International Conference on Control, Instrumentation and Mechatronics Engineering (CIM'07),
Johor Bahru, Johor, Malaysia, May 28-29,2007

Simulation of Direct Model Reference Adaptive Control on a Coupled-Tank System using Nonlinear Plant Model

Muhammad Nasiruddin Mahyuddin[1], Mohd. Rizal Arshad[1] and Zaharuddin Mohamed[2]

[1]School of Electrical and Electronic, Engineering, Universiti Sains, Malaysia, Penang, Malaysia.
nasiruddin@eng.usm.my, rizal@eng.usm.my

[2]Faculty of Electrical, Universiti Teknologi Malaysia, Johor, Malaysia
zahar@fke.utm.my

Abstract

This paper presents the application of direct model reference adaptive control (DMRAC) on a nonlinear model of coupled-tank liquid level control system through simulation. The coupled-tank liquid level control system is regarded as the relevant plant to emulate the process control in petrol and chemical industries. The processing plants in these industries largely involve in controlling the liquid level and the flow rate from one reservoir to another in the presence of nonlinearity and disturbance. This requires the use of adaptive techniques such as DMRAC in the process control system. Coupled-tank which resembles the model of the chemical or mixing process plant is used to evaluate the performance of DMRAC under various conditions. A simulation is carried out using MATLAB® and Simulink® to control the modeled nonlinear coupled-tank using the adaptive control algorithm. It is also utilised to show that the controller can produce the appropriate control signals to the coupled-tank system to control the liquid level in the presence of plant nonlinearity, disturbance and measurement noise.

1. Introduction

Process in petrol and chemical industries largely involves controlling the liquid level and the flow rate from one reservoir to another in the presence of nonlinearity and disturbance. This requires the use of adaptive techniques such as DMRAC in the process control system as fixed controllers are often not capable of controlling a process whose system parameters are varying and disturbances acting upon the system during the operation. The simulated nonlinear plant model is based on the principle of mass balance, i.e. the rate of change of liquid volume in each tank equals to the net flow of liquid into the tank. This nonlinear plant model is used instead of linearised perturbation model of coupled tank as to evaluate the controller performance in the presence of nonlinearity.

The DMRAC algorithm proposed by Sobel, Kaufman, and Barkana [1] provides an attractive adaptive control approach. Its control structure adopts the use of linear combination of feedforward model states, command inputs and the error feedback between the plant outputs and the model reference outputs. One of the properties that make the algorithm relatively easy to be implemented is that it only requires the plant and reference model outputs and reference model states to be available for measurement. Other related works such as Landau [2], termed the approach as an adaptive model following control. Another attractive characteristic of this algorithm that provides design convenience is that the order of the reference model can be made lower than that of the order of the plant to be controlled. This complements its ability of not needing the identification of process parameter.

The performance of PID controller is also presented as to show a brief comparison. A series of tracking performance test, disturbance rejection and plant parameter variations are then introduced in the simulation.

2. Nonlinear model of coupled-tank

The mathematical model of the coupled-tank liquid level control system in the simulation is formulated based on a real laboratory-scale coupled-tank system, developed by Augmented Innovation Ltd. The plant is currently used as an experimental apparatus in the Faculty of Electrical, UTM, Malaysia. The coupled-tank consists of two tower-type tanks with an internal baffle in between, as shown in Figure 1. The schematic of the coupled-tank is similar to the attempt made by Lian *et al* [4] in modeling the Kent Ridge Instrument's coupled-tank. The height of the baffle can be adjusted to vary the leakage closure between the two tanks. Both tanks are equipped with an outlet whose opening can be varied by means of adjustable clamps. These vary the discharge coefficient of the liquid flowing out of the respective tank. The water level of the liquid in each tank is measured by a capacitive probe sensor that converts the capacitive measurement to an electrical signal in voltage unit. The control objective is to control the liquid level in Tank 2 by manipulating the flow rate of the liquid into Tank 1 by means of pump (Pump 1) voltage.

The simulated plant dynamics are based on the principle of mass balance which states that the rate of change of liquid volume in each tank equals the net of liquid flows into the tank.

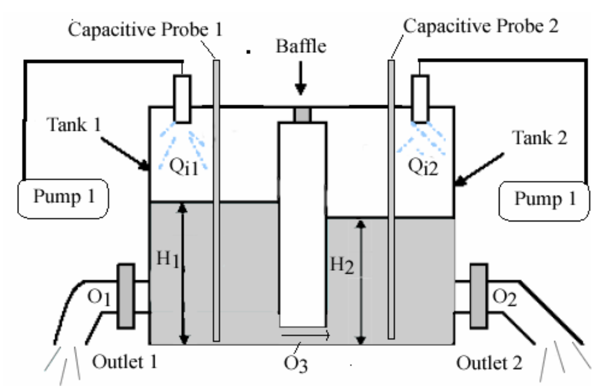

Figure 1. A schematic of the coupled-tank level-control system

The liquid used in the plant is assumed to be steady, non-viscous, incompressible type of liquid which leads to the use of Bernoulli's equation to obtain a set of nonlinear state equations [4]:

$$A_1 \frac{dH_1}{dt} = Q_{i1} - \alpha_1 \sqrt{H_1} - \alpha_3 \sqrt{H_1 - H_2}, \qquad (1)$$

$$A_2 \frac{dH_2}{dt} = Q_{i2} - \alpha_2 \sqrt{H_2} + \alpha_3 \sqrt{H_1 - H_2} \qquad (2)$$

Where H_1 and H_2 are the height of fluid in Tank 1 and Tank 2 respectively. The volumetric flow rates of Pump 1 and Pump 2 are represented by Q_{i1} and Q_{i2} respectively. Each outlet drain can be modeled as a simple orifice. The parameters α_1, α_2 and α_3 are the proportionality constants of the corresponding $\sqrt{H_1}, \sqrt{H_2}$ and $\sqrt{H_1 - H_2}$ terms which depends on the coefficients of discharge, the cross sectional area of each orifice (O_1, O_2 and O_3) and the gravitational constant.

Table 1 shows the parameters of the coupled-tank system used in the plant modeling and simulation.

Table 1. Plant's parameters

Description	Value		
Cross sectional area of each tank	32 cm^2		
Proportionality constant, α_i subscript *i* denotes which tank it refers. α_3 corresponds to the opening between the tanks.	α_1	α_2	α_3
	($cm^{3/2}$ / sec)		
	14.30	14.30	20.00
Sensor gain	0.157 *V/cm*		
Pump gain	13.571 $cm^3 / s / volt$		
Pump motor's time constant	1 sec		

3. Formulation of DMRAC algorithm

The linear time-invariant model reference adaptive control problem is considered for the nonlinear plant

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p u_p(t) \\ y_p(t) &= C_p x_p(t) \end{aligned} \qquad (2)$$

where $x_p(t)$ is the state vector, $u_p(t)$ is the control vector, $y_p(t)$ is the plant output vector, and A_p, B_p are matrices with appropriate dimensions. The range of plant parameters is assumed to be known and bounded with

$$\begin{aligned} &\underline{a}_{ij} \le a_p(i,j) \le \bar{a}_{ij}, \quad i,j = 1,\dots,n \\ &\underline{b}_{ij} \le b_p(i,j) \le \bar{b}_{ij}, \quad i = 1,\dots,n \\ &\qquad\qquad\qquad\qquad\quad j = 1,\dots,m \end{aligned} \tag{3}$$

The objective is to find, without explicit knowledge of A_p and B_p, the control $u_p(t)$ such that the plant output vector $y_p(t)$ follows the reference model

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= C_m x_m(t) \end{aligned} \tag{4}$$

The model incorporates the desired behavior of the plant. The order of the actual plant is allowed to be higher than the order of the reference model which simplifies the characterisation of the reference model. The ideal control law $u_p^*(t)$, generating perfect output tracking and the ideal state trajectories $x_p^*(t)$ is assumed to be a linear combination of the model states and model input:

$$\begin{bmatrix} x_p^* \\ u_p^* \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} x_m(t) \\ u_m \end{bmatrix} \tag{5}$$

where u_m is presently set to a constant.

The S_{ij} submatrices satisfy the following conditions

$$\begin{aligned} S_{11}A_m &= A_p S_{11} + B_p S_{21} \\ S_{11}B_m &= A_p S_{12} + B_p S_{22} \\ C_m &= C_p S_{11}, \qquad 0 = C_p S_{12} \end{aligned} \tag{6}$$

When perfect output tracking occurs, $x_p(t) = x_p^*(t)$, and the ideal control is given by

$$u_p^*(t) = S_{21}x_m(t) + S_{22}u_m \tag{7}$$

If when perfect output tracking does not occur, $y_p(t) \ne y_m(t)$, asymptotic tracking is achievable provided stabilizing output feedback is included in the control law

$$\begin{aligned} u_p(t) = S_{21}x_m(t) + S_{22}u_m(t) + K_e(y_m(t) \\ - y_p(t)) \end{aligned} \tag{8}$$

Therefore, the adaptive control law based on this command generator tracker (CGT) approach is given as [1]

$$\begin{aligned} u_p(t) = K_e(t)\lfloor y_m(t) - y_p(t) \rfloor + K_x(t)x_m(t) \\ + K_u(t)u_m(t) \end{aligned} \tag{9}$$

where $K_e(t)$, $K_x(t)$, and $K_u(t)$ are adaptive gains and concatenated into matrix *K(t)* as follows

$$K(t) = \begin{bmatrix} K_e(t) & K_x(t) & K_u(t) \end{bmatrix} \tag{10}$$

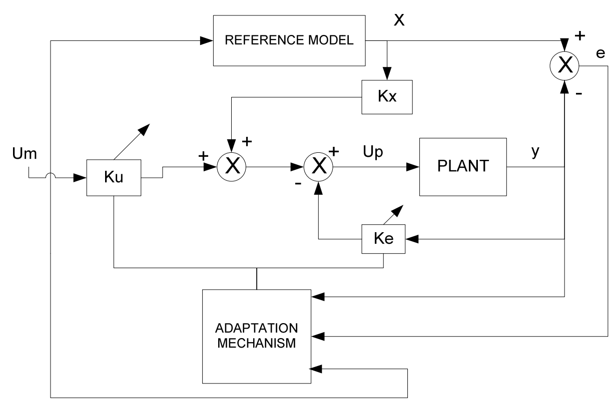

Figure 2. DMRAC structure

Figure 2 depicts the general structure of the DMRAC control scheme in a form of block diagram.

Defining the vector *r(t)* as

$$r(t) = \begin{bmatrix} y_m(t) - y_p(t) \\ x_m(t) \\ u_m(t) \end{bmatrix} \tag{11}$$

the control $u_p(t)$ is written in a compact form as follows

$$u_p(t) = K(t)r(t) \tag{12}$$

The adaptive gains are obtained as combination of an integral gain and a proportional gain as shown in [1].

$$K(t) = K_p(t) + K_i(t)$$
$$K_p(t) = [y_m(t) - y_p(t)]r^T(t)T_p, \quad T_p \geq 0$$
$$\dot{K}_i(t) = [y_m(t) - y_p(t)]r^T(t)T_i - \sigma K_i(t), \quad T_i > 0 \tag{13}$$

The sufficiency conditions for asymptotic tracking are

1. There exists a solution to the CGT problem [1].
2. The plant is ASPR; that is there exists a positive definite constant gain matrix K_e, not needed for implementation, such that the closed loop transfer function

$$G(s) = [I + G_p(s)K_e]^{-1}G_p(s) \tag{14}$$

is strictly positive real (SPR) [3].

The term σ in the expression of $\dot{K}_i(t)$ in equation (13) introduced by Ioannou and Kokotovic [7] helps to avoid the divergence of the integral gains in the presence of disturbances. $K_i(t)$ is the perfect integrator and may reach unnecessarily large values when the perfect following ($e_y(t) = y_m(t) - y_p(t) = 0$) is not possible during steady increase. With the σ-term, $K_i(t)$ obtained from the first-order filtering of $e_y(t)r^T(t)T$ cannot diverge. This aids to system stability.

4. Adaptation Mechanism

The adaptation weight for the proportional adaptation, $K_p(t)$ and integral adaptation, $\dot{K}_I(t)$ (see equation 13) are determined using the method outlined by Howard [1]. It is to base the initial weight selection on the CGT solution for the nominal process parameters ($A_p^{\circ}, B_p^{\circ}, C_p^{\circ}$). A linearized perturbation model of the second order SISO plant is used to obtain the initial adaptive weight.

4.1. Adaptation weights for nominal response

Table 2 shows the adaptation weights for nominal response system.

Table 2. Adaptation weights

Reference Model state variables and model following output errors	Adaptation weights (T for $K_p(t), \overline{T}$ for $K_i(t)$)	
	$K_p(t)$	$K_i(t), T = \overline{T}/\tau$
X_{m1}	2	0.06
X_{m2}	2	0.06
U_m	2	0.06
e	20	0.06

τ = settling time of the reference model response

5. Reference model

Different types of reference model can be formulated with each governs different response specifications. The transient response of reference models depends on the plant to be controlled which is in this case, a coupled-tank. Therefore, the selection of reference model is usually preceded by a careful study on the plant's boundary and physical limits of which it can operate.

In this simulation, a reference model is formulated according to the nominal operation of the coupled-tank equipment as outlined in the equipment's manual. From the nominal reference model, slow and fast response can be also gauged and formulated within the limits. In this study, the order of the formulated reference model is of second order.

Some considerations are made when setting up a reference model in the adaptation mechanism. For each component of output vector, the initial position state of the model transfer function would be set to the initial value of the output itself. If a second order model is used, the velocity state component should be set to zero. The second consideration is that all initial plant and model states would be set to zero if the plant starts from rest. The initial conditions of the reference model would have to be set such that the initial plant reference model output vectors have the same value.

5.1. Nominal response reference model

The reference model with nominal specification is formulated to have the following performance specification:

Rise Time, T_r :	10 seconds
Peak Time, T_p :	23 seconds
Settling Time, T_s :	30 seconds
Percentage Overshoot, OS%:	5%

After the setting up of reference model, the state space representation of the reference model with nominal specification is decomposed as follows

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -0.2667 & -0.0363 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} q_1 \tag{15}$$

$$y = [0 \quad 0.0361] \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

The parameter q_1 refers to the volumetric flow rate of the water pumped into the first tank.The state space of the reference model as shown in (13) need to be modified to accommodate for the controlled variable regulation defined as such

$$y = H_2 + \alpha \dot{H}_2 \tag{16}$$

This method is regarded as the regulation of an output by means of weighted combination of H_2 (the controlled variable) and its derivative.

5.2. Slow response reference model

As it has been mentioned before, the system to be controlled by DMRAC can be tested under different dynamic characteristic by defining the governed reference model. Slow or fast response can be simply gauged from the nominal reference model, paying careful attention given to the physical limit of the plant.

A simple notion to formulate the slower response reference model is that it will have a longer settling time of 50 seconds and rise time of 27 seconds. The percentage of overshoot will be at 0.5%. The following is the state space representation for such response specification

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -0.16 & -0.0087 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} q_1 \tag{17}$$

$$y = [0 \quad 0.0087] \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

5.3. Fast response reference model

The faster response reference model exposes the system response to a slightly demanding response specification. The settling time is set to 15 seconds whilst the rise time is set to 7 seconds with an allowable overshoot of 1%.

The corresponding state space representation for the fast response specification is as follow

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -0.533 & 1 \\ 0.1042 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} q_1 \tag{18}$$

$$y = [0 \quad 0.1042] \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

6. Simulation Results

Responses of the system with nominal, slow and fast reference model are observed.

As depicted in Figure 3, DMRAC with nominal reference model achieves consistent performance and maintains the desired transient response characteristic throughout all operating points.

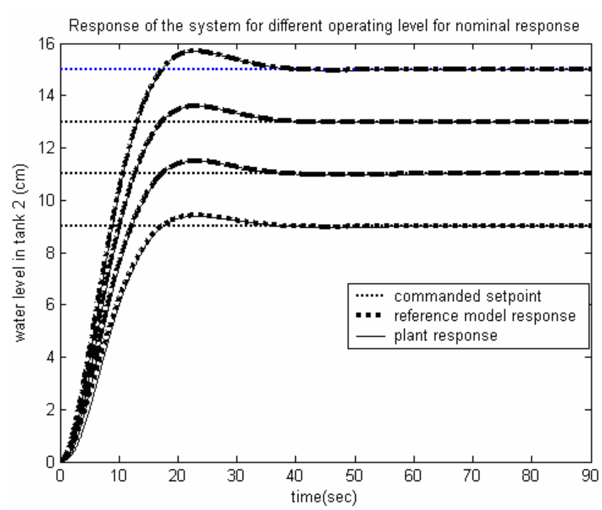

Figure 3. Response of the system with nominal reference model

This result also applies to the case with the slow response reference model as shown in Figure 4.

Figure 5 illustrates controller's difficulty in achieving the set point (15 cm of water level) with desired fast transient response. However, the controller still achieves good tracking performance for all water levels lower than 15 cm.

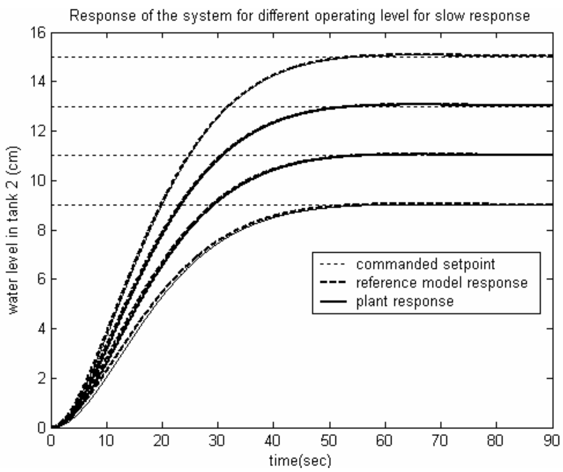

Figure 4. Response of the system with slow response reference model

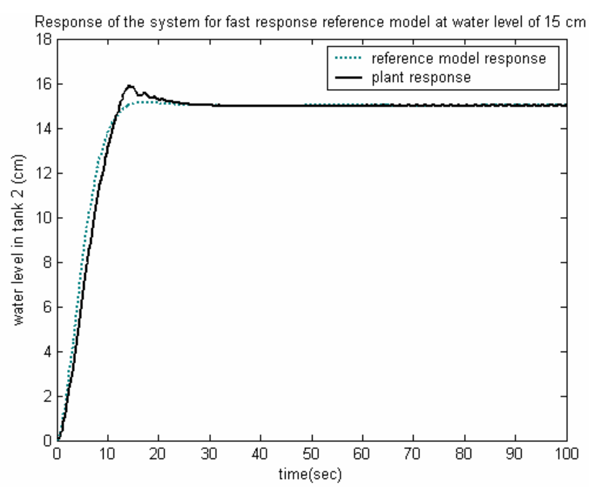

Figure 5. Response of the system with fast response reference model at water level of 15cm

6.1. Set point tracking

The set point tracking test consists of changing the set point consecutively during the operation. Figure 6 shows the performance of DMRAC and PID in the set point tracking test. The set point change is done at every 100 seconds by a magnitude of 1 cm height in water level for the first two changes and 2 cm height for the last two changes.

From Figure 6, it can be seen that DMRAC tracks the set point changes in water level accurately as compared to the fixed controller, PID. Besides from the excellent tracking, the DMRAC sustains the characteristic of transient response specified by the nominal reference model for every set point changes. In contrast, the PID controller exhibits inconsistent transient response performance throughout the set point changes. The PID controller requires its parameters to be tuned for every operating range because the plant characteristic is nonlinear whereas DMRAC performance can be determined once by the formulation of reference model.

6.2. Disturbance rejection

The disturbance rejection test is performed in the simulation by introducing a load disturbance. In the real plant, the load disturbance is in a form of water inflow into Tank 2. In the simulation, the volumetric flow rate of the water inflow is set to 40 $cm^3/\sec$. In Figure 7, the DMRAC is able to recover to the set point much quicker than that of PID upon the introduction of load disturbance.

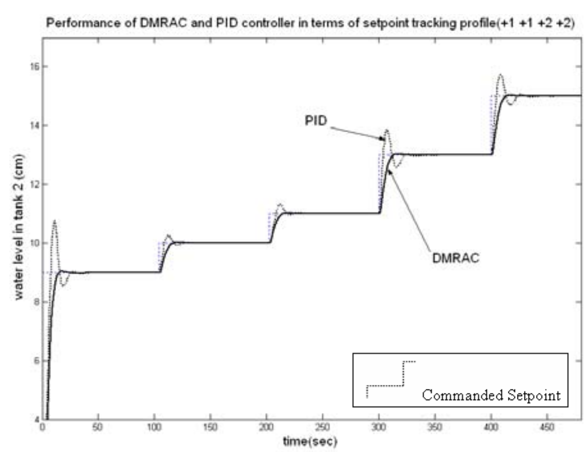

Figure 6. Set point tracking performance between DMRAC and PID

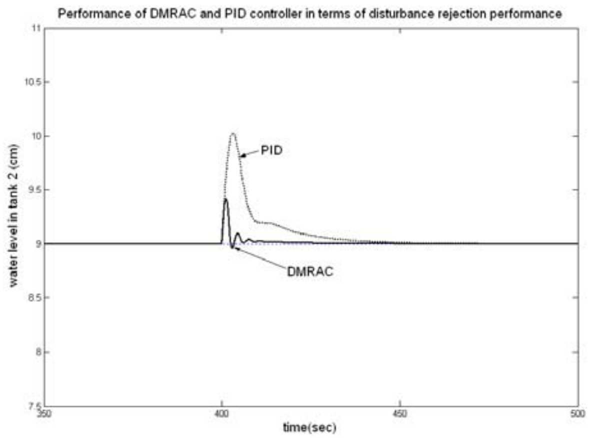

Figure 7. Disturbance rejection performance

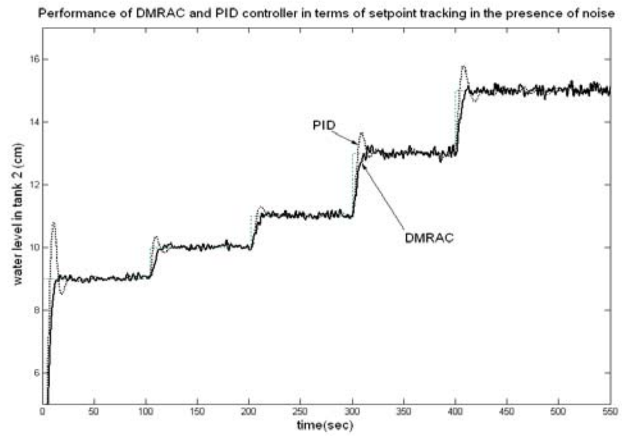

Figure 8. Tracking in the presence of modeled measurement noise

Table 3. Performance index comparison in ITAE for set point tracking in the presence of noise

Controller	Performance Index, ITAE
DMRAC	43570
PID	89480

6.3. Robustness in the presence of measurement noise

The measurement noise is modeled as band-limited white noise and it is installed at the controlled variable feedback for both controllers in the simulation. The result of the simulation of tracking performance in the presence of measurement noise is shown in Figure 8. Although the effect of measurement noise to the DMRAC is relatively significant, the tracking performance is still satisfactory. This can be proven by computing Integral Time Absolute Error (ITAE) over a time range of 9 minutes like shown in Table 3. DMRAC gives a much lower ITAE as compared to PID controller.

6.4. Plant parameter variation

The plant parameter is changed by closing the outlet at Tank 1 during the steady state operation. Figure 9 shows that DMRAC is very robust to the plant parameter variation as compared to PID. This verifies the fact that the DMRAC algorithm does not require explicit identification of the plant model.

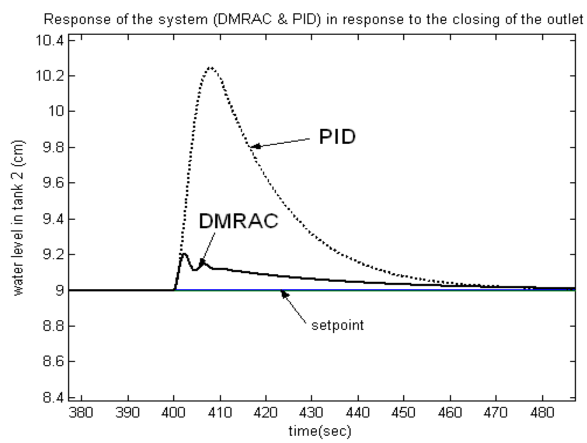

Figure 9. Response with plant parameter varied

7. Conclusion

It has been shown that DMRAC can cope with the coupled-tank nonlinear characteristic at all operating points (water level or height in Tank 2). There exists a design convenience of defining different sets of reference model to represent the desired transient response with reference model order much lower than the plant's order. This complements the algorithm's special characteristic of not requiring explicit identification of the process or plant parameters. The DMRAC controller is able to sustain the desired transient response throughout the set point changes both with and without the measurement noise conditions. The DMRAC is also robust to the load disturbance and sudden changes in the plant characteristic.

Conversely, PID controller exhibits inconsistent transient performance at each set point change. This shows that fixed controllers are not able to sustain the predefined transient performance at all operating points in the presence of plant nonlinearities. Although PID controller's parameters can be tuned by Ziegler-Nichols on-line tuning method, the tuning is required for each operating point which is inconvenient. However, according to Rosbi [6], although Ziegler-Nichols correlation is the best among of other tuning method, set point responses are very oscillatory which may lead to instability. This problem can be solved if auto-tuning PID controller is used in the future.

The DMRAC's susceptibility to measurement noise can be solved by prefiltering at the controlled variable feedback. In the real plant, this can be accomplished by means of adequate filtering of the noise originated from the capacitive probe.

The future improvement to the DMRAC algorithm can be made by having feedforward compensator design [3] and can be further extended to accommodate MIMO type of settings in the coupled-tank plant.

7. References

[1] H. Kaufman, I. Barkana, and K. Sobel, Direct Adaptive Control Algorithms - Theory and Applications. New York, NY: Springer-verlag, 1994.

[2] I. D. Landau, Adaptive Control – The Model Reference Approach. New York: Marcel Decker, 1979.

[3] S. Ozcelik and H. Kaufman, "Design of MIMO robust direct model reference adaptive controllers," in 36th IEEE Conference on Decision & Control, (San Diego, CA), pp. 1890-1895, 1997.

[4] S.T. Lian, K. Marzuki and Y. Rubiyah, "Tuning of a neuro-fuzzy controller by genetic algorithms with an application to a coupled-tank liquid-level control system.," in Engineering Application of Artificial Intelligence, vol. II, pp. 517-529, 1998.

[5] Augmented Innovations Ltd., Coupled-Tank Liquid Level Computer-Controlled Laboratory Teaching Package – Experimental and Operational (service) Manual. Kuala Lumpur: CAIRO, 2000.

[6] Rosbi Mamat, "Comparative performance of some tuning methods for industrial PID controllers," in UTM's Elektrika, vol. V, pp. 38-45, 2002.

[7] P. Ioannou and P. Kokotovic, *Adaptive Systems with Reduced Models.* Berlin: Springer-Verlag, 1983.