UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 1998/99

Ogos/September 1998

MSG 423 - Kaedah Unsur Terhingga

Masa: [3 jam]

ARAHAN KEPADA CALON:

Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA soalan di dalam DUA halaman dan EMPAT halaman Lampiran yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab **SEMUA** soalan.

1.(a) Cari penyelesaian hampiran bagi masalah berikut:

$$y'' - y + 20 = 0, \quad 0 < x < 3$$
$$y(0) = 30, \quad y(3) = 40$$

dengan menggunakan kaedah unsur terhingga dengan tiga unsur linear.

(40/100)

(b) Cari

(i) $\int_\Omega N_i N_j \, dA$ (ii) $\int_\Omega \frac{\partial N_i}{\partial x} \cdot \frac{\partial N_j}{\partial x} \, dA$ (iii) $\int_\Gamma N_i N_j \, d\Gamma$

di mana Ω ialah rantau di dalam segiempat $i(1, 1)$, $j(9, 1)$, $k(9, 5)$, $m(1, 5)$, Γ ialah sempadan ij dan N_i, N_j ialah fungsi bentuk bilinear.

(30/100)

(c) Katakan N_i, N_j, N_k ialah fungsi bentuk linear untuk segitiga $i(0, 0)$, $j(5, 0)$, $k(0, 6)$ dan $\phi = 2N_i + 3N_j + 4N_k$. Cari $\phi(1, 1)$, $\frac{\partial \phi}{\partial x}(1, 1)$ dan $\frac{\partial \phi}{\partial y}(1, 1)$.

(30/100)

2.(a) Pertimbangkan masalah haba berikut:

$$\frac{\partial^2 \phi}{\partial x^2} = 6\frac{\partial \phi}{\partial t}, \quad 0 < x < 3, \quad t > 0$$

$$\phi(0, t) = 100, \quad \phi(3, t) = 100, \quad t > 0$$

$$\phi(x, 0) = 20(x^2 - 3x) + 100, \quad 0 \le x \le 3$$

Dengan menggunakan tiga unsur linear dengan perumusan konsisten, $\theta = \frac{1}{2}$ dan $\Delta t = 1$, dapatkan ϕ untuk dua langkah masa.

(40/100)

...2/-

(b) Selesaikan masalah haba berikut:

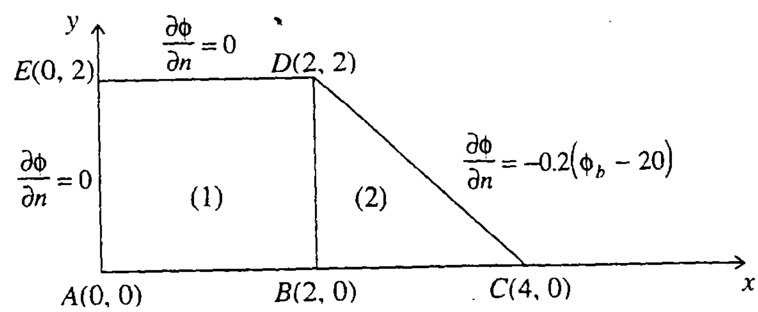
$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0, \quad \text{di dalam } \Omega$$

$$\phi(x,\ 0)=20x+10, \quad 0\le x\le 4$$

$$\frac{\partial\phi}{\partial n}=0 \quad \text{pada sisi } DE \text{ dan } EA$$

$$\frac{\partial\phi}{\partial n}=-0.2(\phi_b-20), \quad \text{pada sisi } CD$$

di mana Ω ialah rantau $ABCDE$ berikut:



(60/100)

3.(a) Dengan menggunakan unsur kuadratik dapatkan matriks unsur yang digunakan untuk menyelesaikan masalah berikut:

$$Dy''+Q=0, \quad 0<x<\ell$$
$$y(0)=A, \quad y(\ell)=B$$

di mana D, Q, A, B dan ℓ adalah pemalar.

(40/100)

(b) Pertimbangkan penyelesaian persamaan haba berikut:

$$\frac{\partial^2\phi}{\partial x^2}=\frac{\partial\phi}{\partial t}, \quad 0<x<\ell, \quad t>0$$

$$\phi(0,\ t)=A, \quad \phi(\ell,\ t)=B, \quad t>0$$

$$\phi(x,\ 0)=f(x)$$

melalui kaedah unsur terhingga dengan skema beza ke depan dan perumusan bergumpal. Dapatkan syarat atas Δt supaya skema pengiraan berangka ini adalah stabil.

(40/100)

(c) Dengan menggunakan kuadratur Gauss dengan tiga titik pensampelan, nilaikan

$$\int_1^5 \frac{dx}{1+x^4}$$

(20/100)

-oooOooo-

LAMPIRAN (MSG 423)

Unsur Linear 1-D

$$\left[k^{(e)}\right] = \frac{D}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Unsur Segitiga Linear

$$N_i = [a_i + b_i x + c_i y]/(2A), \quad N_j = [a_j + b_j x + c_j y]/(2A)$$
$$N_k = [a_k + b_k x + c_k y]/(2A)$$

dengan

$$2A = \begin{vmatrix} 1 & X_i & Y_i \\ 1 & X_j & Y_j \\ 1 & X_k & Y_k \end{vmatrix}$$

dan

$$a_i = X_j Y_k - X_k Y_j, \quad b_i = Y_j - Y_k, \quad c_i = X_k - X_j$$
$$a_j = X_k Y_i - X_i Y_k, \quad b_j = Y_k - Y_i, \quad c_j = X_i - X_k$$
$$a_k = X_i Y_j - X_j Y_i, \quad b_k = Y_i - Y_j, \quad c_k = X_j - X_i$$

$$\left[k_D^{(e)}\right] = \frac{D_x}{4A}\begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_i b_j & b_j^2 & b_j b_k \\ b_i b_k & b_j b_k & b_k^2 \end{bmatrix} + \frac{D_y}{4A}\begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_i c_j & c_j^2 & c_j c_k \\ c_i c_k & c_j c_k & c_k^2 \end{bmatrix}$$

$$\left[k_G^{(e)}\right] = \frac{GA}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QA}{3}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\left[k_M^{(e)}\right] = \frac{ML_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \text{dll.}$$

$$\int_A L_1^a L_2^b L_3^c \, dA = \frac{a!\,b!\,c!}{(a+b+c+2)!} 2A$$

Unsur Segiempat Tepat Bilinear

$$N_i = \frac{1}{4}(1-\xi)(1-\eta), \qquad N_j = \frac{1}{4}(1+\xi)(1-\eta)$$
$$N_k = \frac{1}{4}(1+\xi)(1+\eta), \qquad N_m = \frac{1}{4}(1-\xi)(1+\eta)$$

$$N_i = \left(1 - \frac{s}{2b}\right)\left(1 - \frac{t}{2a}\right), \qquad N_j = \frac{s}{2b}\left(1 - \frac{t}{2a}\right)$$

$$N_k = \frac{st}{4ab}, \qquad N_m = \frac{t}{2a}\left(1 - \frac{s}{2b}\right)$$

$$\left[k_D^{(e)}\right] = \frac{D_x a}{6b}\begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{D_y b}{6a}\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

$$\left[k_G^{(e)}\right] = \frac{GA}{36}\begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}; \qquad \left\{f^{(e)}\right\} = \frac{QA}{4}\begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\left[k_M^{(e)}\right] = \frac{ML_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \text{dll.}$$

Unsur Kuadratik 1-D

$$N_1 = \frac{1}{2}\xi(\xi-1), \qquad N_2 = -(\xi+1)(\xi-1), \qquad N_3 = \frac{1}{2}\xi(\xi+1)$$

Unsur Segitiga Kuadratik 6-Nod

$$N_1 = L_1(2L_1-1), \qquad N_2 = 4L_1L_2,$$

$$N_3 = L_2(2L_2-1), \qquad N_4 = 4L_2(1-L_1-L_2)$$

$$N_5 = 1 - 3(L_1+L_2) + 2(L_1+L_2)^2, \qquad N_6 = 4L_1(1-L_1-L_2)$$

Unsur Segiempat Kuadratik 8-Nod

$$N_1 = -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta), \qquad N_2 = \frac{1}{2}(1-\xi^2)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1), \qquad N_4 = \frac{1}{2}(1-\eta^2)(1+\xi)$$

$$N_5 = \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1), \qquad N_6 = \frac{1}{2}(1-\xi^2)(1+\eta)$$

$$N_7 = -\frac{1}{4}(1-\xi)(1+\eta)(\xi-\eta+1), \qquad N_8 = \frac{1}{2}(1-\eta^2)(1-\xi)$$

Kuadratur Gauss-Legendre

$n=1$	$\xi_i = 0.0$	$W_i = 2.0$
$n=2$	$\xi_i = \pm 0.577350$	$W_i = 1.0$
$n=3$	$\xi_i = 0.0$	$W_i = 8/9$
	$\xi_i = \pm 0.774597$	$W_i = 5/9$
$n=4$	$\xi_i = \pm 0.861136$	$W_i = 0.347855$
	$\xi_i = \pm 0.339981$	$W_i = 0.652145$

Untuk Domain Segitiga

n	Titik	L_1	L_2	W_i
2	a	1/3	1/3	1/2
3	a	1/2	0	1/6
	b	1/2	1/2	1/6
	c	0	1/2	1/6

Masalah Bersandarkan Masa

$$\left([C] + \theta\Delta t[K]\right)\{\Phi\}_b = \left([C] - (1-\theta)\Delta t[K]\right)\{\Phi\}_a + \Delta t\left((1-\theta)\{F\}_a + \theta\{F\}_b\right)$$

Perumusan Konsisten

$$\left[C^{(e)}\right] = \frac{\lambda L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \qquad \left[C^{(e)}\right] = \frac{\lambda A}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\left[C^{(e)}\right] = \frac{\lambda A}{36}\begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}$$

$$\Delta t > \frac{\lambda L^2}{6D\theta}, \qquad \Delta t < \frac{\lambda L^2}{12D(1-\theta)}$$

Perumusan Tergumpal

$$\left[C^{(e)}\right] = \frac{\lambda L}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \left[C^{(e)}\right] = \frac{\lambda A}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[C^{(e)}\right] = \frac{\lambda A}{4}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Delta t < \frac{\lambda L^2}{4D(1-\theta)}$$

-ooo0ooo-