UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

MAA 101 – Calculus for Science Students I
[Kalkulus untuk Pelajar Sains I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

[Arahan: *Jawab **semua sepuluh** [10] soalan.]*

1. Solve

(a) $|x-2| = 3x+1$

(b) $\dfrac{x+1}{x+2} - \dfrac{x+5}{x+4} < 0$

[8 marks]

2. (a) Given that $f(x) = x^2 - 5; \quad x \geq 0$

(i) State the domain and range of f.

(ii) Is f one to one function?. If yes, then find the inverse funtion of f.

(iiii) Hence sketch the graph of f and f^{-1}.

(b) Given that $f(x) = x^2 + 1$ and $g(x) = 3x + 2$, find all values of x such that $f(g(x)) = g(f(x))$.

[10 marks]

3. Evaluate the limit. The L'Hospital rule can be applied whenever applicable.

(a) $\lim\limits_{x \to 2} \dfrac{x^4 - 16}{x^2 - 4}$

(b) $\lim\limits_{x \to 2} \dfrac{\sqrt{x+2} - 2}{x-2}$

(c) $\lim\limits_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

[11 marks]

4. Given $g(x) = \begin{cases} x^2 - C, & x < 4 \\ -\sqrt{C}\,x + 20, & x \geq 4 \end{cases}$

(a) Find C that makes g is continuous on $(-\infty, \infty)$

(b) With the value C obtained in (i), does g differentiable? Explain your answer.

[10 marks]

1. *Selesaikan*

(a) $|x-2| = 3x+1$

(b) $\dfrac{x+1}{x+2} - \dfrac{x+5}{x+4} < 0$

[8 markah]

2. (a) *Diberi bahawa* $f(x) = x^2 - 5; \quad x \geq 0$

(i) *Nyatakan domain and julat f.*

(ii) *Adakah f fungsi satu ke satu?. Jika ya, cari fungsi songsangan f.*

(iiii) *Seterusnya lakarkan graf untuk f and* f^{-1}.

(b) *Diberi bahawa* $f(x) = x^2 + 1$ and $g(x) = 3x + 2$, *cari semua nilai x supaya* $f(g(x)) = g(f(x))$.

[10 markah]

3. *Nilaikan had berikut. Hukum L'Hospital boleh digunakan di tempat yang sesuai.*

(a) $\lim\limits_{x \to 2} \dfrac{x^4 - 16}{x^2 - 4}$

(b) $\lim\limits_{x \to 2} \dfrac{\sqrt{x+2} - 2}{x-2}$

(c) $\lim\limits_{x \to 0^+} (1 + \sin 4x)^{\cot x}$

[11 markah]

4. *Diberi* $g(x) = \begin{cases} x^2 - C, & x < 4 \\ -\sqrt{C}\,x + 20, & x \geq 4 \end{cases}$

(a) *Cari C supaya g selanjar pada* $(-\infty, \infty)$

(b) *Adakah g terbezakan jika menggunakan C yang diperolehi dari (i)? Jelaskan jawapan anda.*

[10 markah]

5. Find the derivative of functions.

(a) $y = \ln\sqrt{\dfrac{x^2+1}{x^3+5}}$

(b) $f(x) = \sin 3x\, e^{3x-x^2}$

(c) $h(x) = \dfrac{\tan 3x}{x^2}$

[10 marks]

6. (a) Show that the tangent line at point (a,b) on the curve whose equation is $2x^2+3xy+y^2=-2$ is horizontal if $4a+3b=0$. Find two points on the curve such that the tangent line horizontal.

(b) Let $f(x)=x^2-x$
Find all value c such that f satisfy the conclusion of Mean-Value Theorem on the interval [0,2].

[9 mark]

7. Let $f(x)=\dfrac{3x^2-8}{x^2-4}$. Find

(a) all asymptotes.
(b) the interval on which f is increasing or decreasing.
(c) the local maximum and minimum values of f if any.
(d) the interval of concavity and the inflection points if exist.
(e) Sketch the graph of f.

[13 marks]

8. Evaluate the integral.

(a) $\int_1^4 x|x-2|\,dx$

(b) $\int x\sqrt{16-x^2}$

(c) $\int\dfrac{3x^2+x+1}{x(x^2+1)}\,dx$

[11 marks]

5. *Cari pembezaan fungsi berikut.*

(a) $y = \ln\sqrt{\dfrac{x^2+1}{x^3+5}}$

(b) $f(x) = \sin 3x\, e^{3x-x^2}$

(c) $h(x) = \dfrac{\tan 3x}{x^2}$

[10 markah]

6. (a) *Tunjukkan bahawa garis tangen pada titik* (a,b) *di atas lengkungan* $2x^2+3xy+y^2=-2$ *mendatar jika* $4a+3b=0$. *Dapatkan dua titik pada lengkungan itu supaya garis tangen adalah mendatar.*

(b) *Biarkan* $f(x)=x^2-x$
Cari semua nilai c *supaya* f *memenuhi kesimpulan Teorem Nilai Min pada selangl* $[0,2]$.

[9 markah]

7. *Biarkan* $f(x)=\dfrac{3x^2-8}{x^2-4}$. *Cari*

(a) *semua asimptot.*
(b) *selang* f *menokok atau menyusut.*
(c) *nilai maximum and minimum tempatan jika ada.*
(d) *selang kecekungan* f *dan titik lengkuk balas jika wujud.*
(e) *Lakarkan graf untuk* f.

[13 markah]

8. *Nilaikan kamiran berikut.*

(a) $\int_1^4 x|x-2|\,dx$

(b) $\int x\sqrt{16-x^2}$

(c) $\int\dfrac{3x^2+x+1}{x(x^2+1)}\,dx$

[11 markah]

9. Find an equation for the tangent line to the curve $y = F(x)$ at a point $P(x,y)$ where $x = 1$ and

$$F(x) = \int_1^{x^2} \frac{t^2+1}{t-2}\, dt$$

[7 marks]

10. Sketch the region **R** bounded by x-axis , $y = 3x^2$ and $y = 4 - x^2$, $x \geq 0$.
Hence find
(a) the area of the region **R**
(b) the volume of the solid obtained by rotating the region **R** about the y-axis

[11 marks]

9. *Cari persamaan garis tangent ke lengkungan* $y = F(x)$ *di titik* $P(x,y)$ *apabila* $x = 1$ *dan*

$$F(x) = \int_1^{x^2} \frac{t^2+1}{t-2}\, dt$$

[7 markah]

10. *Lakarkan kawasan* ***R*** *yang di batasi oleh paksi-x,* $y = 3x^2$ *dan* $y = 4 - x^2$, $x \geq 0$. *Seterusnya cari*

(a) *luas kawasan tersebut*

(b) *isipadu bongkah kisaran yang terhasil apabila kawasan* ***R*** *dikisarkan terhadap paksi-y.*

[11 markah]