
UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2008/2009

Jun 2009

MAA 161 – Statistics for Science Students
[Statistik untuk Pelajar Sains]

Duration : 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of EIGHT pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all ten** [10] questions.

Arahan : Jawab **semua sepuluh** [10] soalan.]

1. Stores A, B and C have 50, 75 and 100 employees respectively, and 50, 60 and 70 percent of these are women. The resignations are equally likely among all employees, regardless of sex.
- (a) If one employee resigns and this is a woman (W), what is the probability that she works in the store C?
- (b) Find $P(A|\bar{W})$, where A is an employee from store A.

[10 marks]

2. Let the density for the continuous random variable X be given by

$$f(x) = \begin{cases} \frac{1}{2}e^{-x} & x \geq 0 \\ \frac{1}{2}e^x & x < 0 \end{cases}$$

- (a) Show that $\int_{-\infty}^{\infty} f(x) dx = 1$.
- (b) Find the mean and the standard deviation of the above distribution.

[10 marks]

3. Let the probability density function of X be given by

$$f_X(x) = \begin{cases} c(4x^2 - 2x^3) & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find

- (a) the value of c
- (b) $P\left(\frac{1}{2} < X < \frac{3}{2}\right)$
- (c) $E(X)$ and $Var(X)$.

By using the Central Limit Theorem, find $P\left(\frac{3}{4} < \bar{X} < \frac{5}{4}\right)$ for a sample size of 4.

[10 marks]

...3/-

1. Stor-stor A, B dan C mempunyai 50, 75 dan 100 pekerja, dan masing-masing, 50, 60 dan 70 peratus adalah perempuan. Pemberhentian pekerja bagi A, B dan C adalah sama dan tidak mengikut jantina.
- (a) Sekiranya seorang pekerja meletak jawatan, dan ia adalah perempuan (W), apakah kebarangkalian dia bekerja di stor C?
- (b) Cari $P(A|\bar{W})$, yang mana A ialah pekerja dari stor A.

[10 markah]

2. Katakan fungsi ketumpatan bagi pembolehubah rawak selanjar, X diberi seperti berikut:

$$f(x) = \begin{cases} \frac{1}{2}e^{-x} & x \geq 0 \\ \frac{1}{2}e^x & x < 0 \end{cases}$$

- (a) Tunjukkan bahawa $\int_{-\infty}^{\infty} f(x) dx = 1$.
- (b) Cari min dan sisihan piawai bagi taburan di atas.

[10 markah]

3. Katakan taburan kebarangkalian bagi fungsi X seperti berikut

$$f_x(x) = \begin{cases} c(4x^2 - 2x^3) & 0 < x < 2 \\ 0 & \text{selainnya} \end{cases}$$

Cari

- (a) nilai c
- (b) $P\left(\frac{1}{2} < X < \frac{3}{2}\right)$
- (c) $E(X)$ dan $Var(X)$.

Dengan menggunakan Teorem Had Memusat, cari $P\left(\frac{3}{4} < \bar{X} < \frac{5}{4}\right)$ bagi saiz sampel 4.

[10 markah]

...4/-

4. A random variable, X takes the value of x with the probability $P(x)$ where,

$$P(X = x) = \begin{cases} kx & x = 1, 2, 3, 4, 5 \\ k(10 - x) & x = 6, 7, 8, 9 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the value of k .
 (b) Calculate $P(3/2 < X < 13/2)$
 (c) Calculate the mean and variance of X
 (d) Calculate $E(2X - 3)$ and $Var(2X - 3)$

[10 marks]

5. Given the fuel consumption (in Litre/100 km) of a car in the table below.

Fuel Consumption (Litre/100km)	Frequency
7 – 8	9
8 – 9	10
9 – 10	6
10 – 11	7
11 – 12	2
12 – 13	4
13 – 14	1
14 – 15	0
15 onwards	1

- (a) Calculate the mean, variance and standard deviation.
 (b) Compute Q_1 and Q_3 .
 (c) What is the value of $P(X \leq 10)$?

[10 marks]

6. A sample of the scores of 35 students has a mean of 82. The standard deviation of the sample is 15.

- (a) Find the 95% confidence interval of the mean scores of all the students.
 (b) Find the 99% confidence interval of the mean scores of all the students.
 (c) Which interval is larger? Explain why.

[10 marks]

...5/-

4. Suatu pembolehubah rawak, X mengambil nilai x dengan kebarangkalian $P(x)$ di mana,

$$P(X = x) = \begin{cases} kx & x = 1, 2, 3, 4, 5 \\ k(10 - x) & x = 6, 7, 8, 9 \\ 0 & \text{selainnya} \end{cases}$$

- (a) Cari nilai k .
 (b) Kira $P(3/2 < X < 13/2)$
 (c) Kira min and varians bagi X
 (d) Kira $E(2X - 3)$ dan $Var(2X - 3)$

[10 markah]

5. Diberi penggunaan petrol bagi sebuah kereta di dalam jadual di bawah (dalam Litre/100 km).

Penggunaan petrol (Litre/100km)	Kekerapan
7 – 8	9
8 – 9	10
9 – 10	6
10 – 11	7
11 – 12	2
12 – 13	4
13 – 14	1
14 – 15	0
15 ke atas	1

- (a) Kira min, varians dan sisihan piawai.
 (b) Kira Q_1 dan Q_3 .
 (c) Apakah nilai $P(X \leq 10)$?

[10 markah]

6. Suatu sampel skor 35 pelajar mempunyai min sebanyak 82. Sisihan piawai bagi sampel adalah 15.

- (a) Cari selang keyakinan 95% bagi min skor untuk semua pelajar.
 (b) Cari selang keyakinan 99% bagi min skor untuk semua pelajar.
 (c) Selang yang manakah lebih besar? Terangkan mengapa.

[10 markah]

7. A company claims that the variance of the sugar content of its product is less than or equal to 25. (The sugar content is measured in milligrams per ounce). A sample of size 20 is selected, and the sugar content is measured. The variance of the sample is found to be 36. At $\alpha = 0.10$, is there enough evidence to reject the claim?

[10 marks]

8. The data on the number of miles travelled by two bus companies are shown below. Is there a difference in average miles travelled for each of the two bus companies during a randomly selected week? Use $\alpha = 0.05$. Assume that the populations are normally distributed.

Company A	$\bar{X}_1 = 837$	$\sigma_1 = 30$	$n_1 = 25$
Company B	$\bar{X}_2 = 753$	$\sigma_2 = 40$	$n_2 = 20$

[10 marks]

9. Three coins are tossed 72 times, and the number of heads obtained is shown. At $\alpha = 0.05$, test the null hypothesis that the coins are 'fair' and randomly tossed.

No. of heads	0	1	2	3
Frequency	3	10	17	42

[10 marks]

10. In a corporation, female and male workers were matched according to years of experience working for the company. Their salaries were then compared. The data (in thousands of ringgit) are shown in the table below.

Males	18	43	32	27	15	45	21	22
Females	16	38	35	29	15	46	25	28

Using the Wilcoxon signed-rank test at the 10% level of significance, is there a difference in the salaries of the males and females?

[10 marks]

7. Suatu syarikat mendakwa bahawa varians kandungan gula bagi produknya adalah kurang atau sama dengan 25. (Kandungan gula diukur dalam milligram perouns). Suatu sampel bersaiz 20 dipilih dan kandungan gula diukur. Varians sampel didapati sebanyak 36. Pada $\alpha = 0.10$, adakah bukti cukup untuk menolak dakwaan?

[10 markah]

8. Data bagi jarak dalam batu yang dilalui oleh dua syarikat bas diberikan di bawah. Adakah terdapat perbezaan purata jarak yang dilalui oleh dua buah syarikat bas pada suatu minggu yang dipilih secara rawak? Guna $\alpha = 0.05$. Andaikan bahawa kedua-dua populasi tersebut adalah bertaburan normal.

$$\begin{array}{llll} \text{Syarikat A} & \bar{X}_1 = 837 & \sigma_1 = 30 & n_1 = 25 \\ \text{Syarikat B} & \bar{X}_2 = 753 & \sigma_2 = 40 & n_2 = 20 \end{array}$$

[10 markah]

9. Tiga siling dilambung sebanyak 72 kali dan bilangan kepala yang diperolehi ditunjukkan. Pada $\alpha = 0.05$, uji hipotesis nol bahawa siling-siling tersebut adalah seimbang dan dilambung secara rambang.

Bilangan Kepala	0	1	2	3
Kekerapan	3	10	17	42

[10 markah]

10. Dalam suatu syarikat korporat, pekerja-pekerja lelaki dan perempuan dinilai berdasarkan kepada bilangan tahun pengalaman bekerja dengan syarikat. Gaji mereka kemudian dibandingkan. Data tersebut (dalam ribu ringgit) adalah ditunjukkan di dalam jadual di bawah.

Lelaki	18	43	32	27	15	45	21	22
Perempuan	16	38	35	29	15	46	25	28

Gunakan ujian pangkat bertanda Wilcoxon pada 10% aras keertian, adakah terdapat perbezaan di antara gaji lelaki dan perempuan?

[10 markah]

APPENDIX/FORMULA

Confidence Interval:

$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ $\bar{d} \pm t_{\frac{\alpha}{2}} \frac{s_d}{\sqrt{n_d}}$	$\bar{X} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$ $b \pm t_{\frac{\alpha}{2}} s_b$	$(\bar{X} - \bar{Y}) \pm t_{\alpha/2} S_p \sqrt{\frac{1}{n_x} + \frac{1}{n_y}}$
$\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $(\hat{p}_x - \hat{p}_y) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_x(1-\hat{p}_x)}{n_x} + \frac{\hat{p}_y(1-\hat{p}_y)}{n_y}}$ $(\bar{X} - \bar{Y}) \pm Z_{\alpha/2} \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}$ $(\bar{X} - \bar{Y}) \pm t_{\alpha/2} \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}$	$\left(\frac{(n-1)s^2}{\chi_{\frac{\alpha}{2}, n-1}^2}, \frac{(n-1)s^2}{\chi_{1-\frac{\alpha}{2}, n-1}^2} \right)$ $\left(\frac{s}{Z_{\frac{\alpha}{2}} \sqrt{1 + \frac{\alpha}{2n}}}, \frac{s}{Z_{\frac{\alpha}{2}} \sqrt{1 - \frac{\alpha}{2n}}} \right)$ $\left(\frac{s_1^2}{s_2^2} F_{1-\frac{\alpha}{2}, (v_2, v_1)}, \frac{s_1^2}{s_2^2} F_{\frac{\alpha}{2}, (v_2, v_1)} \right)$	$\left(\frac{(n-1)s^2}{\chi_{\frac{\alpha}{2}, n-1}^2}, \frac{(n-1)s^2}{\chi_{1-\frac{\alpha}{2}, n-1}^2} \right)$ $\left(\frac{s}{Z_{\frac{\alpha}{2}} \sqrt{1 + \frac{\alpha}{2n}}}, \frac{s}{Z_{\frac{\alpha}{2}} \sqrt{1 - \frac{\alpha}{2n}}} \right)$ $\left(\frac{s_1^2}{s_2^2} F_{1-\frac{\alpha}{2}, (v_2, v_1)}, \frac{s_1^2}{s_2^2} F_{\frac{\alpha}{2}, (v_2, v_1)} \right)$

Test Statistic:

$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$ $T = \frac{\bar{X} - \mu}{s / \sqrt{n}}$ $T = \frac{\bar{d} - \mu_d}{s_d / \sqrt{n_d}}$ $T = \frac{b - \beta_1}{s_b}$ $T = r \sqrt{\frac{n-2}{1-r^2}}$ $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$	$Z = \frac{s - \sigma}{\sigma / \sqrt{2n}}$ $Z = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}}$ $T = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2 \left(\frac{1}{n_x} + \frac{1}{n_y} \right)}}$ $S_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}$ $F = \frac{s_x^2}{s_y^2}$	$Z = \frac{(\hat{p}_x - \hat{p}_y) - (p_x - p_y)}{\sqrt{\hat{p}(1-\hat{p}) \left(\frac{1}{n_x} + \frac{1}{n_y} \right)}}$ $T = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}}$ $dk = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y} \right)^2}{\frac{\left(\frac{s_x^2}{n_x} \right)^2}{n_x - 1} + \frac{\left(\frac{s_y^2}{n_y} \right)^2}{n_y - 1}}$ $\chi^2 = \sum \frac{(O - E)^2}{E}, \quad E = np$
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