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UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang  
Sidang Akademik 2008/2009

Jun 2009

**MAA 101 – Calculus for Science Students I**  
***[Kalkulus untuk Pelajar Sains I]***

Duration : 3 hours  
*[Masa: 3 jam]*

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Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions** : Answer **all eight** [8] questions.

**Arahan** : Jawab **semua lapan** [8] soalan.]

1. Solve

(a)  $|3x-2| = |5x+1|$

(b)  $\frac{2x-5}{x-2} < 1$

(c) Find the domain of  $f(x) = \frac{\sqrt{-x+1}}{x+3}$

[11 marks]

2. (a) Given that  $f(x) = \frac{x+1}{x-1}$

(i) Show that  $f$  is one to one function

(ii) Find the inverse function of  $f$ . Hence, state the domain of  $f^{-1}$ .

(b) Let  $f(x) = x^2 + 5x - 9$ . For what value of  $x$  it is true that  $f(2x) = f(3x)$ .

[12 marks]

3. Evaluate the limit. The L'Hospital rule can be applied whenever applicable.

(a)  $\lim_{x \rightarrow 1.5^-} \frac{2x^2 - 3x}{|2x - 3|}$

(b)  $\lim_{x \rightarrow 3^+} \frac{x^2 - 4x + 3}{x^2 - 6x + 9}$

(c)  $\lim_{x \rightarrow 0^+} (\cos x)^{1/x}$

[12 marks]

1. Selesaikan

(a)  $|3x-2| = |5x+1|$

(b)  $\frac{2x-5}{x-2} < 1$

(c) Cari domain bagi  $f(x) = \frac{\sqrt{-x+1}}{x+3}$

[11 markah]

2. (a) Diberi bahawa  $f(x) = \frac{x+1}{x-1}$

(i) Tunjukkan bahawa  $f$  fungsi satu ke satu.

(ii) Cari fungsi songsangan  $f$ . Seterusnya nyatakan domain bagi  $f^{-1}$ .

(b) Diberi bahawa  $f(x) = x^2 + 5x - 9$ . Apakah nilai  $x$  supaya  $f(2x) = g(3x)$ .

[12 markah]

3. Nilaikan had berikut. Hukum L'Hospital boleh digunakan ditempat yang sesuai.

(a)  $\text{had}_{x \rightarrow 1.5^-} \frac{2x^2 - 3x}{|2x - 3|}$

(b)  $\text{had}_{x \rightarrow 3^+} \frac{x^2 - 4x + 3}{x^2 - 6x + 9}$

(c)  $\text{had}_{x \rightarrow 0^+} (\cos x)^{1/x}$

[12 markah]

4. Consider the function  $g(x) = \begin{cases} a+bx, & x > 2 \\ 3 & x = 2 \\ b-ax^2, & x < 2 \end{cases}$

- (a) Determine the values of constants  $a$  and  $b$  so that  $\lim_{x \rightarrow 2} g(x)$  exists.  
 (b) Show that if  $a = 3$  and  $b = 1$  then  $g(x)$  does not continuous at  $x = 2$ .

[10 marks]

5. Find the derivative of functions.

- (i)  $y = (\sin x)^{x^3}$   
 (ii)  $y = \tan(x^3) + e^{-x^2}$   
 (iii)  $f(x) = \frac{1+3x^2}{x + \cos(2x)}$

[12 marks]

6. (a) Two curves are said to be *orthogonal* at a point of intersection if they have perpendicular tangent lines at that point. Prove that the curves  $2x^2 + y^2 = 24$  and  $y^2 = 8x$  are orthogonal at the point  $(2, 4)$  of intersection.

(b) Given a graph of a function satisfied the following properties:

$$f(0) = 3; \quad f(2) = 1; \quad f(3) = 0 = f(8); \quad f(5) = -2$$

$$f'(2) = 0 = f'(5), \quad f'(8) \text{ does not exist}$$

$$f'(x) < 0 \text{ on } (-\infty, 3), (3, 5) \text{ and } (8, \infty); \quad f'(x) > 0 \text{ on } (5, 8)$$

$$f''(x) < 0 \text{ on } (2, 3); \quad f''(x) > 0 \text{ on } (-\infty, 2), (3, 8) \text{ and } (8, \infty)$$

State

- (i) the interval on which  $f$  is increasing or decreasing.  
 (ii) the local maximum and minimum numbers if any.  
 (iii) the interval of concavity and the inflection points if exist.

Sketch the graph of  $f$  with the above properties.

[12 marks]

...5/-

4. Diberi fungsi  $g(x) = \begin{cases} a+bx, & x > 2 \\ 3 & x = 2 \\ b-ax^2, & x < 2 \end{cases}$

(a) Tentukan nilai pemalar  $a$  dan  $b$  supaya  $\lim_{x \rightarrow 2} g(x)$  wujud.

(b) Tunjukkan jika  $a = 3$  dan  $b = 1$  maka  $g(x)$  tidak selanjar pada  $x = 2$ .

[10 markah]

5. Cari pembezaan fungsi berikut.

(i)  $y = (\sin x)^{x^3}$

(ii)  $y = \tan(x^3) + e^{-x^2}$

(iii)  $f(x) = \frac{1+3x^2}{x + \cos(2x)}$

[12 markah]

6. (a) Dua lengkung dikatakan orthogonal pada suatu titik persilangan jika garis tangent kedua-dua lengkung pada titik itu berserenjang. Buktikan bahawa garis-garis lengkung  $2x^2 + y^2 = 24$  and  $y^2 = 8x$  orthogonal pada titik persilangan  $(2, 4)$ .

(b) Diberi graf suatu fungsi memenuhi ciri-ciri berikut.

$$f(0) = 3; \quad f(2) = 1; \quad f(3) = 0 = f(8); \quad f(5) = -2$$

$$f'(2) = 0 = f'(5), \quad f'(8) \text{ does not exist}$$

$$f'(x) < 0 \text{ on } (-\infty, 3), (3, 5) \text{ and } (8, \infty); \quad f'(x) > 0 \text{ on } (5, 8)$$

$$f''(x) < 0 \text{ on } (2, 3); \quad f''(x) > 0 \text{ on } (-\infty, 2), (3, 8) \text{ and } (8, \infty)$$

Nyatakan

(i) selang bagi  $f$  menokok atau menyusut.

(ii) Nilai  $x$  bilamana  $f$  maximum and minimum tempatan, jika ada.

(iii) selang kecekungan  $f$  dan titik lengkung balas jika wujud.

Seterusnya lakarkan graf tersebut.

[12 markah]

...6/-

7. (a) Evaluate the integral.

(i)  $\int_0^1 \frac{e^x}{e^x + 1} dx$

(ii)  $\int_{-1}^1 |x^3| dx$

(iii)  $\int \frac{1}{x(x-1)} dx$

(b) Given that  $F(x) = \int_1^{2x} \frac{e^t}{t+1} dt$ . Find  $F'(4)$

[16 marks]

8. Sketch the region **R** bounded by  $y = \sqrt{x}$ ,  $y = 6 - x$  and  $y = 0$ . Hence, find

(a) the area of the region **R**

(b) the volume of the solid obtained by rotating the region **R** about the  $y$ -axis

[15 marks]

7. (a) *Nilaikan kamiran berikut.*

(i)  $\int_0^1 \frac{e^x}{e^x + 1} dx$

(ii)  $\int_{-1}^1 |x^3| dx$

(iii)  $\int \frac{1}{x(x-1)} dx$

- (b) *Diberi bahawa  $F(x) = \int_1^{2x} \frac{e^t}{t+1} dt$ . Cari  $F'(4)$*

[16 markah]

8. *Lakarkan kawasan  $R$  yang dibatasi oleh  $y = \sqrt{x}$ ,  $y = 6 - x$  and  $y = 0$ .  
Seterusnya cari*

- (a) *luas kawasan tersebut*

- (b) *Isipadu bongkah kisanan yang terhasil apabila kawasan  $R$  dikisarkan terhadap paksi-y.*

[15 markah]