UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 2008/2009

November 2008

EEE228 – ISYARAT DAN SISTEM

Masa: 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN muka surat dan DUA BELAS muka surat LAMPIRAN yang bercetak sebelum anda memulakan peperiksaan ini.

Kertas soalan ini mengandungi ENAM soalan.

Jawab LIMA soalan.

Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru.

Agihan markah bagi setiap soalan diberikan di sudut sebelah kanan soalan berkenaan.

Jawab semua soalan dalam bahasa Malaysia atau bahasa Inggeris atau kombinasi kedua-duanya.

1. (a) Tentukan jika sistem yang digambarkan oleh
Determine if the system described by

$$y(t) = \ln[x(t)]$$

adalah
is

(i) mempunyai ingatan atau tiada ingatan,
with memory or memoryless,

(ii) boleh disongsang atau tidak boleh disongsang,
invertible or non-invertible,

(iii) kausal atau tidak kausal,
causal or non-causal,

(iv) boleh berubah mengikut masa atau tidak boleh berubah,
time-invariant or time-varying,

(v) lelurus atau tidak lelurus.
linear or non-linear.

(20%)

(b) Diberi $x(t) = 5u(t+2) - u(t) + 3u(t-2) - 7u(t-4)$. Cari dan lakarkan $x(-2t-4)$.

Given $x(t) = 5u(t+2) - u(t) + 3u(t-2) - 7u(t-4)$. *Find and sketch* $x(-2t-4)$.

(40%)

(c) Tentukan samada isyarat di bawah adalah isyarat tenaga atau kuasa dan carikan tenaga atau kuasa isyarat tersebut.

Determine whether the following signal is a power or an energy signal and find the energy or power of the signal.

$$x(t) = 10\sin(5t)\cos(10t)$$

(20%)

(d) Carikan bahagian genap dan ganjil bagi isyarat
Find the even and odd parts of the signal

$$x(t) = 2\cos t + 4\sin t + 5\sin t\cos t$$

(20%)

2. (a) Bagi isyarat diskret-masa
For the discrete-time signal

$$x[n] = \left\{-1, \frac{1}{2}, 0, 1, -\frac{3}{2}, -1\right\}$$

lakarkan $x(0.5n+1)$
sketch $x(0.5n+1)$

(40%)

(b) Carikan penjumlahan pelingkaran $y[n] = h[n] * x[n]$ bagi pasangan jujukan terhingga berikut:

Find the convolution summation $y[n] = h[n] * x[n]$ *for the following pair of finite sequences:*

$$x[n] = \{1,2,3,0,-1\},\ h[n] = \{2,-1,3,1,-2\}$$

(40%)

(c) Nyatakan tiga sifat pelingkaran.
State three properties of convolution.

(20%)

3. Pertimbangkan suatu sistem diskret-masa, LTI yang digambarkan oleh persamaan kebezaan.

Consider a discrete-time, LTI system described by the difference equation.

$$y[n]-0.7y[n-1]=2.5[n]-x[n-1]$$

(a) Lukiskan gambarajah blok bagi sistem ini.
Draw the block diagram of this system.

(25%)

(b) Tentukan rangkap langkah h[n], $0 \le n \le 4$, bagi sistem ini.
Determine the impulse response h[n], $0 \le n \le 4$, for the system.

(25%)

(c) Katakan masukan kepada sistem ini diberikan oleh
Suppose that the system input is given by

$$x[n]=\begin{cases} 1 & , \quad n=-2 \\ -3 & , \quad n=0 \\ 2 & , \quad n=1 \end{cases}$$

dan x[n] adalah sifar bagi semua nilai n yang lain. Nyatakan keluaran y[n] dalam sebutan h[n].

and x[n] is zero for all other values of n. Express the output y[n] as a function of h[n].

(25%)

(d) Kirakan keluaran y[n] bagi n = -3,-1 dan 1 untuk x[n] di bahagian (c) dan menggunakan keputusan di bahagian (b).

Calculate the output y[n] for n = -3,-1 and 1 for x[n] in part (c) and using the results of part (b).

(25%)

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4. (a) Jelaskan secara ringkas,
Explain briefly,

(i) Proses modulasi
Modulation process

(ii) Transmisi tanpa herotan
Distortionless transmission

(iii) Teori persampelan
The sampling theorem

(25%)

(b) Dengan menggunakan definisi jelmaan Fourier, cari FT bagi,
Using the definition of the Fourier transform, find the FT of,

(i) $\delta(t)$

(ii) $\cos(\omega_0 t)$

(25%)

(c) Tentukan kadar persampelan Nyquist dan julat persampelan Nyquist bagi isyarat $0.01\ sinc^2\left(100\pi t\right)$.

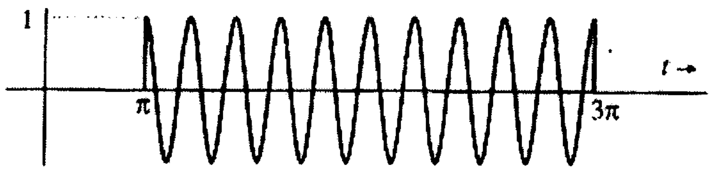
Determine the Nyquist sampling rate and the Nyquist sampling interval for the signal of $0.01\ sinc^2\left(100\pi t\right)$.

(25%)

(d) Isyarat dalam Rajah 1 merupakan isyarat yang telah dimodulat dengan pembawa cos 10t. Cari Jelmaan Fourier bagi isyarat tersebut dengan menggunakan ciri-ciri Jelmaan Fourier yang sesuai.

The signal in Figure 1 is a modulated signal with carrier cos 10t. Find the Fourier transform of this signal using appropriate properties of the Fourier transform.

Hint: $f(t)\cos\omega_0 t \Leftrightarrow \frac{1}{2}\left[F(\omega-\omega_0)+F(\omega+\omega_0)\right]$



Rajah 1
Figure 1

(25%)

5. (a) (i) Cari sambutan keadaan kosong bagi sistem LTID dengan fungsi pindah.

Find the zero-state response of an LTID system with transfer function.

$$H[z]=\frac{z}{(z+0.2)(z-0.8)}$$

dengan masukan

the input $f[k]=e^{(k+1)}u[k]$

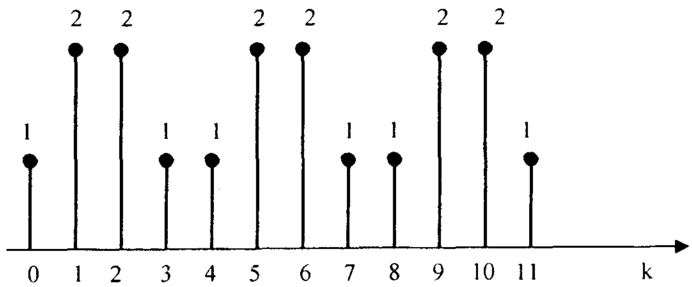
(ii) Nyatakan persamaan pembezaan yang mengaitkan keluaran y[k] dengan masukan f[k].

Write the difference equation relating the output y[k] to input f[k].

(40%)

(b) Kirakan 4-titik DFT dan IDFT bagi gelombang yang ditunjukkan dalam Rajah 2.

Compute the 4-point DFT and IDFT for the waveform shown in Figure 2.



Rajah 2
Figure 2

Diberi:

Given that:

$$F_r = \sum_{k=0}^{N_0-1} f[k] e^{-j\Omega_0 rk} \qquad , \Omega_0 = \frac{2\pi}{N_0}$$

$$f[k] = \frac{1}{N_0} \sum_{r=0}^{N_0-1} F_r e^{j\Omega_0 rk}$$

(60%)

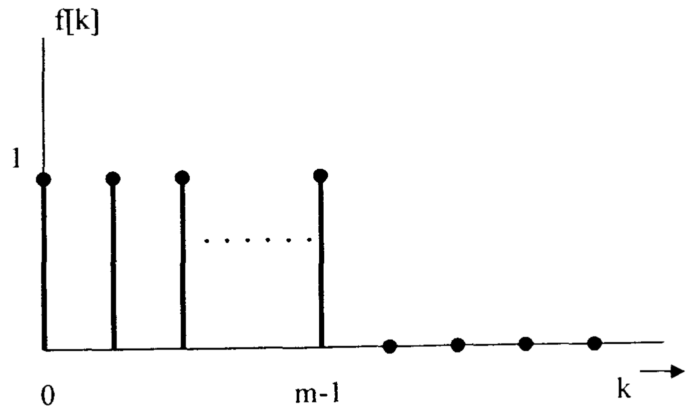
6. (a) Bagi isyarat masa diskret dalam Rajah 3, tunjukkan bahawa,

For a discrete-time signal shown in Figure 3, show that,

$$F[z] = \frac{1-z^{-m}}{1-z^{-1}}$$

(20%)

...8/-



Rajah 3
Figure 3

(b) Diberi fungsi seperti berikut,
Given the function following,

$$X[z] = \frac{2 + z^{-1}}{1 - \frac{1}{2} z^{-1}} \qquad \text{with ROC } |z| > ½$$

(i) Cari jelmaan-z songsang fungsi tersebut dengan menggunakan pengembangan siri kuasa.
Find the inverse z-transform of the function using a power series expansion.

(ii) Dapatkan empat ungkapan pertama bagi x[n].
Find the first four terms of x[n].

(30%)

(c) Selesaikan persamaan pembezaan berikut:
Solve the difference equation:

$$y[k+2] - 3y[k+1] + 2y[k] = f[k+1]$$

Diberi keadaan awalan,
Given the initial condition,

$y[-1]=2$ and $y[-2] = 3$, $f[k]= (3)^k u[k]$

(50%)

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Lampiran

Appendix

A Short Table of Fourier Transforms

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Fourier Transform Operations

Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$kf(t)$	$kF(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling (a real)	$f(at)$	$\frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Frequency shift (ω_0 real)	$f(t)e^{j\omega_0 t}$	$F(\omega - \omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi}F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(x)\,dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

Lampiran

Appendix

Convolution Table

No	$f_1(t)$	$f_2(t)$	$f_1(t) * f_2(t) = f_2(t) * f_1(t)$
1	$f(t)$	$\delta(t-T)$	$f(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\dfrac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\dfrac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^n u(t)$	$e^{\lambda t}u(t)$	$\dfrac{n!\,e^{\lambda t}}{\lambda^{n+1}}u(t) - \sum_{j=o}^{n}\dfrac{n!\,t^{n-j}}{\lambda^{j+1}(n-j)!}u(t)$
8	$t^m u(t)$	$t^n u(t)$	$\dfrac{m!n!}{(m+n+1)!}t^{m+n+1}u(t)$
9	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$
10	$t^m e^{\lambda t}u(t)$	$t^n e^{\lambda t}u(t)$	$\dfrac{m!\,n!}{(n+m+1)!}t^{m+n+1}e^{\lambda t}u(t)$
11	$t^m e^{\lambda_1 t}u(t)$	$t^n e^{\lambda_2 t}u(t)$	$\sum_{j=0}^{m}\dfrac{(-1)^j m!(n+j)!\,t^{m-j}e^{\lambda_1 t}}{j!(m-j)!(\lambda_1-\lambda_2)^{n+j+1}}u(t)$
	$\lambda_1 \neq \lambda_2$		$+\sum_{k=0}^{n}\dfrac{(-1)^k n!(m+k)!\,t^{n-k}e^{\lambda_2 t}}{k!(n-k)!(\lambda_2-\lambda_1)^{m+k+1}}u(t)$
12	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\dfrac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$
			$\phi = \tan^{-1}[-\beta/(\alpha+\lambda)]$
13	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \quad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$
14	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$

Convolution Sums

No.	$f_1[k]$	$f_2[k]$	$f_1[k] * f_2[k] = f_2[k] * f_1[k]$	
1	$\delta[k-j]$	$f[k]$	$f[k-j]$	
2	$\gamma^k u[k]$	$u[k]$	$\left[\frac{1-\gamma^{k+1}}{1-\gamma}\right] u[k]$	
3	$u[k]$	$u[k]$	$(k+1)u[k]$	
4	$\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\left[\frac{\gamma_1^{k+1}-\gamma_2^{k+1}}{\gamma_1-\gamma_2}\right] u[k] \quad \gamma_1 \neq \gamma_2$	
5	$\gamma_1^k u[k]$	$\gamma_2^k u[-(k+1)]$	$\frac{\gamma_1}{\gamma_2-\gamma_1}\gamma_1^k u[k] + \frac{\gamma_2}{\gamma_2-\gamma_1}\gamma_2^k u[-(k+1)]$	$\lvert\gamma_2\rvert > \lvert\gamma_1\rvert$
6	$k\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\frac{\gamma_1\gamma_2}{(\gamma_1-\gamma_2)^2}\left[\gamma_2^k - \gamma_1^k + \frac{\gamma_1-\gamma_2}{\gamma_2} k\gamma_1^k\right] u[k]$	$\gamma_1 \neq \gamma_2$
7	$ku[k]$	$ku[k]$	$\frac{1}{6}k(k-1)(k+1)u[k]$	
8	$\gamma^k u[k]$	$\gamma^k u[k]$	$(k+1)\gamma^k u[k]$	
9	$\gamma^k u[k]$	$ku[k]$	$\left[\frac{\gamma(\gamma^k-1)+k(1-\gamma)}{(1-\gamma)^2}\right] u[k]$	
10	$\lvert\gamma_1\rvert^k \cos(\beta k+\theta)u[k]$	$\gamma_2^k u[k]$	$\frac{1}{R}\left[\lvert\gamma_1\rvert^{k+1}\cos[\beta(k+1)+\theta-\phi] - \gamma_2^{k+1}\cos(\theta-\phi)\right] u[k]$	γ_2 real
			$R = \left[\lvert\gamma_1\rvert^2 + \gamma_2^2 - 2\lvert\gamma_1\rvert\gamma_2\cos\beta\right]^{1/2}$	
			$\phi = \tan^{-1}\left[\frac{(\lvert\gamma_1\rvert\sin\beta)}{(\lvert\gamma_1\rvert\cos\beta-\gamma_2)}\right]$	

$\mathcal{Z}$- Transform Operations

Operation	$f[k]$	$F[z]$
Addition	$f_1[k] + f_2[k]$	$F_1[z] + F_2[z]$
Scalar multiplication	$af[k]$	$aF[z]$
Right-shift	$f[k-m]u[k-m]$	$\frac{1}{z^m}F[z]$
	$f[k-m]u[k]$	$\frac{1}{z^m}F[z] + \frac{1}{z^m}\sum_{k=1}^{m} f[-k]z^k$
	$f[k-1]u[k]$	$\frac{1}{z}F[z] + f[-1]$
	$f[k-2]u[k]$	$\frac{1}{z^2}F[z] + \frac{1}{z}f[-1] + f[-2]$
	$f[k-3]u[k]$	$\frac{1}{z^3}F[z] + \frac{1}{z^2}f[-1] + \frac{1}{z}f[-2] + f[-3]$
Left-shift	$f[k+m]u[k]$	$z^mF[z] - z^m\sum_{k=0}^{m-1} f[k]z^{-k}$
	$f[k+1]u[k]$	$zF[z] - zf[0]$
	$f[k+2]u[k]$	$z^2F[z] - z^2f[0] - zf[1]$
	$f[k+3]u[k]$	$z^3F[z] - z^3f[0] - z^2f[1] - zf[2]$
Multiplication by γ^k	$\gamma^k f[k]u[k]$	$F\left[\frac{z}{\gamma}\right]$
Multiplication by k	$kf[k]u[k]$	$-z\frac{d}{dz}F[z]$
Time Convolution	$f_1[k] * f_2[k]$	$F_1[z]F_2[z]$
Frequency Convolution	$f_1[k]f_2[k]$	$\frac{1}{2\pi j}\oint F_1[u]F_2\left[\frac{z}{u}\right]u^{-1}\,du$
Initial value	$f[0]$	$\lim_{z\to\infty} F[z]$
Final value	$\lim_{N\to\infty} f[N]$	$\lim_{z\to 1}(z-1)F[z]$ poles of $(z-1)F[z]$ inside the unit circle.

(Unilateral) z-Transform Pairs

	$f[k]$	$F[z]$
1	$\delta[k-j]$	z^{-j}
2	$u[k]$	$\frac{z}{z-1}$
3	$ku[k]$	$\frac{z}{(z-1)^2}$
4	$k^2u[k]$	$\frac{z(z+1)}{(z-1)^3}$
5	$k^3u[k]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^{k-1}u[k-1]$	$\frac{1}{z-\gamma}$
7	$\gamma^k u[k]$	$\frac{z}{z-\gamma}$
8	$k\gamma^k u[k]$	$\frac{\gamma z}{(z-\gamma)^2}$
9	$k^2\gamma^k u[k]$	$\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\frac{k(k-1)(k-2)\cdots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$	$\frac{z}{(z-\gamma)^{m+1}}$
11a	$\|\gamma\|^k \cos\beta k\, u[k]$	$\frac{z(z-\|\gamma\|\cos\beta)}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
11b	$\|\gamma\|^k \sin\beta k\, u[k]$	$\frac{z\|\gamma\|\sin\beta}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
12a	$r\|\gamma\|^k\cos(\beta k+\theta)u[k]$	$\frac{rz[z\cos\theta-\|\gamma\|\cos(\beta-\theta)]}{z^2-(2\|\gamma\|\cos\beta)z+\|\gamma\|^2}$
12b	$r\|\gamma\|^k\cos(\beta k+\theta)u[k]$ $\quad \gamma=\|\gamma\|e^{j\beta}$	$\frac{(0.5re^{j\theta})z}{z-\gamma}+\frac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\|\gamma\|^k\cos(\beta k+\theta)u[k]$ $r=\sqrt{\frac{A^2\|\gamma\|^2+B^2-2AaB}{\|\gamma\|^2-a^2}}$ $\beta=\cos^{-1}\frac{-a}{\|\gamma\|},\ \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{\|\gamma\|^2-a^2}}$	$\frac{z(Az+B)}{z^2+2az+\|\gamma\|^2}$

The Laplace Transform Properties

Operation	$f(t)$	$F(s)$
Addition	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
Scalar multiplication	$kf(t)$	$kF(s)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - \dot{f}(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - s\dot{f}(0^-) - \ddot{f}(0^-)$
Time integration	$\int_{0^-}^{t} f(\tau)\,d\tau$	$\frac{1}{s}F(s)$
	$\int_{-\infty}^{t} f(\tau)\,d\tau$	$\frac{1}{s}F(s) + \frac{1}{s}\int_{-\infty}^{0^-} f(t)\,dt$
Time shift	$f(t-t_0)u(t-t_0)$	$F(s)e^{-st_0}$ $t_0 \geq 0$
Frequency shift	$f(t)e^{s_0t}$	$F(s-s_0)$
Frequency differentiation	$-tf(t)$	$\frac{dF(s)}{ds}$
Frequency integration	$\frac{f(t)}{t}$	$\int_{s}^{\infty} F(z)\,dz$
Scaling	$f(at),\ a \geq 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi j}F_1(s) * F_2(s)$
Initial value	$f(0^+)$	$\lim_{s\to\infty} sF(s)$ $(n > m)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$ (poles of $sF(s)$ in LHP)

B.7 Miscellaneous

B.7-1 L'Hôpital's Rule

If $\lim f(x)/g(x)$ results in the indeterministic form $0/0$ or ∞/∞, then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{\dot{f}(x)}{\dot{g}(x)}$$

B.7-2 The Taylor and Maclaurin Series

$$f(x) = f(a) + \frac{(x-a)}{1!}\dot{f}(a) + \frac{(x-a)^2}{2!}\ddot{f}(a) + \cdots$$

$$f(x) = f(0) + \frac{x}{1!}\dot{f}(0) + \frac{x^2}{2!}\ddot{f}(0) + \cdots$$

B.7-3 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \binom{n}{k}x^k + \cdots + x^n$$

$$\approx 1 + nx \qquad |x| \ll 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$

B.7-4 Sums

$$\sum_{m=0}^{k} r^m = \frac{r^{k+1} - 1}{r - 1} \qquad r \neq 1$$

$$\sum_{m=M}^{N} r^m = \frac{r^{N+1} - r^M}{r - 1} \qquad r \neq 1$$

$$\sum_{m=0}^{k} \left(\frac{a}{b}\right)^m = \frac{a^{k+1} - b^{k+1}}{b^k(a-b)} \qquad a \neq b$$

B.7-5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$a + jb = re^{j\theta} \qquad r = \sqrt{a^2+b^2}, \qquad \theta = \tan^{-1}\left(\tfrac{b}{a}\right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1e^{j\theta_1})(r_2e^{j\theta_2}) = r_1r_2e^{j(\theta_1+\theta_2)}$$

B.7-6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j\sin x$$

$$\cos x = \tfrac{1}{2}[e^{jx} + e^{-jx}]$$

$$\sin x = \tfrac{1}{2j}[e^{jx} - e^{-jx}]$$

$$\cos\left(x \pm \tfrac{\pi}{2}\right) = \mp\sin x$$

$$\sin\left(x \pm \tfrac{\pi}{2}\right) = \pm\cos x$$

$$2\sin x\cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \tfrac{1}{4}(3\cos x + \cos 3x)$$

$$\sin^3 x = \tfrac{1}{4}(3\sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$

$$\sin x\sin y = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)]$$

$$\cos x\cos y = \tfrac{1}{2}[\cos(x-y) + \cos(x+y)]$$

$$\sin x\cos y = \tfrac{1}{2}[\sin(x-y) + \sin(x+y)]$$

$$a\cos x + b\sin x = C\cos(x+\theta)$$

in which $C = \sqrt{a^2+b^2}$ and $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$

B.7-7 Indefinite Integrals

$$\int u\,dv = uv - \int v\,du$$

$$\int f(x)\dot{g}(x)\,dx = f(x)g(x) - \int \dot{f}(x)g(x)\,dx$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x\sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2\cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

$$\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \sin ax\cos bx\,dx = -\left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)}\right] \qquad a^2 \neq b^2$$

$$\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln(x^2+a^2)$$

B.7-8 Differentiation Table

$\frac{d}{dx}f(u) = \frac{d}{du}f(u)\frac{du}{dx}$	$\frac{d}{dx}a^{bx} = b(\ln a)a^{bx}$
$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	$\frac{d}{dx}\sin ax = a\cos ax$
$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}\cos ax = -a\sin ax$
$\frac{dx^n}{dx} = nx^{n-1}$	$\frac{d}{dx}\tan ax = \frac{a}{\cos^2 ax}$
$\frac{d}{dx}\ln(ax) = \frac{1}{x}$	$\frac{d}{dx}(\sin^{-1} ax) = \frac{a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx}\log(ax) = \frac{\log e}{x}$	$\frac{d}{dx}(\cos^{-1} ax) = \frac{-a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx}e^{bx} = be^{bx}$	$\frac{d}{dx}(\tan^{-1} ax) = \frac{a}{1+a^2x^2}$

B.7-9 Some Useful Constants

$\pi \approx 3.1415926535$

$e \approx 2.7182818284$

$\frac{1}{e} \approx 0.3678794411$

$\log_{10} 2 = 0.30103$

$\log_{10} 3 = 0.47712$

B.7-10 Solution of Quadratic and Cubic Equations

Any quadratic equation can be reduced to the form

$$ax^2 + bx + c = 0$$

The solution of this equation is provided by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A general cubic equation

$$y^3 + py^2 + qy + r = 0$$

may be reduced to the depressed cubic form

$$x^3 + ax + b = 0$$

by substituting

$$y = x - \frac{p}{3}$$

This yields

$$a = \frac{1}{3}(3q - p^2) \qquad b = \frac{1}{27}(2p^3 - 9pq + 27r)$$

Now let

$$A = \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}, \qquad B = \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}$$

The solution of the depressed cubic is

$$x = A + B, \qquad x = -\frac{A+B}{2} + \frac{A-B}{2}\sqrt{-3}, \qquad x = -\frac{A+B}{2} - \frac{A-B}{2}\sqrt{-3}$$

and

$$y = x - \frac{p}{3}$$

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