergent.

[20 marks]

5. Let $f(x, y) = \frac{x-2y}{2x+y}$, $2x+y \neq 0$.

(a) Find $\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y)$ and $\lim_{y \rightarrow 0} \lim_{x \rightarrow 0} f(x, y)$.

(b) What can you say about $\lim_{(x,y) \rightarrow (0,0)} \frac{x-2y}{2x+y}$?

[8 marks]

1. Cari had berikut. Guna petua L'Hospital jika sesuai.

$$(a) \lim_{x \rightarrow \infty} \left(1 + \frac{5}{x}\right)^x$$

$$(b) \lim_{x \rightarrow 0} \frac{x - \tan x}{x - \sin x}$$

[16 markah]

2. (a) (i) Apakah maksud suatu jujukan $\{a_n\}_{n=1}^{\infty}$ menumpu?

(ii) Apakah maksud suatu siri $\sum_{n=1}^{\infty} a_n$ menumpu?

(b) Pertimbangkan $a_n = \frac{(-1)^n}{n3^n}$.

(i) Tentukan sama ada jujukan $\{a_n\}_{n=1}^{\infty}$ terbatas, berekonada dan menumpu.

(ii) Tentukan sama ada siri $\sum_{n=1}^{\infty} a_n$ menumpu atau mencapah.

[26 markah]

3. (a) Cari jejari penumpuan dan selang penumpuan untuk siri

$$\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$$

(b) Dengan menggunakan perwakilan siri kuasa yang sesuai, cari siri kuasa untuk

$$\frac{3}{1-x^4}$$

[20 markah]

4. (a) Nilai kamiran berikut.

$$(i) \int_1^2 \frac{1}{2x-1} dx$$

$$(ii) \int_0^1 \frac{1}{2x-1} dx$$

(b) Tentukan sama $\int_{10}^{\pi/2} \frac{1}{x \sin x} dx$ menumpu atau mencapah.

[20 markah]

5. Andaikan $f(x, y) = \frac{x-2y}{2x+y}$, $2x+y \neq 0$.

(a) Cari $\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y)$ dan $\lim_{y \rightarrow 0} \lim_{x \rightarrow 0} f(x, y)$.

(b) Apa anda boleh kata tentang $\lim_{(x,y) \rightarrow (0,0)} \frac{x-2y}{2x+y}$?

[8 markah]

6. Consider the function $f(x, y) = \sqrt{9x^2 - 4y^2 - 1}$, $9x^2 - 4y^2 - 1 \geq 0$.

- Find the directional derivative $D_{\vec{v}}f(3, -2)$ in the direction $\vec{v} = (1, 5)$.
- Find all the first order partial derivatives of f .
- Does the graph of f intersect the y -axis? Give your reason.

[18 marks]

7. The equation $xe^{yz} + (3z - 2y)e^{xy} = 2$ defines implicitly a function $z = f(x, y)$.

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

[10 marks]

8. Reverse the order of the iterated integral $\int_0^2 \int_{y^2}^4 y \cos(x^5) dx dy$ and then evaluate it.

[12 marks]

9. *True or False*

(a) If $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$ exists, then $\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$ also exists.

(b) If $f(x) = \sum_{n=1}^{\infty} c_n (x-a)^n$ converges on the interval $(a-R, a+R)$, then $f'(x) = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1}$ also converges on $(a-R, a+R)$.

(c) If the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ is differentiable, then f is continuous.

(d) If $f(x) \leq g(x)$ for $x \in [1, \infty)$ and $\int_1^{\infty} f(x) dx$ diverges, then $\int_1^{\infty} g(x) dx$ diverges.

(e) If the sequence $\{a_n\}$ converges, then $\lim_{n \rightarrow \infty} a_n$ exists.

[10 marks]

6. Pertimbangkan fungsi $f(x, y) = \sqrt{9x^2 - 4y^2 - 1}$, $9x^2 - 4y^2 - 1 \geq 0$.

- (a) Cari terbitan berarah $D_{\vec{v}}f(3, -2)$ pada arah $\vec{v} = (1, 5)$.
 (b) Cari semua terbitan separa peringkat pertama untuk f .
 (c) Adakah graf untuk f memintas paksi- y ? Beri alasan anda.

[18 markah]

7. Persamaan $xe^{yz} + (3z - 2y)e^{xy} = 2$ menakrif secara tersirat suatu fungsi

$$z = f(x, y). \text{ Cari } \frac{\partial z}{\partial x} \text{ dan } \frac{\partial z}{\partial y}.$$

[10 markah]

8. Tukar tertib kamiran terlelar $\int_0^2 \int_y^4 y \cos(x^5) dx dy$ dan kemudian nilaikannya.

[12 markah]

9. Benar atau Salah

(a) Jika $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$ wujud, maka $\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)}$ juga wujud.

(b) Jika $f(x) = \sum_{n=1}^{\infty} c_n (x-a)^n$ menumpu pada selang $(a-R, a+R)$, maka $f'(x) = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1}$ juga menumpu pada $(a-R, a+R)$.

(c) Jika fungsi $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ adalah terbezakan, maka f adalah selanjar.

(d) Jika $f(x) \leq g(x)$ untuk $x \in [1, \infty)$ dan $\int_1^{\infty} f(x) dx$ mencapah, maka $\int_1^{\infty} g(x) dx$ mencapah.

(e) Jika jujukan $\{a_n\}$ menumpu, maka $\lim_{n \rightarrow \infty} a_n$ wujud.

[10 markah]