UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 1992/93

Oktober/November 1992

FMT 202 Statistik

Masa: (2 jam)

Kertas ini mengandungi ENAM (6) soalan dan 11 muka surat yang bertaip.

Jawab LIMA (5) soalan sahaja.

Semua soalan mesti dijawab di dalam Bahasa Malaysia.

1. (A) Sekiranya $X \sim N(100, 36)$ dan $P(X > b) = 0.1093$. Carikan nilai b.

(5 markah)

(B) Sebuah beg mengandungi 4 biji tablet merah dan 3 biji tablet kuning. Sebuah beg yang lain mengandungi 3 biji tablet merah dan 4 biji tablet kuning. Katakan sebiji tablet dikeluarkan dari beg pertama dan dimasukkan ke dalam beg kedua. Beg kedua itu dicampur baik dan kemudian sebiji tablet dikeluarkan serta dikembalikan semula ke beg pertama. Sekiranya sebiji tablet dikeluarkan sekarang dari beg pertama, apakah kebarangkalian bahawa ia sebiji tablet merah? (Gunakan gambarajah pohon untuk menjawab soalan ini).

(15 markah)

2. (A) Di dalam sebuah jawatankuasa adalah 8 orang perempuan dan 10 orang lelaki. Sekiranya 6 orang akan dipilih daripada jawatankuasa itu, berapakah cara boleh terjadi jika terdiri daripada

(a) 3 orang perempuan dan 3 orang lelaki?
(b) sekurang-kurangnya 4 orang lelaki?

...3/-

Di antara 18 orang tersebut, terdapat sepasang anak kembar (twins), seorang perempuan dan seorang lelaki. Katakan 3 orang perempuan dan 3 orang lelaki dipilih. Carikan kebarangkalian bahawa kedua-dua anak kembar itu dipilih.

(10 markah)

(B) Bilangan kemalangan yang berlaku di dalam sebuah kilang farmaseutik mengikut taburan Poisson dengan kevarinan 3.0.

Carikan kebarangkalian bahawa

(a) tiada kemalangan yang berlaku di dalam seminggu.
(b) lebih daripada 4 kemalangan berlaku di dalam seminggu.
(c) kurang daripada 3 kemalangan berlaku di dalam setengah bulan.
(d) 7 kemalangan berlaku di dalam setengah bulan.

(10 markah)

...4/-

3\. (A) Pejabat Pendaftaran Hospital Besar Pulau Pinang telah menyatakan bahawa min umur pesakit (μ) yang mendaftar ialah 42 tahun dan sisihan piawai populasi, σ, ialah 8 tahun. Untuk menentukan dakwaan tersebut satu sampel rawak seramai 120 pesakit telah diambil daripada rekod pejabat pendaftaran tersebut dan min sampel yang diperolehi ialah 44.2.

(i) Di peringkat $P < 0.05$, pilih satu ujian yang sesuai untuk menentukan sama ada terdapat perbezaan yang signifikan atau tidak di antara min sampel dan μ.

(10 markah)

(ii) Nyatakan asas-asas yang menyebabkan anda memilih ujian tersebut.

(5 markah)

(B) Bincangkan prinsip-prinsip rekabentuk ujikaji yang baik.

(5 markah)

...5/-

4. Kesan amfetamin dan klorpromazin terhadap berat badan telah diuji dengan menyuntik drug-drug tersebut ke atas 2 kumpulan tikus. Keputusan yang diperolehi adalah seperti berikut:

Penurunan berat badan (gm)	
Amfetamin	Klorpromazin
50	45
45	30
40	25
46	34
35	40
25	31
33	26
42	22

(a) Di peringkat $P < 0.01$, pilih satu ujian yang sesuai untuk menentukan sama ada terdapat perbezaan penurunan berat badan di antara kedua-dua drug tersebut.

(10 markah)

(b) Tentukan selang keyakinan min penurunan berat badan bagi kumpulan amfetamin di peringkat 99%.

(10 markah)

...6/-

5. Suatu kajian dikendalikan untuk membandingkan dua Formulasi A dan B yang mengandungi drug dan dos yang sama. Dua belas (12) subjek manusia dibahagikan secara rawak kepada dua kumpulan dengan bilangan subjek yang sama. Kumpulan I diberikan Formulasi A dan kumpulan II diberikan Formulasi B. Berikut ialah data yang diperolehi:

Amaun yang diserap (mg)	
Formulasi A	Formulasi B
78	95
92	88
65	76
73	84
50	70
66	47

Data yang diperolehi tidak bertaburan normal.

(A) Pilih suatu ujian statistik dan tentukan sama ada penyerapan dari Formulasi A dan B itu adalah sama atau tidak.

(10 markah)

(B) Apakah kelemahan rekabentuk ujian yang digunakan di atas. Bincangkan rekabentuk-rekabentuk lain yang boleh digunakan serta kebaikan dan kelemahannya.

(10 markah)

6. Tiga kaedah pengajaran matematik dibandingkan. Lapan belas (18) pelajar dipilih dan dibahagikan secara rawak kepada tiga kumpulan yang mengandungi bilangan pelajar yang sama. Satu kumpulan diajar dengan satu kaedah pengajaran. Selepas 3 bulan, suatu ujian matematik dikendalikan bagi semua pelajar itu. Berikut ialah markah-markah yang diperolehi.

Kaedah I	Kaedah II	Kaedah III
71	76	75
79	70	82
80	90	60
72	80	66
88	75	74
64	82	58

(A) Adakah varians bagi data-data itu homogenous?

(4 markah)

(B) Pilih suatu ujian statistik untuk menentukan sama ada markah-markah yang diperolehi oleh tiga kumpulan itu berbeza secara statistik atau tidak.

(10 markah)

(C) Apakah jenis-jenis ralat yang berlaku dalam suatu kaedah analisis kimia? Terangkan secara ringkas suatu ujian statistik yang dapat menilaikan ralat-ralat itu.

(6 markah)

<u>FORMULA</u>

1. Median (m) = $b + c \times \frac{d}{f}$

2. $u_i = Ax_i + B$

3. $\bar{x} = \frac{1}{A}(\bar{u} - B)$

4. $S_x{}^2 = \frac{1}{A^2} S_u{}^2$

5. $S_u{}^2 = \frac{\Sigma ui^2 fi - n\bar{u}^2}{n - 1}$

6. Trimean = $\frac{\text{kuartil atas} + (2 \times \text{median}) + \text{kuartil bawah}}{4}$

7. <u>Ujian-t</u>

$$t = \frac{\bar{x} - \mu_o}{S/\sqrt{n}}$$

$$s = \sqrt{\frac{x^2 - \frac{(\ x)^2}{n}}{n - 1}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S\sqrt{1/n_1 + 1/n_2}}$$

$$s = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

$$t = \frac{\bar{D}}{S/\sqrt{n}}$$

$$s = \sqrt{\frac{\Sigma D^2 - \frac{(\Sigma D)^2}{n}}{n - 1}}$$

...9/-

8. Ujian Wilcoxon (independent samples)

$$U = n_1 \; n_2 + \frac{n_2(n_2 + 1)}{2} - \Sigma_R$$

$$U' = n_1 \; n_2 - U$$

9. Ujian Sign

$$P\ (s \geq k) = 1 - P\ (s \geq k-1)$$

10. ANOVA (1-way)

$$SS_{Total} = \Sigma X^2 - \frac{(\Sigma X)^2}{n_T}$$

$$SS_{Treatments} = \frac{(\Sigma X_A)^2}{n_A} + \frac{(\Sigma X_B)^2}{n_B} \; \ldots\ldots - \frac{(\Sigma X)^2}{n_T}$$

$$SS_{Error} = SS_{Total} - SS_{Treatments}$$

$$d.f.\ (Total) = (n_T - 1)$$

$$d.f.\ (Treatment) = (k - 1)$$

$$d.f.\ (Error) = (n_1 + n_2 \; \ldots\ldots + n_k - k)$$

$$HSD = \frac{q\sqrt{MS_{error}}}{\sqrt{n}}$$

$$n_{nm} = \frac{2\ n_1 n_2}{n_1 + n_2}$$

...10/-

11. Ujian Kruskal-Wallis

$$H = \frac{12}{N(N + 1)}\left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \ldots\ldots + \frac{R_k^2}{n_k}\right) - 3\,(N + 1)$$

$$N = n_1 + n_2 + \ldots\ldots + n_k$$

$$d.f. = k - 1$$

Ujian perbandingan berganda:

$$\Delta\bar{R} = Z_{(\alpha/k\ (k-1))}\sqrt{\frac{N(N+1)}{12}\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

12. Ujian Friedman

$$Q = \frac{12}{n_k(k + 1)}(R_1^2 + R_2^2 + \ldots\ldots + R_k^2) - 3\,n\,(k + 1)$$

$$d.f. = k - 1$$

Ujian perbandingan berganda:

$$\Delta R = Z_{(\alpha/k\ (k-1))}\sqrt{\frac{b\ k\ (K + 1)}{6}}$$

13. Formula Sturges

$$k = 1 + 3.3\ \log_{10} n$$

...11/-

14. Ujian Korelasi

$$R = \frac{n\ \Sigma xy - \Sigma x\ \Sigma y}{\sqrt{n\ \Sigma x^2 - (\Sigma x)^2}\ \sqrt{n\ \Sigma y^2 - (\Sigma y)^2}}$$

15. Analisis Regresi

$$y = mx + c$$

$$m = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

$$c = \frac{\Sigma y - m(\Sigma x)}{n}$$

$$SS_E = \Sigma y^2 - m\ \Sigma xy - \frac{(\Sigma y)^2}{n} + \frac{m\ \Sigma x \Sigma y}{n}$$

$$S_{yx} = \sqrt{\frac{SS_E}{n-2}}$$

16. $\chi^2 = \frac{N(AD - BC)^2}{(A+B)(C+D)(A+C)(B+D)}$

17. $99\%\ CI\ \mu = \bar{X} \pm \left(t \times \frac{s}{\sqrt{n}}\right)$

18. $99\%\ CI\ \mu = \bar{X} \pm \left(z \times \frac{s}{\sqrt{n}}\right)$

19. $Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

-oo0oo-

THE CORRELATION COEFFICIENT

Values of the correlation Coefficient for Different Levels of Significance (2 tail)

d.f.	.1	.05	.02	.01	.001
1.	.98769	.99692	.999507	.999877	.9999988
2.	.90000	.95000	.98000	.990000	.99900
3.	.8054	.8783	.93433	.95873	.99116
4.	.7293	.8114	.8822	.91720	.97406
5.	.6694	.7545	.8329	.8745	.95074
6.	.6215	.7067	.7887	.8343	.92493
7.	.5822	.6664	.7498	.7977	.8982
8.	.5494	.6319	.7155	.7646	.8721
9.	.5214	.6021	.6851	.7348	.8471
10.	.4973	.5760	.6581	.7079	.8233
11.	.4762	.5529	.6339	.6835	.8010
12.	.4575	.5324	.6120	.6614	.7800
13.	.4409	.5139	.5923	.6411	.7603
14.	.4259	.4973	.5742	.6226	.7420
15.	.4124	.4821	.5577	.6055	.7246

d.f. = degrees of freedom

TABLE XIII Table of q (0.05 level)

d.f. \ k	2	3	4	5	6	7	8	9	10	11
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17
6	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49	6.65
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05
9	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74	5.87
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39	5.51
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26
17	2.98	3.63	4.02	4.30	4.52	4.71	4.86	4.99	5.11	5.21
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.17
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04	5.14
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.11
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01
30	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82	4.92
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73	4.82
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73
120	2.80	3.36	3.68	3.92	4.10	4.24	4.36	4.47	4.56	4.64
∞	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55

From H.L. Harker in *Annals of Mathematical Statistics*, 31 (1960): 1122-1147. Reprinted by permission of the publishers, The Institute of Mathematical Statistics.

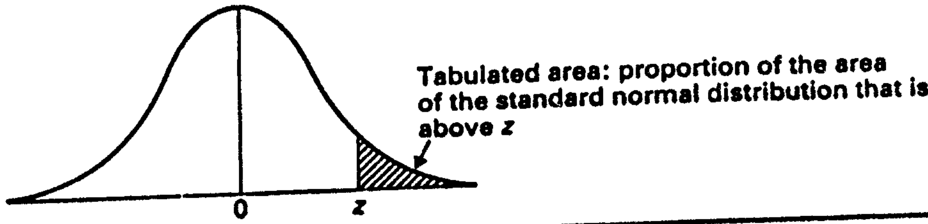
Table A3 Percentage points of the *t* distribution.

Adapted from Table 7 of White *et al.* (1979) with permission of authors and publishers.

	One-sided *P* value								
	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
	Two-sided *P* value								
d.f.	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
1	1.00	3.08	6.31	12.71	31.82	63.66	127.32	318.31	636.62
2	0.82	1.89	2.92	4.30	6.96	9.92	14.09	22.33	31.60
3	0.76	1.64	2.35	3.18	4.54	5.84	7.45	10.21	12.92
4	0.74	1.53	2.13	2.78	3.75	4.60	5.60	7.17	8.61
5	0.73	1.48	2.02	2.57	3.36	4.03	4.77	5.89	6.87
6	0.72	1.44	1.94	2.45	3.14	3.71	4.32	5.21	5.96
7	0.71	1.42	1.90	2.36	3.00	3.50	4.03	4.78	5.41
8	0.71	1.40	1.86	2.31	2.90	3.36	3.83	4.50	5.04
9	0.70	1.38	1.83	2.26	2.82	3.25	3.69	4.30	4.78
10	0.70	1.37	1.81	2.23	2.76	3.17	3.58	4.14	4.59
11	0.70	1.36	1.80	2.20	2.72	3.11	3.50	4.02	4.44
12	0.70	1.36	1.78	2.18	2.68	3.06	3.43	3.93	4.32
13	0.69	1.35	1.77	2.16	2.65	3.01	3.37	3.85	4.22
14	0.69	1.34	1.76	2.14	2.62	2.98	3.33	3.79	4.14
15	0.69	1.34	1.75	2.13	2.60	2.95	3.29	3.73	4.07
16	0.69	1.34	1.75	2.12	2.58	2.92	3.25	3.69	4.02
17	0.69	1.33	1.74	2.11	2.57	2.90	3.22	3.65	3.96
18	0.69	1.33	1.73	2.10	2.55	2.88	3.20	3.61	3.92
19	0.69	1.33	1.73	2.09	2.54	2.86	3.17	3.58	3.88
20	0.69	1.32	1.72	2.09	2.53	2.84	3.15	3.55	3.85
21	0.69	1.32	1.72	2.08	2.52	2.83	3.14	3.53	3.82
22	0.69	1.32	1.72	2.07	2.51	2.82	3.12	3.50	3.79
23	0.68	1.32	1.71	2.07	2.50	2.81	3.10	3.48	3.77
24	0.68	1.32	1.71	2.06	2.49	2.80	3.09	3.47	3.74
25	0.68	1.32	1.71	2.06	2.48	2.79	3.08	3.45	3.72
26	0.68	1.32	1.71	2.06	2.48	2.78	3.07	3.44	3.71
27	0.68	1.31	1.70	2.05	2.47	2.77	3.06	3.42	3.69
28	0.68	1.31	1.70	2.05	2.47	2.76	3.05	3.41	3.67
29	0.68	1.31	1.70	2.04	2.46	2.76	3.04	3.40	3.66
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.38	3.65
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.31	3.55
60	0.68	1.30	1.67	2.00	2.39	2.66	2.92	3.23	3.46
120	0.68	1.29	1.66	1.98	2.36	2.62	2.86	3.16	3.37
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.09	3.29

Table A1 Areas in tail of the standard normal distribution.

Adapted from Table 3 of White *et al.* (1979) with permission of the authors and publishers.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
	Second decimal place of z									
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003

Taburan Poisson

X menandakan suatu pembolehubah dengan suatu taburan Poisson yang mempunyai jangkaan μ. Sifir ini memberikan nilai $P(X = k) = (e^{-\mu}\mu^k/k!)$ bagi berbagai nilai *k* dan μ.

Contoh. Untuk $\mu = 1.0$,

$P(X = 1) = .368$

$P(X = 3) = .061$

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= .368 + .368 + .184$

$= .920$

k	$\mu = .1$	$\mu = .5$	$\mu = 1.0$	$\mu = 2.0$
0	.905	.607	.368	.135
1	.090	.303	.368	.271
2	.005	.076	.184	.271
3		.013	.061	.180
4		.002	.015	.090
5			.003	.036
6			.001	.012
7				.003
8				.001

k	$\mu = 3.0$	$\mu = 4.0$	$\mu = 5.0$	$\mu = 6.0$
0	.050	.018	.007	.002
1	.149	.073	.034	.015
2	.224	.147	.084	.045
3	.224	.195	.140	.089
4	.168	.195	.175	134
5	.101	.156	.175	.161
6	.050	.104	.146	.161
7	.022	.060	.104	.138
8	.008	.030	.065	.103
9	.003	.013	.036	.069
10	.001	.005	.018	.041
11		.002	.008	.023
12		.001	.003	.011
13			.001	.005
14				.002
15				.001

k	$\mu = 7.0$	$\mu = 8.0$	$\mu = 9.0$	$\mu = 10.0$
0	.001			
1	.006	.003	.001	
2	.022	.011	.005	.002
3	.052	.029	.015	.008
4	.091	.057	.034	.019
5	.128	.092	.061	.038
6	.149	.122	.091	.063
7	.149	.140	.117	.090
8	.130	.140	.132	.113
9	.101	.124	.132	.125
10	.071	.099	.119	.125
11	.045	.072	.097	.114
12	.026	.048	.073	.095
13	.014	.030	.050	.073
14	.007	.017	.032	.052
15	.003	.009	.019	.035
16	.001	.005	.011	.022
17	.001	.002	.006	.013
18		.001	.003	.007
19			.001	.004

TABLE V Chi square

Column headings indicate probability of chance deviation between O and E.

D.F. \ P	0.25	0.10	0.05	0.025	0.01	0.005
1.	1.323	2.706	3.841	5.024	6.635	7.879
2.	2.773	4.605	5.991	7.378	9.210	10.597
3.	4.108	6.251	7.815	9.348	11.345	12.838
4.	5.385	7.779	9.488	11.143	13.277	14.860
5.	6.626	9.236	11.071	12.833	15.086	16.750
6.	7.841	10.645	12.592	14.449	16.812	18.548
7.	9.037	12.017	14.067	16.013	18.475	20.278
8.	10.219	13.362	15.507	17.535	20.090	21.955
9.	11.389	14.684	16.919	19.023	21.666	23.589
10.	12.549	15.987	18.307	20.483	23.209	25.188
11.	13.701	17.275	19.675	21.920	24.725	26.757
12.	14.845	18.549	21.026	23.337	26.217	28.299
13.	15.984	19.812	22.362	24.736	27.688	29.819
14.	17.117	21.064	23.685	26.119	29.141	31.319
15.	18.245	22.307	24.996	27.488	30.578	32.801

Adapted from table of χ^2 appearing in *Handbook of Statistical Tables* by D. B. Owen, Addison-Wesley, 1962, p. 50. Reprinted by permission of the U.S. Atomic Energy Commission.

TABLE IV Normal curve areas

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.49903									
3.2	.49931									
3.3	.49952									
3.4	.49966									
3.5	.49977									
3.6	.49984									
3.7	.49989									
3.8	.49993									
3.9	.49995									
4.0	.50000									

TABLE VII (*continued*)

0.05 (light row) and 0.01 (dark row) points for the distribution of F

Degrees of freedom for lesser mean square	Degrees of freedom for greater mean square: 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞
32	4.15	3.30	2.90	2.67	2.51	2.40	2.32	2.25	2.19	2.14	2.10	2.07	2.02	1.97	1.91	1.86	1.82	1.76	1.74	1.69	1.67	1.64	1.61	1.59
	7.50	5.34	4.46	3.97	3.66	3.42	3.25	3.12	3.01	2.94	2.86	2.80	2.70	2.62	2.51	2.42	2.34	2.25	2.20	2.12	2.08	2.02	1.98	1.96
34	4.13	3.28	2.88	2.65	2.49	2.38	2.30	2.23	2.17	2.12	2.08	2.05	2.00	1.95	1.89	1.84	1.80	1.74	1.71	1.67	1.64	1.61	1.59	1.57
	7.44	5.29	4.42	3.93	3.61	3.38	3.21	3.08	2.97	2.89	2.82	2.76	2.66	2.58	2.47	2.38	2.30	2.21	2.15	2.08	2.04	1.98	1.94	1.91
36	4.11	3.26	2.86	2.63	2.48	2.36	2.28	2.21	2.15	2.10	2.06	2.03	1.89	1.93	1.87	1.82	1.78	1.72	1.69	1.65	1.62	1.59	1.56	1.55
	7.39	5.25	4.38	3.89	3.58	3.35	3.18	3.04	2.94	2.86	2.78	2.72	2.62	2.54	2.43	2.35	2.26	2.17	2.12	2.04	2.00	1.94	1.90	1.87
38	4.10	3.25	2.85	2.62	2.46	2.35	2.26	2.19	2.14	2.09	2.05	2.02	1.96	1.92	1.85	1.80	1.76	1.71	1.67	1.63	1.60	1.57	1.54	1.53
	7.35	5.21	4.34	3.86	3.54	3.32	3.15	3.02	2.91	2.82	2.75	2.69	2.59	2.51	2.40	2.32	2.22	2.14	2.08	2.00	1.97	1.90	1.86	1.84
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.07	2.04	2.00	1.95	1.90	1.84	1.79	1.74	1.69	1.66	1.61	1.59	1.55	1.53	1.51
	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.88	2.80	2.73	2.66	2.56	2.49	2.37	2.29	2.20	2.11	2.05	1.97	1.94	1.88	1.84	1.81
42	4.07	3.22	2.83	2.59	2.44	2.32	2.24	2.17	2.11	2.06	2.02	1.90	1.94	1.89	1.82	1.78	1.73	1.68	1.64	1.60	1.57	1.54	1.51	1.49
	7.27	5.15	4.29	3.80	3.49	3.26	3.10	2.96	2.86	2.77	2.70	2.64	2.54	2.46	2.35	2.26	2.17	2.08	2.02	1.94	1.91	1.85	1.80	1.78
44	4.06	3.21	2.82	2.58	2.43	2.31	2.23	2.16	2.10	2.05	2.01	1.98	1.92	1.88	1.81	1.76	1.72	1.66	1.63	1.58	1.56	1.52	1.50	1.48
	7.24	5.12	4.26	3.78	3.46	3.24	3.07	2.94	2.84	2.75	2.68	2.62	2.52	2.44	2.32	2.24	2.15	2.06	2.09	1.92	1.88	1.82	1.78	1.75
46	4.05	3.20	2.81	2.57	2.42	2.30	2.22	2.14	2.09	2.04	2.00	1.97	1.91	1.87	1.80	1.75	1.71	1.65	1.62	1.57	1.54	1.51	1.48	1.46
	7.21	5.10	4.24	3.76	3.44	3.22	3.05	2.92	2.82	2.73	2.66	2.60	2.50	2.42	2.30	2.22	2.13	2.04	1.98	1.90	1.86	1.80	1.76	1.72
48	4.04	3.19	2.80	2.56	2.41	2.30	2.21	2.14	2.08	2.03	1.99	1.96	1.90	1.86	1.79	1.74	1.70	1.64	1.61	1.56	1.53	1.50	1.47	1.45
	7.19	5.08	4.22	3.74	3.42	3.20	3.04	2.90	2.80	2.71	2.64	2.58	2.48	2.40	2.28	2.20	2.11	2.02	1.96	1.88	1.84	1.78	1.73	1.70
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.02	1.98	1.95	1.90	1.85	1.78	1.74	1.69	1.63	1.60	1.55	1.52	1.48	1.46	1.44
	7.17	5.06	4.20	3.72	3.41	3.18	3.02	2.88	2.78	2.70	2.62	2.56	2.46	2.39	2.26	2.18	2.10	2.00	1.94	1.86	1.82	1.76	1.71	1.68
55	4.02	3.17	2.78	2.54	2.38	2.27	2.18	2.11	2.05	2.00	1.97	1.93	1.88	1.83	1.76	1.72	1.67	1.61	1.58	1.52	1.50	1.46	1.43	1.41
	7.12	5.01	4.16	3.68	3.37	3.15	2.98	2.85	2.75	2.66	2.59	2.53	2.43	2.35	2.23	2.15	2.06	1.96	1.90	1.82	1.78	1.71	1.66	1.64
60	4.00	3.15	2.76	2.52	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.86	1.81	1.75	1.70	1.65	1.59	1.56	1.50	1.48	1.44	1.41	1.39
	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.56	2.50	2.40	2.32	2.20	2.12	2.03	1.93	1.87	1.79	1.74	1.68	1.63	1.60
65	3.99	3.14	2.75	2.51	2.36	2.24	2.15	2.08	2.02	1.98	1.94	1.90	1.85	1.80	1.73	1.68	1.63	1.57	1.54	1.49	1.46	1.42	1.39	1.37
	7.04	4.95	4.10	3.62	3.31	3.09	2.93	2.79	2.70	2.61	2:54	2.47	2.37	2.30	2.18	2.09	2.00	1.90	1.84	1.76	1.71	1.64	1.60	1.56
70	3.98	3.13	2.74	2.50	2.35	2.32	2.14	2.07	2.01	1.97	1.93	1.89	1.84	1.79	1.72	1.67	1.62	1.56	1.53	1.47	1.45	1.40	1.37	1.35
	7.01	4.92	4.08	3.60	3.29	3.07	2.91	2.77	2.67	2.59	2.51	2.45	2.35	2.28	2.15	2.07	1.98	1.88	1.82	1.74	1.69	1.62	1.56	1.53
80	3.96	3.11	2.72	2.48	2.33	2.21	2.12	2.05	1.99	1.95	1.91	1.88	1.82	1.77	1.70	1.65	1.60	1.54	1.51	1.45	1.42	1.38	1.35	1.32
	6.96	4.88	4.04	3.56	3.25	3.04	2.87	2.74	2.64	2.55	2.48	2.41	2.32	2.24	2.11	2.03	1.94	1.84	1.78	1.70	1.65	1.57	1.52	1.49

0.05 (light row) and 0.01 (dark row) points for the distribution of F

Degrees of freedom for lesser mean square	Degrees of freedom for greater mean square: 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500	∞
100	3.94	3.09	2.70	2.46	2.30	2.19	2.10	2.03	1.97	1.92	1.88	1.85	1.79	1.75	1.68	1.63	1.57	1.51	1.48	1.42	1.39	1.34	1.30	1.28
	6.90	4.82	3.98	3.51	3.20	2.99	2.82	2.69	2.59	2.51	2.43	2.36	2.26	2.19	2.06	1.98	1.89	1.79	1.73	1.64	1.59	1.51	1.46	1.43
125	3.92	3.07	2.68	2.44	2.29	2.17	2.08	2.01	1.95	1.90	1.86	1.83	1.77	1.72	1.65	1.60	1.55	1.49	1.45	1.39	1.36	1.31	1.27	1.25
	6.84	4.78	3.94	3.47	3.17	2.95	2.79	2.65	2.56	2.47	2.40	2.33	2.23	2.15	2.03	1.94	1.85	1.75	1.68	1.59	1.54	1.46	1.40	1.37
150	3.91	3.06	2.67	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.85	1.82	1.76	1.71	1.64	1.59	1.54	1.47	1.44	1.37	1.34	1.29	1.25	1.22
	6.81	4.75	3.91	3.44	3.13	2.92	2.76	2.62	2.53	2.44	2.37	2.30	2.20	2.12	2.00	1.91	1.83	1.72	1.66	1.56	1.51	1.43	1.37	1.33
200	3.89	3.04	2.65	2.41	2.26	2.14	2.05	1.98	1.92	1.87	1.83	1.80	1.74	1.69	1.62	1.57	1.52	1.45	1.42	1.35	1.32	1.26	1.22	1.19
	6.76	4.71	3.38	3.41	3.11	2.90	2.73	2.60	2.50	2.41	2.34	2.28	1.17	2.09	1.97	1.88	1.79	1.69	1.62	1.53	1.48	1.39	1.33	1.28
400	3.86	3.02	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.81	1.78	1.72	1.67	1.60	1.54	1.49	1.42	1.38	1.32	1.28	1.22	1.16	1.13
	6.70	4.66	3.83	3.36	3.06	2.85	2.69	2.55	2.46	2.37	2.29	2.23	2.12	2.04	1.92	1.84	1.74	1.64	1.57	1.47	1.42	1.32	1.24	1.19
1000	3.85	3.00	2.61	2.38	2.22	2.10	2.02	1.95	1.89	1.84	1.80	1.76	1.70	1.65	1.58	1.53	1.47	1.41	1.36	1.30	1.26	1.19	1.13	1.08
	6.66	4.62	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.26	2.20	2.09	2.01	1.89	1.81	1.71	1.61	1.54	1.44	1.38	1.28	1.19	1.11
∞	3.84	2.99	2.60	2.37	2.21	2.09	2.01	1.94	1.88	1.83	1.79	1.75	1.69	1.64	1.57	1.52	1.46	1.40	1.35	1.28	1.24	1.17	1.11	1.00
	6.64	4.60	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.24	2.18	2.07	1.99	1.87	1.79	1.69	1.59	1.52	1.41	1.36	1.25	1.15	1.00

A-10

The obtained F is significant at a given level if it is equal to or greater than the value shown in the table.

0.05 (light row) and 0.01 (dark row) points for the distribution of F

Degrees of freedom for lesser mean square	Degrees of freedom for greater mean square: 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500
1	161	200	216	225	230	234	237	239	241	242	243	244	245	246	248	249	250	251	252	253	253	254	254
	4052	4999	5403	5625	5764	5859	5928	5981	6022	6056	6082	6106	6142	6169	6208	6234	6258	6286	6302	6323	6334	6352	6361
2	18.51	19.00	19.16	19.25	19.30	19.33	19.36	19.37	19.38	19.39	19.40	19.41	19.42	19.43	19.44	19.45	19.46	19.47	19.47	19.48	19.49	19.49	19.50
	98.49	99.01	99.17	99.25	99.30	99.33	99.34	99.36	99.38	99.40	99.41	99.42	99.43	99.44	99.45	99.46	99.47	99.48	99.48	99.49	99.49	99.49	99.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.88	8.84	8.81	8.78	8.76	8.74	8.71	8.69	8.66	8.64	8.62	8.60	8.58	8.57	8.56	8.54	8.54
	34.12	30.81	29.46	28.71	28.24	27.91	27.67	27.49	27.34	27.23	27.13	27.05	26.92	26.83	26.69	26.60	26.50	26.41	26.30	26.27	26.23	26.18	26.14
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.93	5.91	5.87	5.84	5.80	5.77	5.74	5.71	5.70	5.68	5.66	5.65	5.6
	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.54	14.45	14.37	14.24	14.15	14.02	13.93	13.83	13.74	13.69	13.61	13.57	13.52	13.4
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.78	4.74	4.70	4.68	4.64	4.60	4.56	4.53	4.50	4.46	4.44	4.42	4.40	4.38	4.3
	16.26	13.27	12.06	11.39	10.97	10.67	10.45	10.27	10.15	10.05	9.96	9.89	9.77	9.68	9.55	9.47	9.38	9.29	9.24	9.17	9.13	9.07	9.0
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.96	3.92	3.87	3.84	3.81	3.77	3.75	3.72	3.71	3.69	3.6
	13.74	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.79	7.72	7.60	7.52	7.39	7.31	7.23	7.14	7.09	7.02	6.99	6.94	6.9
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.63	3.60	3.57	3.52	3.49	3.44	3.41	3.38	3.34	3.32	3.29	3.28	3.25	3.2
	12.25	9.55	8.45	7.85	7.46	7.19	7.00	6.84	6.71	6.62	6.54	6.47	6.35	6.27	6.15	6.07	5.98	5.90	5.85	5.78	5.75	5.70	5.6
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.34	3.31	3.28	3.23	3.20	3.15	3.12	3.08	3.05	3.03	3.00	2.98	2.96	2.9
	11.26	8.65	7.59	7.01	6.63	6.37	6.19	6.03	5.91	5.82	5.74	5.67	5.56	5.48	5.36	5.28	5.20	5.11	5.06	5.00	4.96	4.91	4.8
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.13	3.10	3.07	3.02	2.98	2.93	2.90	2.86	2.82	2.80	2.77	2.76	2.73	2.7
	10.56	8.02	6.99	6.42	6.06	5.80	5.62	5.47	5.35	5.26	5.18	5.11	5.00	4.92	4.80	4.73	4.64	4.56	4.51	4.45	4.41	4.36	4.3
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.97	2.94	2.91	2.86	2.82	2.77	2.74	2.70	2.67	2.64	2.61	2.59	2.56	2.5
	10.04	7.56	6.55	5.99	5.64	5.39	5.21	5.06	4.95	4.85	4.78	4.71	4.60	4.52	4.41	4.33	4.25	4.17	4.12	4.05	4.01	3.96	3.9
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.86	2.82	2.79	2.74	2.70	2.65	2.61	2.57	2.53	2.50	2.47	2.45	2.42	2.4
	9.65	7.20	6.22	5.67	5.32	5.07	4.88	4.74	4.63	4.54	4.46	4.40	4.29	4.21	4.10	4.02	3.94	3.86	3.80	3.74	3.70	3.66	3.6
12	4.75	3.88	3.49	3.26	3.11	3.00	2.92	2.85	2.80	2.76	2.72	2.69	2.64	2.60	2.54	2.50	2.46	2.42	2.40	2.36	2.35	2.32	2.3
	9.33	6.93	5.95	5.41	5.06	4.82	4.65	4.50	4.39	4.30	4.22	4.16	4.05	3.98	3.86	3.78	3.70	3.61	3.56	3.49	3.46	3.41	3.3
13	4.67	3.80	3.41	3.18	3.02	2.92	2.84	2.77	2.72	2.67	2.63	2.60	2.55	2.51	2.46	2.42	2.38	2.34	2.32	2.28	2.26	2.24	2.
	9.07	6.70	5.74	5.20	4.86	4.62	4.44	4.30	4.19	4.10	4.02	3.96	3.85	3.78	3.67	3.59	3.51	3.42	3.37	3.30	3.27	3.21	3.
14	4.60	3.74	3.34	3.11	2.96	2.85	2.77	2.70	2.65	2.60	2.56	2.53	2.48	2.44	2.39	2.35	2.31	2.27	2.24	2.21	2.19	2.16	2.
	8.86	6.51	5.56	5.03	4.69	4.46	4.28	4.14	4.03	3.94	3.86	3.80	3.70	3.62	3.51	3.43	3.34	3.26	3.21	3.14	3.11	3.06	3.
15	4.54	3.68	3.29	3.06	2.90	2.79	2.70	2.64	2.59	2.55	2.51	2.48	2.43	2.39	2.33	2.29	2.25	2.21	2.18	2.15	2.12	2.10	2.
	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.73	3.67	3.56	3.48	3.36	3.29	3.20	3.12	3.07	3.00	2.97	2.92	2.

TABLE VII (*continued*)

A-11

0.05 (light row) and 0.01 (dark row) points for the distribution of F

Degrees of freedom for lesser mean square	Degrees of freedom for greater mean square: 1	2	3	4	5	6	7	8	9	10	11	12	14	16	20	24	30	40	50	75	100	200	500
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.45	2.42	2.37	2.33	2.28	2.24	2.20	2.16	2.13	2.09	2.07	2.04	2.02
	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.61	3.55	3.45	3.37	3.25	3.18	3.10	3.01	2.96	2.89	2.86	2.80	2.77
17	4.45	3.59	3.20	2.96	2.81	2.70	2.62	2.55	2.50	2.45	2.41	2.38	2.33	2.29	2.23	2.19	2.15	2.11	2.08	2.04	2.02	1.99	1.97
	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.52	3.45	3.35	3.27	3.16	3.08	3.00	2.92	2.86	2.79	2.76	2.70	2.67
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.29	2.25	2.19	2.15	2.11	2.07	2.04	2.00	1.98	1.95	1.93
	8.28	6.01	5.09	4.58	4.25	4.01	3.85	3.71	3.60	3.51	3.44	3.37	3.27	3.19	3.07	3.00	2.91	2.83	2.78	2.71	2.68	2.62	2.59
19	4.38	3.52	3.13	2.90	2.74	2.63	2.55	2.48	2.43	2.38	2.34	2.31	2.26	2.21	2.15	2.11	2.07	2.02	2.00	1.96	1.94	1.91	1.90
	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.36	3.30	3.19	3.12	3.00	2.92	2.84	2.76	2.70	2.63	2.60	2.54	2.51
20	4.35	3.49	3.10	2.87	2.71	2.60	2.52	2.45	2.40	2.35	2.31	2.28	2.23	2.18	2.12	2.08	2.04	1.99	1.96	1.92	1.90	1.87	1.85
	8.10	5.85	4.94	4.43	4.10	3.87	3.71	3.56	3.45	3.37	3.30	3.23	3.13	3.05	2.94	2.86	2.77	2.69	2.63	2.56	2.53	2.47	2.44
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.20	2.15	2.09	2.05	2.00	1.96	1.93	1.80	1.87	1.84	1.82
	8.02	5.78	4.87	4.37	4.04	3.81	3.65	3.51	3.40	3.31	3.24	3.17	3.07	2.99	2.88	2.80	2.72	2.63	2.58	2.51	2.47	2.42	2.38
22	4.30	3.44	3.05	2.82	2.66	2.55	2.47	2.40	2.35	2.30	2.26	2.23	2.18	2.13	2.07	2.03	1.98	1.93	1.91	1.87	1.84	1.81	1.80
	7.94	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.18	3.12	3.02	2.94	2.83	2.75	2.67	2.58	2.53	2.46	2.42	2.37	2.33
23	4.28	3.42	3.03	2.80	2.64	2.53	2.45	2.38	2.32	2.28	2.24	2.20	2.14	2.10	2.04	2.00	1.96	1.91	1.88	1.84	1.82	1.79	1.77
	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.14	3.07	2.97	2.89	2.78	2.70	2.62	2.53	2.48	2.41	2.37	2.32	2.28
24	4.26	3.40	3.01	2.78	2.62	2.51	2.43	2.36	2.30	2.26	2.22	2.18	2.13	2.09	2.02	1.98	1.94	1.89	1.86	1.82	1.80	1.76	1.74
	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.25	3.17	3.09	3.03	2.93	2.85	2.74	2.66	2.58	2.49	2.44	2.36	2.33	2.27	2.23
25	4.24	3.38	2.99	2.76	2.60	2.49	2.41	2.34	2.28	2.24	2.20	2.16	2.11	2.06	2.00	1.96	1.92	1.87	1.84	1.80	1.77	1.74	1.72
	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.21	3.13	3.05	2.99	2.89	2.81	2.70	2.62	2.54	2.45	2.40	2.32	2.29	2.23	2.19
26	4.22	3.37	2.89	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.18	2.15	2.10	2.05	1.99	1.95	1.90	1.85	1.82	1.78	1.76	1.72	1.70
	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.17	3.09	3.02	2.96	2.86	2.77	2.66	2.58	2.50	2.41	2.36	2.28	2.25	2.19	2.15
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.30	2.25	2.20	2.16	2.13	2.08	2.03	1.97	1.93	1.88	1.84	1.80	1.76	1.74	1.71	1.6
	7.68	5.49	4.60	4.11	3.79	3.56	3.39	3.26	3.14	3.06	2.98	2.93	2.83	2.74	2.63	2.55	2.47	2.38	2.33	2.25	2.21	2.16	2.1
28	4.20	3.34	2.95	2.71	2.56	2.44	2.36	2.29	3.24	2.19	2.15	2.12	2.06	2.02	1.96	1.91	1.87	1.81	1.78	1.75	1.72	1.69	1.6
	7.64	5.45	4.57	4.07	3.76	3.53	3.36	3.23	3.11	3.03	2.95	2.90	2.80	2.71	2.60	2.52	2.44	2.35	2.30	2.22	2.18	2.13	2.0
29	4.18	3.33	2.93	2.70	2.54	2.43	2.35	2.28	2.22	2.18	2.14	2.10	2.05	2.00	1.94	1.90	1.85	1.80	1.77	1.73	1.71	1.68	1.6
	7.60	5.52	4.54	4.04	3.73	3.50	3.32	3.20	3.08	3.00	2.92	2.87	2.77	2.68	2.57	2.49	2.41	2.32	2.27	2.19	2.15	2.10	2.0
30	4.17	3.32	2.92	2.69	2.53	2.42	2.34	2.27	2.21	2.16	2.12	2.09	2.04	1.99	1.93	1.89	1.84	1.79	1.76	1.72	1.69	1.66	1.6
	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.06	2.98	2.90	2.84	2.74	2.66	2.55	2.47	2.38	2.29	2.24	2.16	2.13	2.07	2.0

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TABLE IX Critical values of U

n_1	n_2	0.10	0.05	0.025	0.01	0.005	0.001
3	2	6	-	-	-	-	-
	3	8	9	-	-	-	-
4	2	8	-	-	-	-	-
	3	11	12	-	-	-	-
	4	13	15	16	-	-	-
5	2	9	10	-	-	-	-
	3	13	14	15	-	-	-
	4	16	18	19	20	-	-
	5	20	21	23	24	25	-
6	2	11	12	-	-	-	-
6	3	15	16	17	-	-	-
	4	19	21	22	23	24	-
	5	23	25	27	28	29	-
	6	27	29	31	33	34	-
7	2	13	14	-	-	-	-
7	3	17	19	20	21	-	-
	4	22	24	25	27	28	-
	5	27	29	30	32	34	-
	6	31	34	36	38	39	42
	7	36	38	41	43	45	48
8	2	14	15	16	-	-	-
	3	19	21	22	24	-	-
	4	25	27	28	30	31	-
	5	30	32	34	36	38	40
	6	35	38	40	42	44	47
8	7	40	43	44	49	50	54
	8	45	49	51	55	57	60
9	1	9	-	-	-	-	-
	2	16	17	18	-	-	-
	3	22	23	25	26	27	-
9	4	27	30	32	33	35	-
	5	33	36	38	40	42	44
	6	39	42	44	47	49	52
	7	45	48	51	54	56	60
	8	50	54	57	61	63	67
9	9	56	60	64	67	70	74
10	1	10	-	-	-	-	-
	2	17	19	20	-	-	-
	3	24	26	27	29	30	-
	4	30	33	35	37	38	40
10	5	37	39	42	44	46	49
	6	43	46	49	52	54	57
	7	49	53	56	59	61	65
	8	56	60	63	67	69	74
	9	62	66	70	74	77	82
10	10	68	73	77	81	84	90
11	1	11	-	-	-	-	-
	2	19	21	22	-	-	-
	3	26	28	30	32	33	-
	4	33	36	38	40	42	44
11	5	40	43	46	48	50	53
	6	47	50	53	57	59	62
	7	54	58	61	65	67	71
	8	61	65	69	73	75	80
	9	68	72	76	81	83	89
11	10	74	79	84	88	92	98
	11	81	87	91	96	100	106
12	1	12	-	-	-	-	-
	2	20	22	23	-	-	-
	3	28	31	32	34	35	-
12	4	36	39	41	43	45	48
	5	43	47	49	52	54	58
	6	51	55	58	61	63	68
	7	58	63	66	70	72	77
	8	66	70	74	79	81	87
12	9	73	78	82	87	90	96
	10	81	86	91	96	99	106
	11	88	94	99	104	108	115
	12	95	102	107	113	117	124
13	1	13	-	-	-	-	-
13	2	22	24	25	26	-	-
	3	30	33	35	37	38	-
	4	39	42	44	47	49	51
	5	47	50	53	56	58	62
	6	55	59	62	66	68	73
13	7	63	67	71	75	78	83
	8	71	76	80	84	87	93
	9	79	84	89	94	97	103
	10	87	93	97	103	106	113
	11	95	101	106	112	116	123

TABLE IX (continued)

n_1	n_2	0.10	0.05	0.025	0.01	0.005	0.001
13	12	103	109	115	121	125	133
	13	111	118	124	130	135	143
14	1	14	-	-	-	-	-
	2	24	25	27	28	-	-
	3	32	35	37	40	41	-
14	4	41	45	47	50	52	55
	5	50	54	57	60	63	67
	6	59	63	67	71	73	78
	7	67	72	76	81	83	89
	8	76	81	86	90	94	100
14	9	85	90	95	100	104	111
	10	93	99	104	110	114	121
	11	102	108	114	120	124	132
	12	110	117	123	130	134	143
	13	119	126	132	139	144	153
14	14	127	135	141	149	154	164
15	1	15	-	-	-	-	-
	2	25	27	29	30	-	-
	3	35	38	40	42	43	-
	4	44	48	50	53	55	59
15	5	53	57	61	64	67	71
	6	63	67	71	75	78	83
	7	72	77	81	86	89	95
	8	81	87	91	96	100	106
	9	90	96	101	107	111	118
15	10	99	106	111	117	121	129
	11	108	115	121	128	132	141
	12	117	125	131	138	143	152
	13	127	134	141	148	153	163
	14	136	144	151	159	164	174
15	15	145	153	161	169	174	185
16	1	16	-	-	-	-	-
	2	27	29	31	32	-	-
	3	37	40	42	45	46	-
	4	47	50	53	57	59	62
16	5	57	61	65	68	71	75
	6	67	71	75	80	83	88
	7	76	82	86	91	94	101
	8	86	92	97	102	106	113
	9	96	102	107	113	117	125

TABLE IX (continued)

n_1	n_2	0.10	0.05	0.025	0.01	0.005	0.001
16	10	106	112	118	124	129	137
	11	115	122	129	135	140	149
	12	125	132	139	146	151	161
	13	134	143	149	157	163	173
	14	144	153	160	168	174	185
16	15	154	163	170	179	185	197
	16	163	173	181	190	196	208
17	1	17	-	-	-	-	-
	2	28	31	32	34	-	-
	3	39	42	45	47	49	51
17	4	50	53	57	60	62	66
	5	60	65	68	72	75	80
	6	71	76	80	84	87	93
	7	81	86	91	96	100	106
	8	91	97	102	108	112	119
17	9	101	108	114	120	124	132
	10	112	119	125	132	136	145
	11	122	130	136	143	148	158
	12	132	140	147	155	160	170
	13	142	151	158	166	172	183
17	14	153	161	169	178	184	195
	15	163	172	180	189	195	208
	16	173	183	191	201	207	220
	17	183	193	202	212	219	232
18	1	18	-	-	-	-	-
	2	30	32	34	36	-	-
	3	41	45	47	50	52	54
	4	52	56	60	63	66	69
	5	63	68	72	76	79	84
	6	74	80	84	89	92	98
18	7	85	91	96	102	105	112
	8	96	103	108	114	118	126
	9	107	114	120	126	131	139
	10	118	125	132	139	143	153
	11	129	137	143	151	156	164
18	12	139	148	155	163	169	179
	13	150	159	167	175	181	192
	14	161	170	178	187	194	206
	15	172	182	190	200	206	219
	16	182	193	202	212	218	232

TABLE IX (continued)

n_1	n_2	0.10	0.05	0.025	0.01	0.005	0.001
18	17	193	204	213	224	231	245
	18	204	215	225	236	243	258
19	1	18	19	-	-	-	-
	2	31	34	36	37	38	-
	3	43	47	50	53	54	57
19	4	55	59	63	67	69	73
	5	67	72	76	80	83	88
	6	78	84	89	94	97	103
	7	90	96	101	107	111	118
	8	101	108	114	120	124	132
19	9	113	120	126	133	138	146
	10	124	132	138	146	151	161
	11	136	144	151	159	164	175
	12	147	156	163	172	177	188
	13	158	167	175	184	190	202
19	14	169	179	188	197	203	216
	15	181	191	200	210	216	230
	16	192	203	212	222	230	244
	17	203	214	224	235	242	257
	18	214	226	236	248	255	271
19	19	226	238	248	260	268	284
20	1	19	20	-	-	-	-
	2	33	36	38	39	40	-
	3	45	49	52	55	57	60
	4	58	62	66	70	72	77
20	5	70	75	80	84	87	93
	6	82	88	93	98	102	108
	7	94	101	106	112	116	124
	8	106	113	119	126	130	139
	9	118	126	132	140	144	154
20	10	130	138	145	153	158	168
	11	142	151	158	167	172	183
	12	154	163	171	180	186	198
	13	166	176	184	193	200	212
	14	178	188	197	207	213	226
20	15	190	200	210	220	227	241
	16	201	213	222	233	241	255
	17	213	225	235	247	254	270
	18	225	237	248	260	268	284
	19	237	250	261	273	281	298
	20	249	262	273	286	295	312

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Wilcoxon table

This table gives the significance probabilities for the Wilcoxon signed-rank test for paired comparisons, for various selected values of the test statistic W = sum of all signed ranks. The significance probabilities included in the table are the ones closest to the commonly used levels of significance $\alpha = .10$, $\alpha = .05$, and $\alpha = .01$. Thus the table may be used to obtain the appropriate critical value of W for a given value of α, the level of significance.

The critical values c in the table correspond to the critical value for a one-sided test which rejects for large values of W. If the test is one-sided, and rejects for small (negative) values of W, then the critical value is $-c$, where c is the value in the table for which $P(W \geq c)$ = desired level of significance. If the test is two-sided, then the critical value c is determined by finding the value in the table for which $P(W \geq c) = 1/2\ \alpha$, where α is the desired level of significance. In this case the test is to reject H_0 if $W \leq -c$ or $W \geq c$.

Examples

(*a*) The test is one-sided and rejects for large values of W. Suppose $\alpha = .05$ and $n = 8$. Then the critical value is $c = 24$, since $P(W \geq c) = .055$, and .055 is closest to the desired level $\alpha = .05$. Thus, the test rejects H_0 if $W \geq 24$, and accepts otherwise.

(*b*) The test is one-sided and rejects for small (negative) values of W. Suppose $\alpha = .10$ and $n = 12$. The critical value is -34, since $P(W \geq 34) = .102$, and .102 is the value closest to .10. Thus the test rejects H_0 if $W \leq -34$.

(*c*) The test is two-sided. Suppose $\alpha = .05$ and $n = 20$. Then the critical values are 106 and -106, since $P(W \geq 106) = .024$, and .024 is the value closest to .025 ($= 1/2\alpha$). Thus the test rejects H_0 if $W \leq -106$ or $W \geq 106$.

n	c	$P(W \geq c)$
1	1	.500
2	3	.250
3	6	.125
4	10	.062
	8	.125
5	15	.031
	13	.062
	11	.094
6	21	.016
	19	.031
	17	.047
	13	.109
7	28	.008
	24	.023
	20	.055
	16	.109

n	c	$P(W \geq c)$
8	32	.012
	28	.027
	24	.055
	20	.098
9	39	.010
	33	.027
	29	.049
	23	.102
10	45	.010
	39	.024
	33	.053
	27	.097
11	52	.009
	44	.027
	38	.051
	30	.013

n	c	$P(W \geq c)$
12	58	.010
	50	.026
	44	.046
	34	.102
13	65	.011
	57	.024
	49	.047
	39	.095
14	73	.010
	63	.025
	53	.052
	43	.097
15	80	.011
	70	.024
	60	.047
	46	.104

n	c	$P(W \geq c)$
16	88	.011
	76	.025
	64	.052
	52	.096
17	97	.010
	83	.025
	71	.049
	55	.103
18	105	.010
	91	.024
	77	.049
	61	.098
19	114	.010
	98	.025
	82	.052
	66	.098
20	124	.010
	106	.024
	90	.049
	70	.101

TABLE X Critical values of F_{max}

p a	2	3	4	5	6	7	8	9	10	11	12
2	39.0	87.5	142.	202.	266.	333.	403.	475.	550.	626.	704.
	199.	448.	729.	1036.	1362.	1705.	2063.	2432.	2813.	3204.	3605.
3	15.4	27.8	39.2	50.7	62.0	72.9	83.5	93.9	104.	114.	124.
	47.5	85.	120.	151.	184.	21(6)	24(9)	28(1)	31(0)	33(7)	36(1)
4	9.60	15.5	20.6	25.2	29.5	33.6	37.5	41.1	44.6	48.0	51.4
	23.2	37.	49.	59.	69.	79.	89.	97.	106.	113.	120.
5	7.15	10.8	13.7	16,3	18.7	20.8	22.9	24.7	26.5	28.2	29.9
	14.9	22.	28.	33.	38.	42.	46.	50.	54.	57.	60.
6	5.82	8.38	10.4	12.1	13.7	15.0	16.3	17.5	18.6	19.7	20.7
	11.1	15.5	19.1	22.	25.	27.	30.	32.	34.	36.	37.
7	4.99	6.94	8.44	9.70	10.8	11.8	12.7	13.5	14.3	15.1	15.8
	8.89	12.1	14.5	16.5	18.4	20.	22.	23.	24.	26.	27.
8	4.43	6.00	7.18	8.12	9.03	9.78	10.5	11.1	11.7	12.2	12.7
	7.50	9.9	11.7	13.2	14.5	15.8	16.9	17.9	18.9	19.8	21.
9	4.03	5.34	6.31	7.11	7.80	8.41	8.95	9.45	9.91	10.3	10.7
	6.54	8.5	9.9	11.1	12.1	13.1	13.9	14.7	15.3	16.0	16.6
10	3.72	4.85	5.67	6.34	6.92	7.42	7.87	8.28	8.66	9.01	9.34
	5.85	7.4	8.6	9.6	10.4	11.1	11.8	12.4	12.9	13.4	13.9
12	3.28	4.16	4.79	5.30	5.72	6.09	6.42	6.72	7.00	7.25	7.48
	4.91	6.1	6.9	7.6	8.2	8.7	9.1	9.5	9.9	10.2	10.6
15	2.86	3.54	4.01	4.37	4.68	4.95	5.19	5.40	5.59	5.77	5.93
	4.07	4.9	5.5	6.0	6.4	6.7	7.1	7.3	7.5	7.8	8.0
20	2.46	2.95	3.29	3.54	3.76	3.94	4.10	4.24	4.37	4.49	4.59
	3.32	3.8	4.3	4.6	4.9	5.1	5.3	5.5	5.6	5.8	5.9
30	2.07	2.40	2.61	2.78	2.91	3.02	3.12	3.21	3.29	3.36	3.39
	2.63	3.0	3.3	3.4	3.6	3.7	3.8	3.9	4.0	4.1	4.2
60	1.67	1.85	1.96	2.04	2.11	2.17	2.22	2.26	2.30	2.33	2.36
	1.96	2.2	2.3	2.4	2.4	2.5	2.5	2.6	2.6	2.7	2.7
∞	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

TABLE III Critical values of *t*

For any given df, the table shows the values of *t* corresponding to various levels of probability. Obtained *t* is significant at a given level if it is equal to or greater than the value shown in the table.

	Level of significance for one-tailed test					
	.10	.05	.025	.01	.005	.0005
	Level of significance for two-tailed test					
df	.20	.10	.05	.02	.01	.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.405
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.767
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

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