UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Tambahan
Sidang 1994/95

Jun 1995

MAT 420 Persamaan Pembezaan Separa

Masa : [3 jam]

Jawab **SEMUA** soalan.

1. (a) Tentukan jenis dan cari bentuk berkanun bagi persamaan:

$$yu_{xx} - u_{yy} = 0, \qquad y > 0$$

(30/100)

(b) Selesaikan:

$$u_t = u_{xx} + 4u_x + 6u, \qquad 0 < x < \ell, \quad t > 0$$
$$u(x, 0) = f(x), \quad 0 < x < \ell$$
$$u(0, t) = u(\ell, t) = 0, \quad t > 0$$

(50/100)

(c) Cari siri Fourier bagi fungsi

$$f(x) = x + 1, \quad -\pi < x < \pi$$

(20/100)

2. (a) Cari penyelesaian bagi masalah:

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0, \qquad 0 < r < 1, \quad 0 \le \theta \le 2\pi$$

$$u(1, \theta) = f(\theta), \quad 0 \le \theta \le 2\pi$$

$$|u(r, \theta)| \le M, \quad 0 \le r < 1, \quad 0 \le \theta \le 2\pi$$

dan tulis penyelesaian ini dalam bentuk suatu kamiran.

(40/100)

...2/-

(b) Selesaikan

$$
\begin{aligned}
u_{tt} &= c^2.u_{xx}, \quad 0 < x < 1, \quad t > 0 \\
u(x,0) &= x(1-x), \quad 0 \le x \le 1 \\
u_t(x,0) &= 0, \quad 0 \le x \le 1 \\
u(0,t) &= u(1,t) = 0, \quad t \ge 0
\end{aligned}
$$

(30/100)

(c) Selesaikan

$$
\begin{aligned}
u_t &= ku_{xx} + h(x,t), \quad 0 < x < \ell, \quad t > 0 \\
u(x,0) &= f(x), \quad 0 \le x \le \ell \\
u(0,t) &= u(\ell,t) = 0, \quad t \ge 0
\end{aligned}
$$

(30/100)

3. (a) Cari fungsi Green bagi masalah:

$$
\begin{aligned}
u_{xx} + u_{yy} &= f(x,y), \quad x > 0, \quad y > 0 \\
u(x,0) &= g(x), \quad x \ge 0 \\
u(0,y) &= h(y), \quad y \ge 0
\end{aligned}
$$

Kemudian selesaikan masalah ini.

(40/100)

(b) Selesaikan:

$$
\begin{aligned}
u_t &= u_{xx}, \quad -\infty < x < \infty, \quad t > 0 \\
u(x,0) &= f(x), \quad -\infty < x < \infty \\
|u(x,t)| &\le M, \quad -\infty < x < \infty, \quad t > 0
\end{aligned}
$$

(30/100)

(c) Selesaikan:

$$
\begin{aligned}
u_t &= u_{xx}, \quad x > 0, \quad t > 0 \\
u(x,0) &= 0, \quad x > 0 \\
u(0,t) &= \sin t, \quad t > 0 \\
|u(x,t| &\le M, \quad x > 0, \quad t > 0
\end{aligned}
$$

(30/100)

...3/-

Jadual 1: Jelmaan Fourier	
$f(x)$	$F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{i\alpha x}\,dx$
e^{-cx^2}	$\frac{1}{\sqrt{2c}} e^{-\frac{\alpha^2}{4c}}$
$f^{(n)}(x)$	$(-i\alpha)^n.F(\alpha)$
$\int_{-\infty}^{\infty} f(x-u)g(u)du$	$\sqrt{2\pi}.F(\alpha).G(\alpha)$

Jadual: Jelmaan Laplace	
$f(t)$	$F(s) = \int_0^{\infty} f(t)e^{-st}\,dt$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}, \quad n = 1, 2, ...$
e^{at}	$\frac{1}{s-a}$
$f(t-b)H(t-b)$	$e^{-bs}.F(s)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\int_0^t f(t-u)g(u)du$	$F(s).G(s)$

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