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Digital Image Compression and Decompression Using Three Different Transforms and Comparison of Their Performance

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Abstract

Data compression is an important tool in digital image processing to reduce the burden on the storage and transmission systems. The basic idea of data compression is to reduce the number of the image pixel elements directly, say by sampling, or by using transforms and truncate the transformed image coefficients, so that the total number of picture elements or its coefficients are reduced. The image information now requires lesser storage and also lesser band width for transmission. When ever the image is to be recovered or received after transmission the image information is to be decompressed i.e. brought back to the original size and form. This compression process is essential for images taken by satellites, or unmanned aerial vehicles (UAVs) for remote sensing and weather application. By applying compression algorithm the image data may take one fourth or even less size with out loss of much information. There are different methods for compression and decompression process. In this paper three methods are used for both data compression and decompression process, they are i. Discrete Hartley type transform, ii. Fast Fourier transform (FFT), iii. Discrete cosine transforms (DCT). Algorithms are developed and tested using the three methods on different images. A comparison with respect to mean square error with the original image also presented. The main advantage of discrete Hartley type transform is, it is a real transform. Discrete cosine transform also has similar performance as that of discrete Hartley type transform. An algorithm for compression using FFT method is also presented.

1 Introduction

Active research work is going on, in digital signal processing in recent years. This has good number of applications in remote sensing, medical imaging and telemetry. Digital image processing comprises of many sub areas, some of them are image enhancement, image restoration, image encoding, and data compression. Data compression play's an important role in image processing especially in remote sensing using remotely piloted vehicle (RPV). The image taken by the RPV in digital mode requires huge memory space and higher bandwidths for transmission to ground station. Each raw image in digital form will take few MB (mega bits) hence the transmission and storage in the same form will occupy more bandwidth and larger storage space. Data compression before transmission reduces the channel band width requirement and memory space [1] to [8]. There are different transforms available for data compression. These are Karhumen – Loeve transform, discrete cosine transform, and sampling and interpolation process using fast Fourier transform. [1] to [16].

In this paper Discrete Hartley type transform and Fast Fourier transform are used for data compression in two dimensions, which is directly applicable to digital image processing of RPV photographed images. The performance is compared with the existing discrete cosine transform. The Hartley transform (HT) was developed as a substitute to Fourier transform (FT) in applications where the data is in real domain. [10], [11]. The HT was also defined in two dimensions in which kernel is not separable [12]. Another modified separable Hartley type transform, named the CAS-CAS transform (CCT) was also defined. CAS stands for cos plus sine. And CCT for two dimensional cos plus sine transform. Algorithms are developed for data compression and decompression using the three transforms namely CCT, FFT, and DCT in two dimensions. Comparison of compressed and decompressed image with the original image with respect to mean square error is carried out.

2. CCT Method

Discrete Hartley type transform in two dimensions as separable transform is the CAS-CAS transform and is called as CCT.

Hartley transform relations in 2-D in which the kernel is not separable are [12].

$$H(u,v) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x(m,n) cas 2\pi\left(\frac{mu}{M}+\frac{nv}{N}\right) \tag{1}$$

And reverse transform is

$$X(m,n) = \frac{1}{M \times N} \sum_{u=0}^{M-1}\sum_{v-0}^{N-1} H(u,v) cas 2\pi (\frac{mu}{M} + \frac{nv}{N}) \quad (2)$$

Cas $2\pi(\frac{mu}{M} + \frac{nv}{N})$ is not a separable Kernel.

But for 2-D DFT the kernel is

Exp $[j2\pi(\frac{mu}{M} + \frac{nv}{N})]$ which is separable. Hence the row-column decomposition method [14] of computing 2-D transform from 1-D fast Fourier transform algorithm can be used. In case of 2-D Hartley transform row column can not be applied directly. To take advantage of the fast 1-D HT algorithms, a separable Hartley like transform namely CAS-CAS transform that is CCT is shown.

T (u, v), is defined as [12]

$$T(u,v) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x(m,n) cas \frac{2\pi mu}{M} cas \frac{2\pi nv}{N} \quad (3)$$

$$X(m,n) = \frac{1}{M \times N} \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} T(u,v) cas \frac{2\pi mu}{M} cas \frac{2\pi nv}{N} \quad (4)$$

Where

Cas (α) = cos (α) + sin (α) and

$$H(u,v) = \frac{1}{2} [T(u,v) + T(-u,v) + T(u,-v,) - T(-u,-v)] \quad (5)$$

In this way 2-D Discrete Hartley transform can be computed. For the compression and decompression process only CCT is used instead of Hartley transform.

2.1 The Algorithmic Steps for Compression and De-Compression Using CCT

1. Scan the analog Image to get digital image with a size of pixels (256 x 256) as reference data matrix A. Instead of particular size of the digital image data, general size of the original image matrix A is considered as (M, N).

2. Compute CCT of this matrix A using equation (3) as Matrix B which is of same size as A. Important observation is that the CCT coefficient matrix is having larger amplitudes at the four corners and the value diminishes, negligibly small towards central region

3. Based on the above principle, modify the CCT coefficients matrix B to reduce its size to (M/2, N/2), to achieve a compression factor of 4 by neglecting small amplitude coefficients.

Construct a new matrix C (u, v) from B (u, v) which is reduced in size as follows

C (u, v) = B (u, v) for u = 0, 1 … (M/4) -1
v = 0, 1 … (N/4)-1

= B (u, v+N/2) for u = 0, 1… (M/4)-1
v = (N/4)… (N/2)-1

= B (u+M/2, v) for u =M/4… (M/2)-1
v = 0, 1… N/4

= B (u+M/2, v+N/2) for u =M/4… (M/2)-1
v =N/4…. (N/2)-1

C (u, v) is matrix of size (M/2, N/2), a compression ratio of 4 with respect to original size of the image matrix A of size (M, N) is achieved.

4. This reduced CCT coefficient matrix is to be transmitted instead of the original matrix, so that memory space is saved for storage and burden on the transmission channel is reduced by a factor of 4.

5. At the receiver, after receiving the modified CCT matrix C (u, v), is appended with zeros at respective regions, to make it to reference image size as follows.

D (u, v) = C (u, v) for u = 0, 1… (M/4)-1
v = 0, 1… (N/4)-1

= 0 for u = 0, 1… (M/4)-1
v = N/4 … (3N/4) -1

= C (u, v-N/2) for u = 0, 1… (M/4)-1
v = 3N/4… (N-1)

= 0 for u = M/4 … (3M/4)-1
v = 0, 1, 2 …. (N-1)

= C (u-M/2, v) for u =3M/4 … (M-1)
v = 0, 1… (N/4)-1

=0 for u =3M/4… (M-1)
v = N/4 … (3N/4)-1

= C (u-M/2, v-N/2) for u = 3M/4… (M-1)

v = 3N/4…. (N-1)

6. Multiply each D (u, v) by the square of the compression factor.

7. Perform the inverse CCT using equation (4) on the sequence D (u, v) of size (M, N) to obtain the replica of original image A.

3. Fast Fourier Transform Method

The Fourier transform relations in 2 –D are

$$X(u,v) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x(m,n) e^{-j2\pi(\frac{mu}{M}+\frac{nv}{N})} \quad (6)$$

Where m = 0, 1 … M-1, and n = 0, 1… N-1

$$X(m,n) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} X(u,v) e^{j2\pi(\frac{mu}{M}+\frac{nv}{N})} \quad (7)$$

Where u = 0, 1 … M-1, and v = 0, 1 … N-1

Equation (6) is forward transform and equation (7) is reverse transform. By using the above two equations Fourier transform coefficients and inverse Fourier transform coefficients can be computed for the digital image.

3.1. The Algorithmic Steps for Compression and Decompression Using FFT

The procedure is similar to that of Hartley type transform for data compression and decompression using Fast Fourier transform. The algorithmic steps given above for Hartley transform, are to be used for this also by replacing CCT with FFT. FFT magnitude coefficients are also concentrated at the corners as that of CCT.

4. Discrete Cosine Transform Method

The discrete cosine transform relations in 2 –D are

X (u, v) =

$$\alpha_u \alpha_v \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x(m,n) \cos\frac{\pi(2m+1)u}{2M} \cos\frac{\pi(2n+1)v}{2N} \quad (8)$$

Where m = 0, 1…. M-1; and n = 0, 1… N-1

$$\alpha_u = \frac{1}{\sqrt{M}} \quad \text{For } u = 0$$

$$\alpha_u = \sqrt{\frac{2}{M}} \quad \text{For } u = 1 \ldots M-1$$

$$\alpha_v = \frac{1}{\sqrt{M}} \quad \text{For } v = 0$$

$$\alpha_v = \sqrt{\frac{2}{M}} \quad \text{For } v = 1 \ldots M-1$$

And 2-D inverse discrete cosine Transform is

X (m, n) =

$$\alpha_u \alpha_v \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} X(u,v) \cos\frac{\pi(2m+1)u}{2M} \cos\frac{\pi(2n+1)v}{2N} \quad (9)$$

Where u= 0, 1 … M-1, and v= 0, 1 … N-1

Equation (8) is forward transform and equation (9) is reverse transform. By using the above two equations discrete cosine transform coefficients and inverse discrete cosine transform coefficients can be computed for the digital image.

4.1. The Algorithmic Steps for Compression and Decompression Using DCT

1. Scan the analog Image to get digital image with a size of pixels (256 x 256) as reference data matrix A. Instead of particular size of the digital image data, general size of the original image matrix A is considered as (M, N).

2. Compute DCT of this matrix A using equation (8) as Matrix B which is of same size as A. The DCT coefficient matrix is having larger magnitudes near the origin and is

diminishing as the (m, n) increases.

3. Based on this principle truncate the Matrix B keeping the DCT coefficients for m=0 to (M/2)-1, and n = 0 to (N/2)-1 so that the size of the matrix formulated C is (M/2, N/2) a compression ratio of 4 is achieved.

4. The matrix c is transmitted and received. Now decompression process is applied by appending the C matrix zeros to get back the size of the original matrix A.

5. Compute inverse discrete cosine transform on this modified matrix using equation (9) to get back the replica of the original digital image.

5. Procedure

5.1. CCT Method

The above algorithmic steps referred in chapter 2.1 are applied on the original scanned digital image of size (256,256) pixels shown as figure 1. For this image data CCT coefficients are computed. Then data size is reduced by discarding some of the coefficients by using the algorithmic step (3). Most of CCT coefficients are having larger magnitudes in all the four corners of the matrix instead of central region of the matrix. Based on this principle the discarding of small value data points is achieved. To bring back to the original size matrix, zeros are appended where ever the data points are discarded. Now inverse CCT is applied on this data with a proper multiplier. The resultant replica of the original image is shown as figure 2. These two images figure 1 and figure 2 are looks alike, and there is no noticeable loss of information. In this a reduction of data points to be transmitted is 1/4th of that of original image.

A second example, with a compression ratio of 16 for the same image is applied and the resultant image is shown as figure 3. Figure 3 is not up to the mark, and further compression will completely distorts the image. The advantage of CCT is that it is a real transform and hence the storage and transmission is not complex and FFT algorithm of row column decomposition can be applied.

5. 2. FFT Method

For the same data (figure 1), equation (6) is applied to get the Fourier coefficients. As in the CCT method some of the coefficients are discarded by following same algorithmic steps as in CCT method. The discarded coefficients are substituted with zeros, and with proper multiplier inverse transform equation (7), are applied to get the replica of the original image as shown figure 4. Figures 1 and figure 4 are similar with out appreciable differences. Hence there is a compression ratio of 4 is achieved with out distortion. The only difference in this method in comparison with CCT is that the data to be transmitted is complex instead of real and hence effective reduction is 50 %. As the compression ratio is increased further, decreasing the data points to be transmitted, the recovered image is distorted as shown in figure 5 with a compression ratio of 16. Figure 6 shows the distribution of magnitudes of Fourier coefficients for the figure 1 data points. It shows that the magnitudes are less in the middle region, which are discarded and in the reverse transform they are assumed to be zeros.

5. 3. DCT Method

For the same data (figure 1), equation (8) is applied to get the DCT coefficients. DCT coefficients are concentrated near the origin. To achieve the compression ratio of 4 the DCT coefficient matrix is truncated by retaining (M/2, N/2) from the origin.

Now zeros are appended to get back the original size of the matrix and with proper multiplier inverse transform equation (9), is applied to get back the replica of the original image. Hence there is a compression ratio of 4 is achieved with out distortion. As the compression ratio is increased further say 16, by decreasing the data points to be transmitted, the recovered image is distorted as shown in figure 7.

6. Results and Conclusions

Three methods are tried for different compression factors. The mean square error estimation of resultant images with respect to the original image data (figure 1) are presented in table 1.

Table 1. Mean square error with the three methods for different data compression ratios.

Serial No	Compression factor	CCT Method Mean Square error	FFT Method Mean Square Error	DCT Method Mean Square Error
1	4	0.0019	0.0018	.0019
2	16	0.0050	0.0048	.0050

The mean square error for all the methods is almost the same for different compression factors. It shows that CCT method can be used for image compression process as that of FFT method. The advantage of CCT method over FFT method is that the CCT is a real transform and hence the number of multiplications and additions are less than that of FFT and storage also reduces to half that of FFT. The band width required for transmission also reduces with CCT instead of FFT. Hence digital image data compression and decompression process can be effectively implemented using CCT. DCT method also gives similar performance as that of CCT. Either DCT or CCT can be implemented for any practical system.

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Fgure 1. Original Image

Fig. 2 CCT method
Compression factor of 4

Fig. 3 CCT method compression
Factor of 16

Fig.4 FFT method Compression factor of 4

Fig. 5 FFT method compression factor of 16

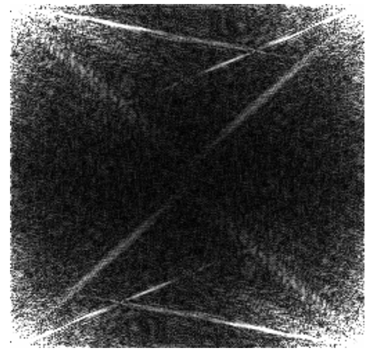

Fig. 6 FFT coefficients magnitudes of Fig. 1

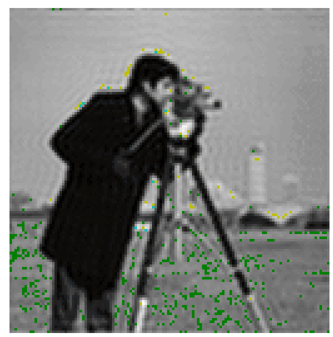

Fig. 7 DCT method compression factor of 16