UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang 1993/94

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<u>MSG442 - Kaedah Unsur Terhingga</u>

Masa: [3 jam]

Jawab **SEMUA** soalan.

1. (a) Cari penyelesaian unsur terhingga bagi:

$$\frac{d^2\phi}{dx^2} - \phi + 1 = 0, \quad 0 < x < 1$$

$$\phi(0) = \phi(1) = 0$$

dengan menggunakan tiga unsur linear.

(30/100)

(b) Jika N_i ialah fungsi bentuk bagi unsur segitiga linear, cari

(i) $$\int_\Omega (x + y + 1) N_i \, dA$$

(ii) $$\int_\Omega \frac{\partial N_i}{\partial x} \cdot \frac{\partial N_i}{\partial y} \, dA$$

(iii) $$\int_{\Gamma_{ij}} (x + y + 1) N_i \, d\Gamma$$

di mana Ω ialah segitiga i(0, 0), j(2, 1), k(1, 3) dan Γ_{ij} ialah sempadan ij.

(30/100)

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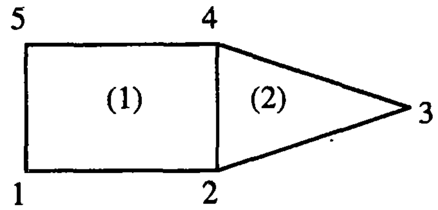
(c) Katakan $\phi^{(1)}$ ialah fungsi interpolasi bilinear dan $\phi^{(2)}$ ialah fungsi interpolasi linear bagi rantau berikut:



Tunjukkan $\phi^{(1)} = \phi^{(2)}$ pada sisi di antara segiempat dan segitiga itu.

(20/100)

(d) Jika $v(x) = \Phi_1 + x.\Phi_2 + x^2.\Phi_3 + x^3.\Phi_4$

dan $I = \int_0^1 \left[v^2 + \left(\frac{dv}{dx}\right)^2 \right] dx,$

cari $\frac{\partial I}{\partial \Phi_i}$ untuk $i = 1, 2, 3, 4.$

(20/100)

2. (a) Dengan membahagikan selang $[0, 3]$ kepada tiga bahagian yang sama dan menggunakan kaedah unsur terhingga dengan perumusan konsisten dan skema beza ke depan, binakan persamaan

$$[A]\{\Phi\}_{t+\Delta t} = [P]\{\Phi\}_t + \{F^*\}$$

untuk masalah berikut:

$$\frac{\partial^2 \phi}{\partial x^2} = 2\frac{\partial \phi}{\partial t}, \quad 0 < x < 3, \quad t > 0$$

$$\phi(x, 0) = 5, \quad 0 \le x \le 3$$

$$\phi(0, t) = 5, \quad t > 0$$

$$\phi(3, t) = 50, \quad t > 0$$

Cari Φ_2 dan Φ_3 pada masa 1 saat.

(30/100)

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(b) Tunjukkan bahawa persamaan matriks untuk penyelesaian unsur terhingga bagi masalah:

$$D\frac{\partial^2\phi}{\partial x^2} + D\frac{\partial^2\phi}{\partial y^2} + Q = \lambda\frac{\partial\phi}{\partial t} \quad \text{dalam } \Omega, \quad t > 0$$

$$\phi(x, y, 0) = \phi_0(x, y), \quad (x, y) \in \Omega$$

$$\phi(x, y, t) = 0, \quad (x, y) \in \Gamma, \quad t > 0$$

boleh ditulis sebagai

$$[C]\left\{\frac{\partial\Phi}{\partial t}\right\} + [K]\{\Phi\} = \{F\}$$

Di sini Ω ialah suatu rantau 2-D dan Γ ialah sempadannya.

(30/100)

(c) Pertimbangkan penyelesaian bagi persamaan:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \frac{\partial\phi}{\partial t}$$

melalui unsur segitiga linear dengan perumusan konsisten.
Untuk segitiga A(0, 0), B(1, 0), C(0, 1), cari syarat atas Δt supaya ayunan berangka dapat dielakkan.

(40/100)

3. (a) Dengan menggunakan kaedah unsur terhingga dan penyegitigaan seperti ditunjukkan di dalam gambar rajah, selesaikan masalah berikut:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + 2 = 0$$

$$\phi = 1 \quad \text{pada } y = 0$$

$$\frac{\partial\phi}{\partial x} = 0 \quad \text{pada } x = 0 \text{ dan pada } x = 1$$

$$\frac{\partial\phi}{\partial x} + \phi = 2 \quad \text{pada } y = 1$$

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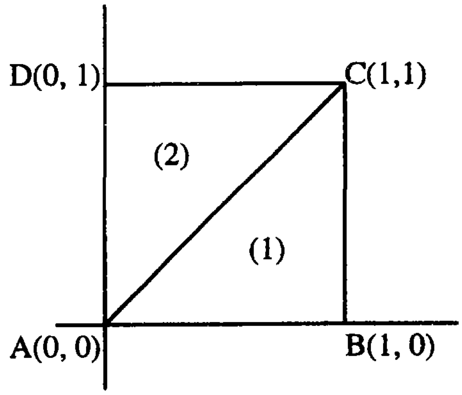

(50/100)

(b) Diberi segitiga A(0, 0), B(5, 0), C(3, 3). Jika $[N] = [N_1\ N_2\ N_3\ N_4\ N_5\ N_6]$ ialah fungsi bentuk bagi unsur kuadratik 6 nod, cari $\dfrac{\partial[N]}{\partial x}$ dan $\dfrac{\partial[N]}{\partial y}$ pada titik $L_1 = L_2 = 0.5$.

(30/100)

(c) Unsur segitiga kuadratik 6 nod digunakan untuk menyelesaikan masalah berikut:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \quad \text{di dalam } \Omega$$

$\phi = \phi_0$ pada sempadan.

Terangkan bagaimana matriks unsur $\left[k^{(e)}\right]$ dihitungkan dengan komputer jika kuadratur Gauss digunakan untuk pengamiran yang terlibat.

(20/100)

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Unsur Linear 1-D

$$\left[k^{(e)}\right] = \frac{D}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Unsur Segitiga Linear

$$N_i = [a_i + b_i x + c_i y]/(2A), \quad N_j = [a_j + b_j x + c_j y]/(2A)$$
$$N_k = [a_k + b_k x + c_k y]/(2A)$$

dengan

$$2A = \begin{vmatrix} 1 & X_i & Y_i \\ 1 & X_j & Y_j \\ 1 & X_k & Y_k \end{vmatrix}$$

dan

$$a_i = X_j Y_k - X_k Y_j, \quad b_i = Y_j - Y_k, \quad c_i = X_k - X_j$$
$$a_j = X_k Y_i - X_i Y_k, \quad b_j = Y_k - Y_i, \quad c_j = X_i - X_k$$
$$a_k = X_i Y_j - X_j Y_i, \quad b_k = Y_i - Y_j, \quad c_k = X_j - X_i$$

$$\left[k_D^{(e)}\right] = \frac{D_x}{4A}\begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_i b_j & b_j^2 & b_j b_k \\ b_i b_k & b_j b_k & b_k^2 \end{bmatrix} + \frac{D_y}{4A}\begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_i c_j & c_j^2 & c_j c_k \\ c_i c_k & c_j c_k & c_k^2 \end{bmatrix}$$

$$\left[k_G^{(e)}\right] = \frac{GA}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QA}{3}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\left[k_M^{(e)}\right] = \frac{ML_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \text{dll.}$$

$$\int_l L_1^a L_2^b L_3^c \, dA = \frac{a!\,b!\,c!}{(a+b+c+2)!}\,2A$$

Unsur Segiempat Tepat Bilinear

$$N_i = \frac{1}{4}(1-\xi)(1-\eta), \qquad N_j = \frac{1}{4}(1+\xi)(1-\eta)$$

$$N_k = \frac{1}{4}(1+\xi)(1+\eta), \qquad N_m = \frac{1}{4}(1-\xi)(1+\eta)$$

$$N_i = \left(1 - \frac{s}{2b}\right)\left(1 - \frac{t}{2a}\right), \qquad N_j = \frac{s}{2b}\left(1 - \frac{t}{2a}\right)$$

$$N_k = \frac{st}{4ab}, \qquad N_m = \frac{t}{2a}\left(1 - \frac{s}{2b}\right)$$

$$\left[k_D^{(e)}\right] = \frac{D_x a}{6b}\begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{D_y b}{6a}\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

$$\left[k_G^{(e)}\right] = \frac{GA}{36}\begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QA}{4}\begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\left[k_M^{(e)}\right] = \frac{ML_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \text{dll.}$$

Unsur Kuadratik 1-D

$$N_1 = \frac{1}{2}\xi(\xi-1), \qquad N_2 = -(\xi+1)(\xi-1), \qquad N_3 = \frac{1}{2}\xi(\xi+1)$$

Unsur Segitiga Kuadratik 6-Nod

$$N_1 = L_1(2L_1-1), \qquad N_2 = 4L_1L_2,$$

$$N_3 = L_2(2L_2-1), \qquad N_4 = 4L_2(1-L_1-L_2)$$

$$N_5 = 1 - 3(L_1+L_2) + 2(L_1+L_2)^2, \qquad N_6 = 4L_1(1-L_1-L_2)$$

Unsur Segiempat Kuadratik 8-Nod

$$N_1 = -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta), \quad N_2 = \frac{1}{2}(1-\xi^2)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1), \quad N_4 = \frac{1}{2}(1-\eta^2)(1+\xi)$$

$$N_5 = \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1), \quad N_6 = \frac{1}{2}(1-\xi^2)(1+\eta)$$

$$N_7 = -\frac{1}{4}(1-\xi)(1+\eta)(\xi-\eta+1), \quad N_8 = \frac{1}{2}(1-\eta^2)(1-\xi)$$

Kuadratur Gauss-Legendre

$n=1$	$\xi_i = 0.0$	$W_i = 2.0$
$n=2$	$\xi_i = \pm 0.577350$	$W_i = 1.0$
$n=3$	$\xi_i = 0.0$	$W_i = 8/9$
	$\xi_i = \pm 0.774597$	$W_i = 5/9$
$n=4$	$\xi_i = \pm 0.861136$	$W_i = 0.347855$
	$\xi_i = \pm 0.339981$	$W_i = 0.652145$

Untuk Domain Segitiga

n	Titik	L_1	L_2	W_i
2	a	1/3	1/3	1/2
3	a	1/2	0	1/6
	b	1/2	1/2	1/6
	c	0	1/2	1/6

Masalah Bersandarkan Masa

$$\Big([C] + \theta\Delta t[\mathrm{K}]\Big)\{\Phi\}_b = \Big([C] - (1-\theta)\Delta t[\mathrm{K}]\Big)\{\Phi\}_a + \Delta t\Big((1-\theta)\{F\}_a + \theta\{F\}_b\Big)$$

Perumusan Konsisten

$$\left[c^{(e)}\right] = \frac{\lambda L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \qquad \left[c^{(e)}\right] = \frac{\lambda A}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\left[c^{(e)}\right] = \frac{\lambda A}{36}\begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}$$

$$\Delta t > \frac{\lambda L^2}{6D\theta}, \qquad \Delta t < \frac{\lambda L^2}{12D(1-\theta)}$$

Perumusan Tergumpal

$$\left[c^{(e)}\right] = \frac{\lambda L}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \left[c^{(e)}\right] = \frac{\lambda A}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[c^{(e)}\right] = \frac{\lambda A}{4}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Delta t < \frac{\lambda L^2}{4D(1-\theta)}$$