UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

MAA 102 – Calculus for Science Students II
[Kalkulus untuk Pelajar Sains II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>FIVE</u> pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>LIMA</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **<u>all ten</u>** [10] questions.

*[**Arahan:**   Jawab **<u>semua sepuluh</u>** [10] soalan.]*

1. Determine whether $\left\{ \dfrac{(-1)^{n-1} n}{n^2 + 1} \right\}$ converges or diverges. If it converges, find its limit.

[4 marks]

2. Test the convergence of the series:

(a) $\displaystyle\sum_{n=1}^{\infty} \dfrac{2^n}{100}$

(b) $\displaystyle\sum_{n=1}^{\infty} \dfrac{\sin n + \cos n}{n^3}$

[10 marks]

3. Let $f(x) = x^{5/2}$.

   (a) Find $T_3(x)$ at $a = 4$.

   (b) Estimate the error between $f(x)$ and $T_3(x)$ for $2 \le x \le 6$.

[12 marks]

4. Find the center, radius and interval of convergence of the power series
$$\sum_{n=1}^{\infty} \dfrac{(x-4)^n}{2^n n}.$$

[13 marks]

5. Determine whether $\displaystyle\int_{2}^{\infty} \dfrac{x^2 + x + 2}{x^4 + x^2 - 1} \, dx$ converges or diverges.

[5 marks]

6. (a) By the definition of partial derivative, find $f_y(x,y)$ if
$f(x,y) = x^2 - xy + 2y^2$. Hence, find $f_{yx}(x,y)$.

   (b) Use the chain rule to find $\dfrac{dw}{dt}$ at $t = \pi$ if $w = x^2 y + y^2 z$, $x = t \sin t$,
   $y = t \cos t$ and $z = 2t$

[13 marks]

1. Tentukan sama ada $\left\{ \dfrac{(-1)^{n-1} n}{n^2 + 1} \right\}$ menumpu atau mencapah. Jika menumpu, dapatkan hadnya.

[4 markah]

2. Uji penumpuan siri:

    (a) $\displaystyle\sum_{n=1}^{\infty} \dfrac{2^n}{100}$ (b) $\displaystyle\sum_{n=1}^{\infty} \dfrac{\sin n + \text{kos}\, n}{n^3}$

[10 markah]

3. Biar $f(x) = x^{5/2}$.

    (a) Dapatkan $T_3(x)$ pada $a = 4$.

    (b) Anggarkan ralat antara $f(x)$ dan $T_3(x)$ bagi $2 \le x \le 6$.

[12 markah]

4. Dapatkan pusat, jejari dan selang penumpuan bagi siri kuasa $\displaystyle\sum_{n=1}^{\infty} \dfrac{(x-4)^n}{2^n n}$

[13 markah]

5. Tentukan sama ada $\displaystyle\int_{2}^{\infty} \dfrac{x^2 + x + 2}{x^4 + x^2 - 1}\, dx$ menumpu atau mencapah.

[5 markah]

6. (a) Menggunakan takrif terbitan separa, dapatkan $f_y(x, y)$ jika $f(x, y) = x^2 - xy + 2y^2$. Seterusnya, dapatkan $f_{yx}(x, y)$.

    (b) Menggunakan petua rantai dapatkan $\dfrac{dw}{dt}$ pada $t = \pi$ jika $w = x^2 y + y^2 z$, $x = t \sin t$, $y = t\, \text{kos}\, t$ dan $z = 2t$.

[13 markah]

7. Let $T(x,y) = \sqrt{5x-4y}$. Find

    (a) the gradient of $T(x,y)$.

    (b) the rate of change of $T(x,y)$ at the point (4,1) in the direction of $\theta = -\dfrac{\pi}{6}$.

    [8 marks]

8. Find the maximum value of $f(x,y) = 2x^2 + 3y^2 - 4x - 5$ on the disk $x^2 + y^2 \le 16$.

    [10 marks]

9. Evaluate the integrals.

    (a) $\displaystyle\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \left(x^3 + xy^2\right) dy\,dx$

    (b) $\displaystyle\iint_D y\, dA$, where D is the region in the first quadrant bounded by $xy = 1$, $y = x$ and $y = 2$.

    [13 marks]

10. (a) Determine whether the differential equation, $y' + \dfrac{y}{x} = 2y^2$ is linear or Bernoulli equation. Hence, find the particular solution when $y(1) = 1$.

    (b) A sample of an isotope initially weighs 90 mg. After 100 days 60 mg remain. After how many days will the sample be down to 20 mg in mass?

    [12 marks]

7. Biar $T(x,y) = \sqrt{5x-4y}$. Dapatkan

   (a)   gradian bagi $T(x,y)$.

   (b)   kadar perubahan $T(x,y)$ pada titik (4,1) di arah $\theta = -\pi/6$

                                                              [8 markah]

8. Dapatkan nilai maksimum $f(x,y) = 2x^2 + 3y^2 - 4x - 5$ di cakera $x^2 + y^2 \leq 16$.

                                                              [10 markah]

9. Nilaikan kamiran.

   (a) $\displaystyle \int_{0}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^3 + xy^2)\, dy\, dx$

   (b) $\displaystyle \iint_{D} y\, dA$, D merupakan rantau di sukuan pertama yang dibatasi oleh

           $xy = 1$, $y = x$ dan $y = 2$.

                                                               [13 markah]

10. (a)   Tentukan sama ada persamaan pembezaan , $y' + \dfrac{y}{x} = 2y^2$ adalah persamaan linear atau Bernoulli. Seterusnya, dapatkan penyelesaian khusus bila $y(1) = 1$.

    (b)   Suatu sampel isotop mempunyai berat awal 90 mg. Selepas 100 hari, bakinya 60 mg. Selepas berapa hari berat sampel akan berkurang kepada 20 mg?

                                                               [12 markah]

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