UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2008/2009

November 2008

ZCT 211/2 – Vector Analysis
[Analisis Vektor]

Duration: 2 hours
[Masa : 2 jam]

Please ensure that this examination paper contains **FOUR** printed pages before you begin the examination.
[Sila pastikan bahawa kertas peperiksaan ini mengandungi EMPAT muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

<u>Instruction</u>: Answer all **FOUR** questions. Students are allowed to answer all questions in Bahasa Malaysia or in English.

[<u>Arahan</u>: Jawab semua EMPAT soalan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]

1.   (a)   (i)   Prove that the area of a parallelogram with sides $\vec{A}$ and $\vec{B}$ is $\left|\vec{A}\times\vec{B}\right|$.

[Buktikan luas bagi parallelogram dengan sisi $\vec{A}$ dan $\vec{B}$ ialah $\left|\vec{A}\times\vec{B}\right|$.]

(10/100)

(ii)   Find the area of a triangle having vertices at P(1, 3, 2), Q(2, −1, 1), R(−1, 2, 3)

[Cari luas suatu segitiga dengan bucu-bucu pada P(1, 3, 2), Q(2, −1, 1), R(−1, 2, 3)]

(10/100)

(iii)   Find the shortest distance from (6, −4, 4) to the line joining (2, 1, 2) and (3, −1, 4)

[Cari jarak terpendek dari (6, −4, 4) ke garis yang menyambung (2, 1, 2) dan (3, −1, 4)]

(25/100)

(b)   Find a set of vectors reciprocal to the set $2\hat{i}+3\hat{j}-\hat{k}$, $\hat{i}-\hat{j}-2\hat{k}$, $-\hat{i}+2\hat{j}+2\hat{k}$

[Cari satu set vektor yang bersalingan kepada set $2\hat{i}+3\hat{j}-\hat{k}$, $\hat{i}-\hat{j}-2\hat{k}$, $-\hat{i}+2\hat{j}+2\hat{k}$]

(30/100)

(c)   If *f(x)* and *g(x)* are differentiable functions, differentiate each of the followings with respect to *x*:

[Jika f(x) dan g(x) adalah fungsi-fungsi terbezakan, bezakan setiap berikut terhadap x:]

(i)   *f(x).g(x)*

(ii)   *f(x)/g(x)*

(iii)   *ln[f(x)]*

(iv) $e^{f(x)}$

(v)   *tan x*

(10/100)

...3/-

(d)   If $\vec{A} = (2x^2y - x^4)\hat{i} + (e^{xy} - y\sin x)\hat{j} + (x^2\cos y)\hat{k}$, find:

[Jika $\vec{A} = (2x^2y - x^4)\hat{i} + (e^{xy} - y\sin x)\hat{j} + (x^2\cos y)\hat{k}$, cari:]

(i) $\dfrac{\partial^2\vec{A}}{\partial x^2}$

(ii) $\dfrac{\partial^2\vec{A}}{\partial y^2}$

(iii) $\dfrac{\partial^2\vec{A}}{\partial x\partial y}$

(15/100)

2.   (a)   Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1, −2, −1) in the direction $2\hat{i} - \hat{j} - 2\hat{k}$

[Cari terbitan berarah $\phi = x^2yz + 4xz^2$ pada (1, −2, −1) dalam arah $2\hat{i} - \hat{j} - 2\hat{k}$]

(20/100)

(b)   (i)   If $\vec{A} = \vec{r}/r$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Find grad div $\vec{A}$.

[Jika $\vec{A} = \vec{r}/r$, di sini $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Cari grad div $\vec{A}$.]

(30/100)

(ii)   Evaluate $\nabla^2\left(\dfrac{1}{r}\right)$ [Nilaikan $\nabla^2\left(\dfrac{1}{r}\right)$]

(50/100)

3.   (a)   If $\vec{F} = (2x + y)\hat{i} + (3y - x)\hat{j}$, evaluate $\int_C \vec{F}\cdot d\vec{r}$ where C is the curve in the xy plane consisting of the straight lines from (0, 0) to (2, 0) and then to (3, 2).

[Jika $\vec{F} = (2x + y)\hat{i} + (3y - x)\hat{j}$, nilaikan $\int_C \vec{F}\cdot d\vec{r}$, di sini C adalah lengkungan dalam satah xy yang terdiri daripada garis-garis lurus dari (0, 0) ke (2, 0) dan kemudian ke (3, 2).]

(50/100)

...4/-

(b) Evaluate $\iint_S \vec{A} \cdot \hat{n}\, dS$, where $A = z\hat{i} + x\hat{j} - 3y^2 z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

[Instruction: $\iint_S \vec{A} \cdot \hat{n}\, dS = \iint_R \vec{A} \cdot \hat{n}\dfrac{dx\, dz}{\left|\hat{n} \cdot \hat{j}\right|}$ **must be used** to solve the question.]

[Nilaikan $\iint_S \vec{A} \cdot \hat{n}\, dS$, *di sini* $A = z\hat{i} + x\hat{j} - 3y^2 z\hat{k}$ *dan S ialah permukaan silinder* $x^2 + y^2 = 16$ *terkandung dalam oktan pertama antara* $z = 0$ *dan* $z = 5$.

[Arahan: $\iint_S \vec{A} \cdot \hat{n}\, dS = \iint_R \vec{A} \cdot \hat{n}\dfrac{dx\, dz}{\left|\hat{n} \cdot \hat{j}\right|}$ ***mesti digunakan*** *untuk menyelesaikan soalan ini.]*

(50/100)

4. (a) (i) State the Green's theorem.
   [Nyatakan teorem Green]

   (10/100)

   (ii) Evaluate $\oint_C (x^2 - 2xy)dx + (x^2 y + 3)dy$ around the boundary of the region defined by $y^2 = 8x$ and $x = 2$ by using Green's theorem.

   [Nilaikan $\oint_C (x^2 - 2xy)dx + (x^2 y + 3)dy$ *yang mengelilingi kawasan sempadan yang ditakrifkan oleh* $y^2 = 8x$ *dan* $x = 2$ *dengan menggunakan teorem Green.]*

   (35/100)

   (b) (i) State the divergence theorem.
   [Nyatakan teorem kecapahan]

   (10/100)

   (ii) Evaluate the divergence theorem for $\vec{A} = 4x\hat{i} - 2y^2 \hat{j} + z^2 \hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$.

   [Nilaikan teorem kecapahan untuk $\vec{A} = 4x\hat{i} - 2y^2 \hat{j} + z^2 \hat{k}$ *bagi kawasan yang diliputi oleh* $x^2 + y^2 = 4$, $z = 0$ *dan* $z = 3$.]

   (45/100)

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