UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

# MAA 111 – Algebra for Science Students
## *[Aljabar untuk Pelajar Sains]*

Duration : 3 hours
*[Masa : 3 jam]*

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Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions**: Answer **all ten** [10] questions.

*[**Arahan:** Jawab **semua sepuluh** [10] soalan.]*

...2/-

1. For which values of $a$ will the following linear system have no solution? Unique solution? And infinite solutions?

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 4 \\ 3x_1 + x_2 - 5x_3 &= 2 \\ 4x_1 + x_2 + \left(a^2 - 14\right)x_3 &= a + 2 \end{aligned}$$

[10 marks]

2. Find the volume of the parallelepiped with one vertex at the origin and with the coordinate of the edges intersecting the origin ending at $(2,1,3),(3,0,1)$ and $(3,0,5)$. Sketch the parallelepiped.

[8 marks]

3. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. Assuming $\det(A) = -7$, find

   (a) $\det(3A)$

   (b) $\det(2A^{-1})$

   (c) $\det\left((2A)^{-1}\right)$

   (d) $\det\begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix}$

[8 marks]

4. (a) Points in the $xy$ – plane are rotated through an angle $\theta$, followed by a reflection about the $x$ – axis and then rotated through an angle $\theta$. Find the matrix representing this combination of transformations.

[5 marks]

   (b) Find the linear transformation of the $xy$ – plane that maps the point $(1,1)$ to $(3,3)$ and the point $(1,0)$ to itself.

[5 marks]

5. Calculate the scalar triple product $\mathrm{u}\cdot(\mathrm{v}\times\mathrm{w})$ of the vectors u = 3i-2j-5k; v = i+4j-4k; w = 3j+2k.

[5 marks]

6. Find the area of the parallelogram determined by u = $(1,-1,2)$ and v = $(0,3,1)$

[8 marks]

1. *Apakah nilai a yang akan menjadikan sistem linear di bawah tidak mempunyai penyelesaian? Penyelesaian unik? Dan penyelesaian infinit?*

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 4 \\ 3x_1 + x_2 - 5x_3 &= 2 \\ 4x_1 + x_2 + \left(a^2 - 14\right)x_3 &= a + 2 \end{aligned}$$

*[10 markah]*

2. *Dapatkan isipadu parallelepipe dengan satu bucunya di asalan serta koordinat penghujungnya bersilang di asalan dan berakhir di titik* $(2,1,3),(3,0,1)$ *dan* $(3,0,5)$. *Lakarkan bentuk parallelepipe tersebut.*

*[8 markah]*

3. *Diberi* $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$. *Andaikan penentu*$(A) = -7$, *dapatkan*

(a) *penentu*$(3A)$

(b) *penentu*$\left(2A^{-1}\right)$

(c) *penentu*$\left((2A)^{-1}\right)$

(d) *penentu*$\begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix}$

*[8 markah]*

4. (a) *Titik-titik di satah-xy diputar melalui sudut* $\theta$, *diikuti dengan refleksi sekitar paksi-x dan seterusnya diputar lagi melalui sudut* $\theta$. *Dapatkan matrik yang mewakili kombinasi transformasi ini.*

*[5 markah]*

(b) *Dapatkan transformasi linear di satah–xy yang memetakan titik* $(1,1)$ *ke* $(3,3)$ *dan titik* $(1,0)$ *dipetakan kepada dirinya sendiri.*

*[5 markah]*

5. *Kira hasil darab scalar gandatiga* $\mathrm{u}\cdot(\mathrm{v}\times\mathrm{w})$ *bagi vektor-vektor berikut* u = 3i-2j-5k; v = i+4j-4k; w = 3j+2k.

*[5 markah]*

6. *Dapatkan luas parallelogram yang ditentukan oleh* u = $(1,-1,2)$ *dan* v = $(0,3,1)$.

*[8 markah]*

7. Determine whether $W$ is a subspace of $V$.

(a) $V = \mathbb{R}^2, \quad W = \left\{ \begin{pmatrix} a \\ b \end{pmatrix};\ a \geq 0 \text{ and } a, b \in \mathbb{R} \right\}$.

(b) $V = \mathbb{R}^3, \quad W = \left\{ \begin{pmatrix} a \\ c \\ c \end{pmatrix};\ a^2 + b^2 + c^2 \leq 1 \text{ and } a, b, c \in \mathbb{R} \right\}$.

[8 marks]

8. Let $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$.

(a) Find the eigenvalues of $A$.

(b) Find a basis for the eigenspaces of $A$.

(c) Is $A$ diagonalizable? If so, find a nonsingular matrix $P$ such that $P^{-1}AP$ is diagonal. Hence, find $A^6$.

[17 marks]

9. Find the rank and nullity of the matrix $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$.

[6 marks]

10. (a) Given that $\{u, v, w\}$ is a linearly independent set of vectors. Show that $\{u + v - 2w, u - v - w, u + w\}$ is linearly independent.

(b) Use the Gram-Schmidt process to transform the basis $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \right\}$ for the subspace $W$ of $\mathbb{R}^3$ into

(i) an orthogonal basis
(ii) an orthonormal basis.
(iii) Write the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ in $\mathbb{R}^3$ as a linear combination of those orthonormal basis in (ii).

[20 marks]

7. *Tentukan sama ada* $W$ *merupakan suatu subruang bagi* $V$.

(a) $V = \mathbb{R}^2, \quad W = \left\{ \begin{pmatrix} a \\ b \end{pmatrix}; \ a \geq 0 \text{ dan } a, b \in \mathbb{R} \right\}.$

(b) $V = \mathbb{R}^3, \quad W = \left\{ \begin{pmatrix} a \\ c \\ c \end{pmatrix}; \ a^2 + b^2 + c^2 \leq 1 \text{ dan } a, b, c \in \mathbb{R} \right\}.$

*[8 markah]*

8. *Biarkan* $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}.$

*(a)* *Dapatkan nilai eigen bagi* $A$.

*(b)* *Dapatkan suatu asas bagi ruang eigen* $A$.

*(c)* *Adakah* $A$ *terpepenjurukan? Jika ya, dapatkan matrik* $P$ *yang tak singular sedemikian* $P^{-1}AP$ *adalah matriks pepenjuru. Seterusnya, dapatkan* $A^6$.

*[17 markah]*

9. *Dapatkan pangkat dan nuliti bagi matriks* $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}.$

*[6 markah]*

10. (a) *Di beri* $\{u, v, w\}$ *merupakan set vektor yang tak bersandar linear. Tunjukkan bahawa* $\{u + v - 2w, u - v - w, u + w\}$ *adalah tak bersandar linear.*

*(b)* *Gunakan proses Gram-Schmidt untuk menukar asas* $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \right\}$ *bagi subruang* $W$ *dari* $\mathbb{R}^3$ *kepada*

*(i)* *asas ortogon*
*(ii)* *asas ortonormal.*
*(iii)* *Tuliskan vektor* $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ *dalam* $\mathbb{R}^3$ *sebagai gabungan linear asas ortonormal di bahagian (ii).*

*[20 markah]*

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