TERJEMAHAN

UNIVERSITI SAINS MALAYSIA

KSCP Examination
1999/2000 Academic Session

April 2000

ZCT 304/3 - Keelektrikan dan Kemagnetan

Time : [3 hours]

Please check that the examination paper consists of **TEN** printed pages before you commence this examination.

Answer **SIX** questions only. Candidates may choose to answer the questions in the Malay Language. If candidates choose to answer questions in the English Language, at least one question must be answered in the Malay Language.

1. (a) Write down Maxwell's vacuum equations in differential form for the case where charge and current densities are present.

(20/100)

(b) Derive the corresponding integral equations and briefly discuss the physical meaning of each of the equations.

(40/100)

(c) For free space, where the charge and current densities are zero, derive the wave equation for the electric-field amplitude $\vec{E}$. Hence find the equation for the wave velocity c in terms of the fundamental constants ε_0 and μ_0.

(40/100)

...7/-

2. (a) Define the potential $V(\vec{r})$ for an electrostatic field and prove that in a charge-free region $V(\vec{r})$ satisfies Laplace's equation $\nabla^2 V = 0$.

(30/100)

(b) The general separated solution of Laplace's equation is $V(\vec{r}) = f_l(r)Y_{lm}(\theta,\phi)$ where l and m are integers and $f_l(r) = r^l$ or $r^{-(l+1)}$. For given l, what values of m are allowed?

(20/100)

(c) Using Cartesian coordinates $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$ derive the expressions for the fields $\vec{E}$ for $l = 1$ and $m = 0$ and the two forms of $f_l(r)$. Sketch the field lines and explain what the field patterns represent physically.

$$[Y_{10}(\theta,\phi) = (3/4\pi)^{1/2}\cos\theta]$$

(50/100)

3. (a) For magnetostatics, define the vector potential $\vec{A}(\vec{r})$.

(20/100)

(b) Explain what is meant by a gauge transformation of $\vec{A}(\vec{r})$ and by a gauge function χ.

(20/100)

(c) Consider a uniform magnetic field in the z direction, $\vec{B} = (0,0,B_0)$. Prove that the three vector potential functions (i) $\vec{A} = (0,B_0x,0)$, (ii) $\vec{A} = (-B_0y,0,0)$ and (iii) $\vec{A} = (-\frac{1}{2}B_0y,\frac{1}{2}B_0x,0)$ all correspond to this field $\vec{B}$.

(30/100)

(d) Find the three gauge functions corresponding to the gauge transformations between (i) and (ii), (ii) and (iii) and (i) and (iii).

(30/100)

...8/-

4. The real-field solution $\vec{E}_R$ of Maxwell's vacuum wave equation for a wave travelling in the z direction is written in terms of a complex vector amplitude $\vec{E}_0$ as

$$\vec{E}_R = \mathrm{Re}\left[\vec{E}_0 \exp(ikz - i\omega t)\right]$$

(a) Explain, using diagrams if necessary, how the quantities k and ω are related to wavelength and frequency.

(30/100)

(b) Writing the real magnetic field $\vec{B}_R$ as $\vec{B}_R = \mathrm{Re}\left[\vec{B}_0 \exp(ikz - i\omega t)\right]$ derive an expression for the complex quantity $\vec{B}_0$ in terms of $\vec{E}_0$, k and ω.

(20/100)

(c) Prove that both $\vec{E}_0$ and $\vec{B}_0$ are transverse, $E_{0z} = B_{0z} = 0$.

(20/100)

(d) Consider the point $z = 0$. Draw sketches to show how $\vec{E}_R$ and $\vec{B}_R$ vary with time t for waves that are (a) plane polarized with $\vec{E}_R$ along x and (b) right and left hand circularly polarized.

(30/100)

5. Assuming that the rate of change of field energy is

$$\frac{\partial U}{\partial t} = \varepsilon_0 \vec{E}.\frac{\partial \vec{E}}{\partial t} + \frac{1}{\mu_0}\vec{B}.\frac{\partial \vec{B}}{\partial t}$$

(a) use Maxwell's equations to derive Poynting's theorem for the electromagnetic field in vacuum when no charges or currents are present.

(20/100)

(b) Explain what physical property of the electromagnetic field is described by Poynting's theorem.

(20/100)

(c) The beam from a 2 mW He-Ne laser is passed through a polarizer and focussed to a spot of 0.5 mm diameter. Assume that half the power is lost in the polarizer and that the power density is uniform across the focussed spot. Taking the z axis as the direction of travel of the beam, use the Poynting vector to find the root-mean-square values of E and B in the focussed region.

(60/100)

...9/-

6. (a) Write down Maxwell's equations in differential form for a general medium with charge and current densities present.

(20/100)

(b) Prove that the differential form of the law of conservation of charge is a consequence of these equations.

(30/100)

(c) Integrate the charge-conservation law through a volume V and apply Gauss's theorem to the term involving the current density $\vec{j}$ to derive the integral form of this law.

(20/100)

(c) Use the integral form to explain why the law does correspond to charge conservation.

(30/100)

7. The optical properties of a monovalent metal like Ag or Na may be described by a frequency-dependent dielectric function $\varepsilon(\omega)$ of the form

$$\varepsilon(\omega) = 1 - \omega_p^2/\omega^2 \qquad \text{where } \omega_p^2 = ne^2/\varepsilon_0 m$$

in which n is the density of electrons in the metal and m is the effective mass of an electron. $f_p = \omega_p/2\pi$ is known as the *plasma frequency*.

(a) Assuming that $n = 2\times10^{29}$ m^{-3} and that m is equal to the free-electron mass m_e, find the numerical value of f_p and the corresponding wavelength $\lambda_p = c/f_p$ and confirm that these lie in the ultraviolet.

(30/100)

(b) Prove that $\varepsilon(\omega) \to -\infty$ as $\omega \to 0$, that $\varepsilon(\omega_p) = 0$ and that $\varepsilon(\omega) \to 1$ as $\omega \to \infty$. Hence sketch the graph of $\varepsilon(\omega)$ versus ω.

(30/100)

(c) Assuming that in a plane wave where all field vectors are proportional to $\exp[i\vec{k}.\vec{r} - i\omega t]$ the magnitude k and ω are related by $k^2 = \varepsilon(\omega)\omega^2/c^2$ use your sketch of $\varepsilon(\omega)$ to sketch the graph of k versus ω..

(40/100)

(Numerical values in SI units are $\varepsilon_0 = 8.854\times10^{-12}$ Fm^{-1} , $m_e = 9.109\times10^{-31}$ kg)

...10/-

8. (a) Explain what is meant by a metallic waveguide. For which part of the electromagnetic spectrum are such waveguides used?

(30/100)

(b) For a wave travelling in the x direction the propagation equation is

$$k_x^2 = \frac{\omega^2}{c^2} - \frac{n^2\pi^2}{d^2}$$

where d is the guide width and $n = 1,2,3,...$ is an integer. Sketch graphs to show the dependence of ω on k_x for a few values of n.

(40/100)

(c) Use your graphs to explain *cut-off frequency* and *monomode region.*

(30/100)

- oooOOooo -