UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2008/2009

Jun 2009

MAA 111 – Algebra for Science Students
[Aljabar untuk Pelajar Sains]

Duration : 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all nine** [9] questions.

*[**Arahan** : Jawab **semua sembilan** [9] soalan.]*

1. Consider the linear system

$$\begin{aligned} x_1 + 3x_2 - 2x_3 &= b_1 \\ -x_1 - 5x_2 + 3x_3 &= b_2 \\ 2x_1 - 8x_2 + 3x_3 &= b_3 \end{aligned}.$$

Determine the conditions (if any) on b_1, b_2 and b_3 so that the linear system is
(a) consistent
(b) inconsistent

[7 marks]

2. (a) Find the solution set for $6x - 8y - 9z = 3$.

[3 marks]

(b) Use row reduction to compute the determinant

$$A = \begin{pmatrix} -2 & 10 & 2 \\ 1 & 0 & 7 \\ 0 & -3 & 5 \end{pmatrix}.$$

[5 marks]

3. Compute the orthogonal projection of $\boldsymbol{u}$ on $\boldsymbol{a}$ and the vector component of $\boldsymbol{u}$ orthogonal to $\boldsymbol{a}$ for each of the following:

(a) $\boldsymbol{u} = (-3,1), \boldsymbol{a} = (7,2)$

(b) $\boldsymbol{u} = (4,0,-1), \boldsymbol{a} = (3,1,-5)$

[10 marks]

4. Determine the new point after applying the transformation to the given point.

(a) $\boldsymbol{x} = (2,-4,1)$ reflected about the xz-plane.

(b) $\boldsymbol{x} = (10,7,-9)$ projected on the x-axis.

(c) Project $\boldsymbol{x} = (4,-1,-3)$ on the y-axis and then dilate by 2.

(d) Project $\boldsymbol{x} = (4,2)$ on the x-axis and the rotate by 45° counter-clockwise.

[18 marks]

...3/-

1. *Pertimbangkan sistem linear*

$$\begin{aligned} x_1 + 3x_2 - 2x_3 &= b_1 \\ -x_1 - 5x_2 + 3x_3 &= b_2 \\ 2x_1 - 8x_2 + 3x_3 &= b_3 \end{aligned}.$$

Tentukan syarat (sekiranya wujud) pada b_1, b_2 *dan* b_3 *supaya sistem linear adalah*

(a) *konsisten*

(b) *tak konsisten.*

[7 markah]

2. (a) *Cari set penyelesaian bagi* $6x - 8y - 9z = 3$

[3 markah]

(b) *Menggunakan kaedah baris terturun dapatkan penentu*

$$A = \begin{pmatrix} -2 & 10 & 2 \\ 1 & 0 & 7 \\ 0 & -3 & 5 \end{pmatrix}.$$

[5 markah]

3. *Dapatkan unjuran ortogon vektor* $\boldsymbol{u}$ *atas* $\boldsymbol{a}$ *dan juga komponen vektor* $\boldsymbol{u}$ *ortogon kepada* $\boldsymbol{a}$ *untuk setiap yang berikut:*

(a) $\boldsymbol{u} = (-3,1), \boldsymbol{a} = (7,2)$

(b) $\boldsymbol{u} = (4,0,-1), \boldsymbol{a} = (3,1,-5)$

[10 markah]

4. *Dapatkan titik baru setelah transformasi dikenakan pada titik yang diberikan.*

(a) $\boldsymbol{x} = (2,-4,1)$ *direfleksikan pada satah-xz.*

(b) $\boldsymbol{x} = (10,7,-9)$ *diunjurkan pada paksi-x.*

(c) *Unjur titik* $\boldsymbol{x} = (4,-1,-3)$ *pada paksi-y dan dibesarkan sebanyak* 2.

(d) *Unjur titik* $\boldsymbol{x} = (4,2)$ *pada paksi-x dan diputar* 45° *mengikut arah lawan jam.*

[18 markah]

5. Find the area of the parallelogram determined by $u=(1,-1,2)$ and $v=(0,3,1)$.

[8 marks]

6. Determine whether

(a) $V=\left\{\begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix};\ a,b\in\mathbb{R}\right\}$ with the standard matrix addition and scalar multiplication is a vector space or not.

(b) $W=\left\{\begin{pmatrix} a \\ b \\ c \end{pmatrix};\ b=a+c+1 \text{ and } a,b,c\in\mathbb{R}\right\}$ is a subspace of $\mathbb{R}^3$ or not.

[8 marks]

7. (a) Find a 2×2 nondiagonal matrix whose eigenvalues are 2 and -3, and associated eigenvectors are $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, respectively.

(b) Let $A=\begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix}$. Verify the Cayley-Hamilton Theorem for A. Hence, compute A^3.

[16 marks]

8. Let W be the subspace of $\mathbb{R}^4$ spanned by the vectors $u_1=(1,-2,5,-3)$, $u_2=(2,3,1,-4)$, $u_3=(3,8,-3,-5)$.

(a) Find a basis and dimension of W.

(b) Extend the basis of W to a basis of $\mathbb{R}^4$.

[10 marks]

5. *Dapatkan luas parallelogram yang ditentukan oleh* $u=(1,-1,2)$ *dan* $v=(0,3,1)$.

[8 markah]

6. *Tentukan sama ada*

(a) $V=\left\{\begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix};\ a,b\in\mathbb{R}\right\}$ *dengan operasi penambahan dan pendaraban skalar matriks yang biasa merupakan suatu ruang vektor atau bukan.*

(b) $W=\left\{\begin{pmatrix} a \\ b \\ c \end{pmatrix};\ b=a+c+1 \text{ dan } a,b,c\in\mathbb{R}\right\}$ *merupakan suatu subruang atau bukan.*

[8 markah]

7. (a) *Dapatkan suatu matriks* 2×2 *yang bukan matriks pepenjuru yang nilai eigennya adalah* 2 *dan* -3 *dan vektor eigennya adalah* $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ *dan* $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ *masing-masing.*

(b) *Biar* $A=\begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix}$. *Buktikan Teorem Cayley-Hamilton bagi A. Seterusnya, hitung* A^3.

[16 markah]

8. *Katakan W adalah suatu subruang bagi* $\mathbb{R}^4$ *yang direntangi oleh vektor* $u_1=(1,-2,5,-3)$, $u_2=(2,3,1,-4)$, $u_3=(3,8,-3,-5)$.

(a) *Dapatkan asas dan dimensi bagi W.*

(b) *Lanjutkan asas W kepada asas bagi* $\mathbb{R}^4$.

[10 markah]

9. (a) Find the value(s) of k for which $u=\begin{pmatrix}1\\3\\-3\end{pmatrix}$, $v=\begin{pmatrix}-2\\-4\\1\end{pmatrix}$ and $w=\begin{pmatrix}-1\\1\\k\end{pmatrix}$ are linearly dependent.

(b) Show that $\left\{\begin{pmatrix}1\\0\\-1\end{pmatrix},\begin{pmatrix}2\\0\\2\end{pmatrix},\begin{pmatrix}0\\5\\0\end{pmatrix}\right\}$ is an orthogonal set of vectors; then convert it to orthonormal set.

[15 marks]

9. (a) *Dapatkan nilai k supaya* $u = \begin{pmatrix} 1 \\ 3 \\ -3 \end{pmatrix}$, $v = \begin{pmatrix} -2 \\ -4 \\ 1 \end{pmatrix}$ *dan* $w = \begin{pmatrix} -1 \\ 1 \\ k \end{pmatrix}$ *adalah bersandar linear.*

(b) *Tunjukkan bahawa* $\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} \right\}$ *merupakan set vektor yang berortogon; kemudian tukarkannya kepada set ortonormal.*

[15 markah]

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