UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 2008/2009

November 2008

EEE228 – ISYARAT DAN SISTEM

Masa: 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN muka surat dan DUA BELAS muka surat LAMPIRAN yang bercetak sebelum anda memulakan peperiksaan ini.

Kertas soalan ini mengandungi ENAM soalan.

Jawab LIMA soalan.

Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru.

Agihan markah bagi setiap soalan diberikan di sudut sebelah kanan soalan berkenaan.

Jawab semua soalan dalam bahasa Malaysia atau bahasa Inggeris atau kombinasi kedua-duanya.

(c) Tentukan samada isyarat di bawah adalah isyarat tenaga atau kuasa dan carikan tenaga atau kuasa isyarat tersebut.

Determine whether the following signal is a power or an energy signal and find the energy or power of the signal.

$$x(t) = 10\sin(5t)\cos(10t)$$

(20%)

(d) Carikan bahagian genap dan ganjil bagi isyarat

Find the even and odd parts of the signal

$$x(t) = 2\cos t + 4\sin t + 5\sin t\cos t$$

(20%)

2. (a) Bagi isyarat diskret-masa

For the discrete-time signal

$$x[n] = \left\{-1, \frac{1}{2}, 0, 1, -\frac{3}{2}, -1\right\}$$

lakarkan $x(0.5n+1)$

sketch $x(0.5n+1)$

(40%)

(b) Carikan penjumlahan pelingkaran $y[n] = h[n] * x[n]$ bagi pasangan jujukan terhingga berikut:

Find the convolution summation $y[n] = h[n] * x[n]$ *for the following pair of finite sequences:*

$$x[n] = \{1,2,3,0,-1\},\ h[n] = \{2,-1,3,1,-2\}$$

(40%)

(c) Nyatakan tiga sifat pelingkaran.

State three properties of convolution.

(20%)

...4/-

3. Pertimbangkan suatu sistem diskret-masa, LTI yang digambarkan oleh persamaan kebezaan.

Consider a discrete-time, LTI system described by the difference equation.

$$y[n]-0.7y[n-1]=2.5[n]-x[n-1]$$

(a) Lukiskan gambarajah blok bagi sistem ini.
Draw the block diagram of this system.

(25%)

(b) Tentukan rangkap langkah h[n], $0 \le n \le 4$, bagi sistem ini.
Determine the impulse response h[n], $0 \le n \le 4$, for the system.

(25%)

(c) Katakan masukan kepada sistem ini diberikan oleh
Suppose that the system input is given by

$$x[n]=\begin{cases} 1 & , \quad n=-2 \\ -3 & , \quad n=0 \\ 2 & , \quad n=1 \end{cases}$$

dan x[n] adalah sifar bagi semua nilai n yang lain. Nyatakan keluaran y[n] dalam sebutan h[n].

and x[n] is zero for all other values of n. Express the output y[n] as a function of h[n].

(25%)

(d) Kirakan keluaran y[n] bagi n = -3,-1 dan 1 untuk x[n] di bahagian (c) dan menggunakan keputusan di bahagian (b).

Calculate the output y[n] for n = -3,-1 and 1 for x[n] in part (c) and using the results of part (b).

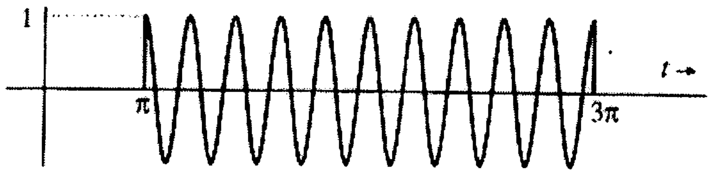
(25%)

...5/-

(d) Isyarat dalam Rajah 1 merupakan isyarat yang telah dimodulat dengan pembawa $\cos 10t$. Cari Jelmaan Fourier bagi isyarat tersebut dengan menggunakan ciri-ciri Jelmaan Fourier yang sesuai.

The signal in Figure 1 is a modulated signal with carrier $\cos 10t$. Find the Fourier transform of this signal using appropriate properties of the Fourier transform.

Hint: $f(t)\cos\omega_0 t \Leftrightarrow \frac{1}{2}\left[F(\omega-\omega_0)+F(\omega+\omega_0)\right]$



Rajah 1
Figure 1

(25%)

5. (a) (i) Cari sambutan keadaan kosong bagi sistem LTID dengan fungsi pindah.

Find the zero-state response of an LTID system with transfer function.

$$H[z]=\frac{z}{(z+0.2)(z-0.8)}$$

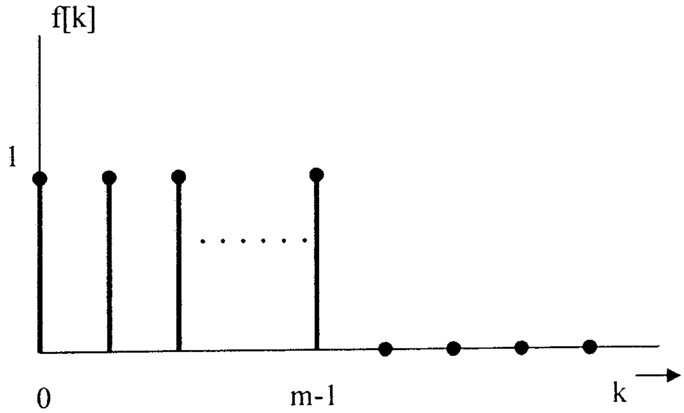
dengan masukan

the input $f[k]=e^{(k+1)}u[k]$

(ii) Nyatakan persamaan pembezaan yang mengaitkan keluaran y[k] dengan masukan f[k].

Write the difference equation relating the output y[k] to input f[k].

(40%)



Rajah 3
Figure 3

(b) Diberi fungsi seperti berikut,
Given the function following,

$$X[z]=\frac{2+z^{-1}}{1-\frac{1}{2}z^{-1}} \qquad \text{with ROC } |z| > ½$$

(i) Cari jelmaan-z songsang fungsi tersebut dengan menggunakan pengembangan siri kuasa.
Find the inverse z-transform of the function using a power series expansion.

(ii) Dapatkan empat ungkapan pertama bagi x[n].
Find the first four terms of x[n].

(30%)

(c) Selesaikan persamaan pembezaan berikut:
Solve the difference equation:

$$y[k+2]-3y[k+1]+2y[k]=f[k+1]$$

Diberi keadaan awalan,
Given the initial condition,

$y[-1]=2$ and $y[-2] = 3$, $f[k]= (3)^k u[k]$

(50%)

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Lampiran

Appendix

A Short Table of Fourier Transforms

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a > 0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a > 0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a > 0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a > 0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Appendix

Convolution Table

No	$f_1(t)$	$f_2(t)$	$f_1(t) * f_2(t) = f_2(t) * f_1(t)$
1	$f(t)$	$\delta(t-T)$	$f(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\frac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\frac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^n u(t)$	$e^{\lambda t}u(t)$	$\frac{n!\,e^{\lambda t}}{\lambda^{n+1}}u(t) - \sum_{j=o}^{n}\frac{n!\,t^{n-j}}{\lambda^{j+1}(n-j)!}u(t)$
8	$t^m u(t)$	$t^n u(t)$	$\frac{m!n!}{(m+n+1)!}t^{m+n+1}u(t)$
9	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\frac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$
10	$t^m e^{\lambda t}u(t)$	$t^n e^{\lambda t}u(t)$	$\frac{m!\,n!}{(n+m+1)!}t^{m+n+1}e^{\lambda t}u(t)$
11	$t^m e^{\lambda_1 t}u(t)$	$t^n e^{\lambda_2 t}u(t)$	$\sum_{j=0}^{m}\frac{(-1)^j m!(n+j)!\,t^{m-j}e^{\lambda_1 t}}{j!(m-j)!(\lambda_1-\lambda_2)^{n+j+1}}u(t)$
	$\lambda_1 \neq \lambda_2$		$+\sum_{k=0}^{n}\frac{(-1)^k n!(m+k)!\,t^{n-k}e^{\lambda_2 t}}{k!(n-k)!(\lambda_2-\lambda_1)^{m+k+1}}u(t)$
12	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\frac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$
			$\phi = \tan^{-1}[-\beta/(\alpha+\lambda)]$
13	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \quad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$
14	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$

Convolution Sums

No.	$f_1[k]$	$f_2[k]$	$f_1[k] * f_2[k] = f_2[k] * f_1[k]$	
1	$\delta[k-j]$	$f[k]$	$f[k-j]$	
2	$\gamma^k u[k]$	$u[k]$	$\left[\frac{1-\gamma^{k+1}}{1-\gamma}\right]u[k]$	
3	$u[k]$	$u[k]$	$(k+1)u[k]$	
4	$\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\left[\frac{\gamma_1^{k+1}-\gamma_2^{k+1}}{\gamma_1-\gamma_2}\right]u[k] \quad \gamma_1 \neq \gamma_2$	
5	$\gamma_1^k u[k]$	$\gamma_2^k u[-(k+1)]$	$\frac{\gamma_1}{\gamma_2-\gamma_1}\gamma_1^k u[k] + \frac{\gamma_2}{\gamma_2-\gamma_1}\gamma_2^k u[-(k+1)]$	$\|\gamma_2\| > \|\gamma_1\|$
6	$k\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\frac{\gamma_1\gamma_2}{(\gamma_1-\gamma_2)^2}\left[\gamma_2^k - \gamma_1^k + \frac{\gamma_1-\gamma_2}{\gamma_2}k\gamma_1^k\right]u[k]$	$\gamma_1 \neq \gamma_2$
7	$ku[k]$	$ku[k]$	$\frac{1}{6}k(k-1)(k+1)u[k]$	
8	$\gamma^k u[k]$	$\gamma^k u[k]$	$(k+1)\gamma^k u[k]$	
9	$\gamma^k u[k]$	$ku[k]$	$\left[\frac{\gamma(\gamma^k-1)+k(1-\gamma)}{(1-\gamma)^2}\right]u[k]$	
10	$\|\gamma_1\|^k \cos(\beta k+\theta)u[k]$	$\gamma_2^k u[k]$	$\frac{1}{R}\left[\|\gamma_1\|^{k+1}\cos[\beta(k+1)+\theta-\phi] - \gamma_2^{k+1}\cos(\theta-\phi)\right]u[k]$	γ_2 real
			$R = \left[\|\gamma_1\|^2 + \gamma_2^2 - 2\|\gamma_1\|\gamma_2\cos\beta\right]^{1/2}$	
			$\phi = \tan^{-1}\left[\frac{(\|\gamma_1\|\sin\beta)}{(\|\gamma_1\|\cos\beta - \gamma_2)}\right]$	

$\mathcal{Z}$- Transform Operations

Operation	$f[k]$	$F[z]$
Addition	$f_1[k]+f_2[k]$	$F_1[z]+F_2[z]$
Scalar multiplication	$af[k]$	$aF[z]$
Right-shift	$f[k-m]u[k-m]$	$\frac{1}{z^m}F[z]$
	$f[k-m]u[k]$	$\frac{1}{z^m}F[z]+\frac{1}{z^m}\sum_{k=1}^{m}f[-k]z^k$
	$f[k-1]u[k]$	$\frac{1}{z}F[z]+f[-1]$
	$f[k-2]u[k]$	$\frac{1}{z^2}F[z]+\frac{1}{z}f[-1]+f[-2]$
	$f[k-3]u[k]$	$\frac{1}{z^3}F[z]+\frac{1}{z^2}f[-1]+\frac{1}{z}f[-2]+f[-3]$
Left-shift	$f[k+m]u[k]$	$z^mF[z]-z^m\sum_{k=0}^{m-1}f[k]z^{-k}$
	$f[k+1]u[k]$	$zF[z]-zf[0]$
	$f[k+2]u[k]$	$z^2F[z]-z^2f[0]-zf[1]$
	$f[k+3]u[k]$	$z^3F[z]-z^3f[0]-z^2f[1]-zf[2]$
Multiplication by γ^k	$\gamma^k f[k]u[k]$	$F\left[\frac{z}{\gamma}\right]$
Multiplication by k	$kf[k]u[k]$	$-z\frac{d}{dz}F[z]$
Time Convolution	$f_1[k]*f_2[k]$	$F_1[z]F_2[z]$
Frequency Convolution	$f_1[k]f_2[k]$	$\frac{1}{2\pi j}\oint F_1[u]F_2\left[\frac{z}{u}\right]u^{-1}\,du$
Initial value	$f[0]$	$\lim_{z\to\infty}F[z]$
Final value	$\lim_{N\to\infty}f[N]$	$\lim_{z\to 1}(z-1)F[z]$ poles of $(z-1)F[z]$ inside the unit circle.

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Appendix

(Unilateral) z-Transform Pairs

	$f[k]$	$F[z]$
1	$\delta[k-j]$	z^{-j}
2	$u[k]$	$\frac{z}{z-1}$
3	$ku[k]$	$\frac{z}{(z-1)^2}$
4	$k^2u[k]$	$\frac{z(z+1)}{(z-1)^3}$
5	$k^3u[k]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^{k-1}u[k-1]$	$\frac{1}{z-\gamma}$
7	$\gamma^k u[k]$	$\frac{z}{z-\gamma}$
8	$k\gamma^k u[k]$	$\frac{\gamma z}{(z-\gamma)^2}$
9	$k^2\gamma^k u[k]$	$\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\frac{k(k-1)(k-2)\cdots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$	$\frac{z}{(z-\gamma)^{m+1}}$
11a	$\lvert\gamma\rvert^k \cos\beta k\, u[k]$	$\frac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
11b	$\lvert\gamma\rvert^k \sin\beta k\, u[k]$	$\frac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12a	$r\lvert\gamma\rvert^k\cos(\beta k+\theta)u[k]$	$\frac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12b	$r\lvert\gamma\rvert^k\cos(\beta k+\theta)u[k]$ $\quad \gamma=\lvert\gamma\rvert e^{j\beta}$	$\frac{(0.5re^{j\theta})z}{z-\gamma}+\frac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\lvert\gamma\rvert^k\cos(\beta k+\theta)u[k]$ $r=\sqrt{\frac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$ $\beta=\cos^{-1}\frac{-a}{\lvert\gamma\rvert},\ \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$	$\frac{z(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$

The Laplace Transform Properties

Operation	$f(t)$	$F(s)$
Addition	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
Scalar multiplication	$kf(t)$	$kF(s)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - \dot{f}(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - s\dot{f}(0^-) - \ddot{f}(0^-)$
Time integration	$\int_{0^-}^{t} f(\tau)\,d\tau$	$\frac{1}{s}F(s)$
	$\int_{-\infty}^{t} f(\tau)\,d\tau$	$\frac{1}{s}F(s) + \frac{1}{s}\int_{-\infty}^{0^-} f(t)\,dt$
Time shift	$f(t-t_0)u(t-t_0)$	$F(s)e^{-st_0} \quad t_0 \geq 0$
Frequency shift	$f(t)e^{s_0t}$	$F(s-s_0)$
Frequency differentiation	$-tf(t)$	$\frac{dF(s)}{ds}$
Frequency integration	$\frac{f(t)}{t}$	$\int_{s}^{\infty} F(z)\,dz$
Scaling	$f(at),\ a \geq 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi j}F_1(s) * F_2(s)$
Initial value	$f(0^+)$	$\lim_{s\to\infty} sF(s) \quad (n > m)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$ (poles of $sF(s)$ in LHP)

B.7 Miscellaneous

B.7-1 L'Hôpital's Rule

If $\lim f(x)/g(x)$ results in the indeterministic form 0/0 or ∞/∞, then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{\dot{f}(x)}{\dot{g}(x)}$$

B.7-2 The Taylor and Maclaurin Series

$$f(x) = f(a) + \frac{(x-a)}{1!}\dot{f}(a) + \frac{(x-a)^2}{2!}\ddot{f}(a) + \cdots$$

$$f(x) = f(0) + \frac{x}{1!}\dot{f}(0) + \frac{x^2}{2!}\ddot{f}(0) + \cdots$$

B.7-3 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \binom{n}{k}x^k + \cdots + x^n$$

$$\approx 1 + nx \qquad |x| \ll 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$

B.7-4 Sums

$$\sum_{m=0}^{k} r^m = \frac{r^{k+1}-1}{r-1} \qquad r \neq 1$$

$$\sum_{m=M}^{N} r^m = \frac{r^{N+1}-r^M}{r-1} \qquad r \neq 1$$

$$\sum_{m=0}^{k} \left(\frac{a}{b}\right)^m = \frac{a^{k+1}-b^{k+1}}{b^k(a-b)} \qquad a \neq b$$

B.7-7 Indefinite Integrals

$$\int u\,dv = uv - \int v\,du$$

$$\int f(x)\dot{g}(x)\,dx = f(x)g(x) - \int \dot{f}(x)g(x)\,dx$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x\sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2\cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

$$\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \sin ax\cos bx\,dx = -\left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)}\right] \qquad a^2 \neq b^2$$

$$\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$$

$$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln(x^2+a^2)$$

B.7-8 Differentiation Table

$\frac{d}{dx}f(u) = \frac{d}{du}f(u)\frac{du}{dx}$	$\frac{d}{dx}a^{bx} = b(\ln a)a^{bx}$
$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	$\frac{d}{dx}\sin ax = a\cos ax$
$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}\cos ax = -a\sin ax$
$\frac{dx^n}{dx} = nx^{n-1}$	$\frac{d}{dx}\tan ax = \frac{a}{\cos^2 ax}$
$\frac{d}{dx}\ln(ax) = \frac{1}{x}$	$\frac{d}{dx}(\sin^{-1} ax) = \frac{a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx}\log(ax) = \frac{\log e}{x}$	$\frac{d}{dx}(\cos^{-1} ax) = \frac{-a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx}e^{bx} = be^{bx}$	$\frac{d}{dx}(\tan^{-1} ax) = \frac{a}{1+a^2x^2}$

B.7-9 Some Useful Constants

$\pi \approx 3.1415926535$

$e \approx 2.7182818284$

$\frac{1}{e} \approx 0.3678794411$

$\log_{10} 2 = 0.30103$

$\log_{10} 3 = 0.47712$

B.7-10 Solution of Quadratic and Cubic Equations

Any quadratic equation can be reduced to the form

$$ax^2 + bx + c = 0$$

The solution of this equation is provided by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Lampiran

Appendix

A general **cubic equation**

$$y^3 + py^2 + qy + r = 0$$

may be reduced to the **depressed cubic** form

$$x^3 + ax + b = 0$$

by substituting

$$y = x - \frac{p}{3}$$

This yields

$$a = \frac{1}{3}(3q - p^2) \qquad b = \frac{1}{27}(2p^3 - 9pq + 27r)$$

Now let

$$A = \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}, \qquad B = \sqrt[3]{-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}$$

The solution of the depressed cubic is

$$x = A + B, \qquad x = -\frac{A+B}{2} + \frac{A-B}{2}\sqrt{-3}, \qquad x = -\frac{A+B}{2} - \frac{A-B}{2}\sqrt{-3}$$

and

$$y = x - \frac{p}{3}$$

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