
UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2008/2009

November 2008

**EEE 512 – ADVANCED DIGITAL SIGNAL AND IMAGE
PROCESSING**

Duration: 3 hours

Please check that this examination paper consists of EIGHT pages of printed material before you begin the examination.

This paper contains SIX questions.

Instructions: Answer FIVE (5) questions.

Answer to any question must start on a new page.

Distribution of marks for each question is given accordingly

All questions must be answered in English.

1. (a) Consider the linear constant-coefficient difference equation

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n-1].$$

Determine $y[n]$ for $n \geq 0$ when $x[n] = \delta[n]$ and $y[n] = 0, n < 0$.

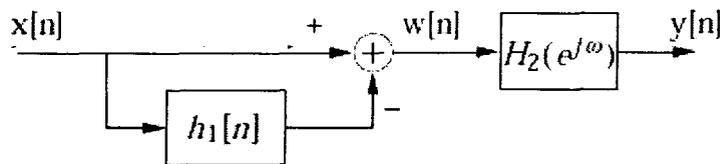
(30 marks)

- (b) Write a difference equation that characterises a system whose frequency response is

$$H(e^{j\omega}) = \frac{1 - \frac{1}{2}e^{-j\omega} + e^{-j3\omega}}{1 + \frac{1}{2}e^{-j\omega} + \frac{3}{4}e^{-j2\omega}}.$$

(30 marks)

- (c) Consider the following interconnection of LTI systems:



where $h_1[n] = \delta[n-1]$ and

$$H_2(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \pi/2 \\ 0 & \pi/2 < |\omega| \leq \pi. \end{cases}$$

Find the frequency response and the unit impulse response of the system.

(40 marks)

2. (a) Determine the transfer function of the digital filter structure of Figure 2.

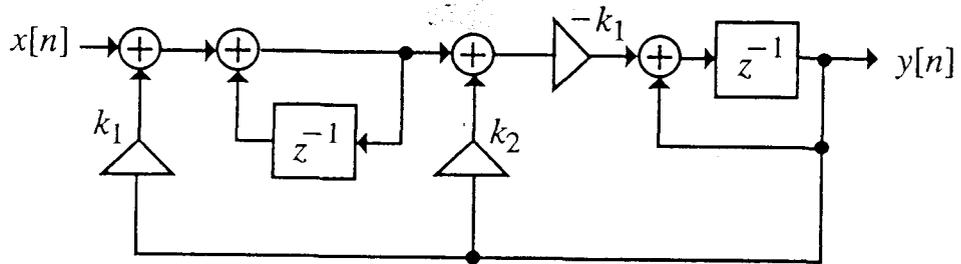


Figure 2

(30 marks)

- (b) Realize the FIR transfer function

$$H(z) = (1 - 0.6z^{-1})^6 = 1 - 3.6z^{-1} + 5.4z^{-2} - 4.32z^{-3} + 1.944z^{-4} - 0.4666z^{-5} + 0.0467z^{-6}$$

in the following forms:

- (i) two different direct forms
- (ii) cascade of six first-order sections
- (iii) cascade of three second-order sections
- (iv) cascade of two third-order sections
- (v) cascade of two second-order sections and two first-order sections

Compare the computational complexity of each of the above realizations.

(70 marks)

3. (a) An IIR low-pass filter is required, based on a Butterworth prototype. The specifications are:

Cut-off frequency	: 4 kHz	Pass-band ripple:	3 dB
Stop-band frequency	: 6 kHz	Stop-band attenuation:	60 dB
Sampling frequency	: 20 kHz		

The bilinear z-transform method of design is to be used.
Calculate the order of the necessary prototype filter.

(40 marks)

- (b) A band-pass digital IIR Filter, based on a prototype Butterworth 1st order filter, having a transfer function $H(s) = 1 / (s + 1)$, is to be designed using the bilinear z-transform. The required parameters are:

Pass-band range: 800 – 1200 Hz
Sampling frequency: 8 kHz

Calculate the pulse transfer function of the required digital filter.

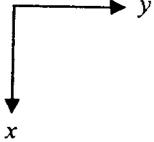
[Low-pass to band-pass transformation is:

$$s = (s^2 + \omega_U \omega_L) / (s(\omega_U - \omega_L))$$

where ω_U and ω_L are the pass band edge frequencies in rad/s]

(60 marks)

4. (a) The filter function is formed by taking the differences in x and y directions and adding the result such as

$$g(x, y) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$


- (i) obtain the filter transfer function $H(u, v)$ in frequency domain,
(30 marks)
- (ii) show that $H(u, v)$ is a high pass filter.
(20 marks)
- (b) The convolution theorem of two dimensional variables $f(x, y)$ and $h(x, y)$ is given by:

$$f(x, y) \otimes h(x, y) = F(u, v)H(u, v)$$

where $F(u, v)$ and $H(u, v)$ are two dimensional Fourier transform of $f(x, y)$ and $h(x, y)$ respectively. Prove the validity of this theorem.

(50 marks)

Given:

$$F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi \left(\frac{ux}{M} + \frac{vy}{N} \right)}$$

$$\Im f(x - x_0, y - y_0) = F(u, v) e^{-j2\pi \left(\frac{ux_0}{M} + \frac{vy_0}{N} \right)}$$

$$2j \sin x = e^{jx} - e^{-jx}$$

$$2 \cos x = e^{jx} + e^{-jx}$$

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5. (a) Briefly explain some unique properties of Haar transform. Describe its main drawback.

(40 marks)

- (b) Consider the 8×8 image as follows:

$$f(x, y) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- (i) construct the 8×8 Haar matrix, (30 marks)
- (ii) perform Haar transformation of $f(x, y)$, (10 marks)
- (iii) reconstruct $f(x, y)$ using the first three basis images, (10 marks)
- (iv) calculate the sum square error of the reconstruction. (10 marks)

Given:

Haar functions are defined as:

$$H_0(t) = 1 ; 0 \leq t \leq 1$$

$$H_1(t) = \begin{cases} 1 & ; 0 \leq t < \frac{1}{2} \\ -1 & ; \frac{1}{2} \leq t < 1 \end{cases}$$

$$H_{2^p+n}(t) = \begin{cases} \sqrt{2^p} & ; \frac{n}{2^p} \leq t < \frac{n+0.5}{2^p} \\ -\sqrt{2^p} & ; \frac{n+0.5}{2^p} \leq t < \frac{n+1}{2^p} \\ 0 & ; \text{elsewhere} \end{cases}$$

$$p = 1, 2, 3, \dots$$

6. (a) The dilation of A by B is defined as

$$A \oplus B = \{z | (B)_z \cap A \neq \emptyset\}$$

Show that this definition is equivalent to

$$A \oplus B = \{w \in Z^2 | w = a + b, \text{ for some } a \in A \text{ and } b \in B\}$$

(40 marks)

- (b) Referring to Figure 6(b), the initial image A consists of all the image components shown in white and set B is the structuring element. Assuming that B is just large enough to enclose each of the noise components, sketch the output image of the following morphological operations

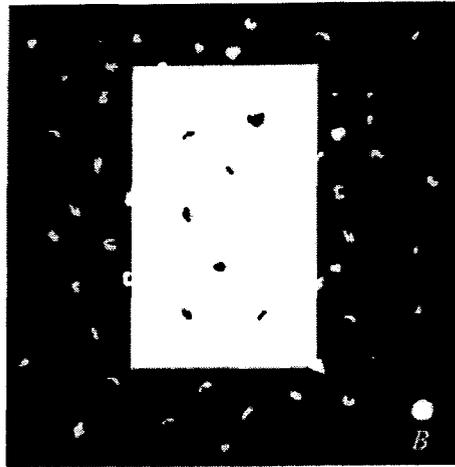


Figure 6(b)

- (i) $C = A \ominus B$, (15 marks)
- (ii) $D = C \oplus B$, (15 marks)
- (iii) $E = D \oplus B$, and (15 marks)
- (iv) $F = E \ominus B$ (15 marks)

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