UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2002/2003

April 2003

ZCT 304E/3 - Keelektrikan dan Kemagnetan II

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUABELAS** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab kesemua LIMA soalan. Pelajar dibenarkan menjawab semua soalan dalam bahasa Inggeris ATAU bahasa Malaysia ATAU kombinasi kedua-duanya.

1. (a) Suatu cakera bulat berjejari R mempunyai ketumpatan cas permukaan yang seragam σ. Carikan medan elektrik pada satu titik pada paksi cakera yang berjarak z dari satahnya. (8/20)

   (b) Suatu silinder bulat yang tegak berjejari R dan panjang L diletakkan di sepanjang paksi z. Silinder tersebut mempunyai ketumpatan isipadu tak seragam yang diberikan dengan persamaan $\rho(z) = \rho_o + \beta z$ merujuk kepada titik asalan pada pusat silinder. Carikan daya keatas satu titik cas q yang diletakkan pada pusat silinder tersebut. (*gunakan jawapan yang diperoleh dari bahagian (a)) (12/20)

2. Suatu cas q ditaburkan secara seragam pada keseluruhan suatu isipadu sferaan bukan pengkonduksi yang mempunyai jejari R.

   (a) Tunjukkan bahawa keupayaan pada titik yang berjarak r dari pusat, di mana $r < R$, diberikan dengan persamaan.

$$V = \frac{q(3R^2 - r^2)}{8\pi\varepsilon_o R^3}$$ (15/20)

   (b) Apakah keupayaan pada titik $r > R$? (5/20)

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3. Dua petala konduktor sfera sepusat berjejari r_1 dan r_2 ditetapkan pada keupayaan φ_1 and φ_2 tiap-tiap satunya. Kawasan antara petala sfera tersebut di penuhi dengan suatu bahan dielektrik.

   (a) Dengan pengiraan terus, tunjukkan tenaga yang tersimpan dalam dielektrik adalah bersamaan dengan $\frac{C(\varphi_1 - \varphi_2)^2}{2}$. (5/20)

   (b) Tentukan C, kapasitans sistem tersebut diatas. (15/20)

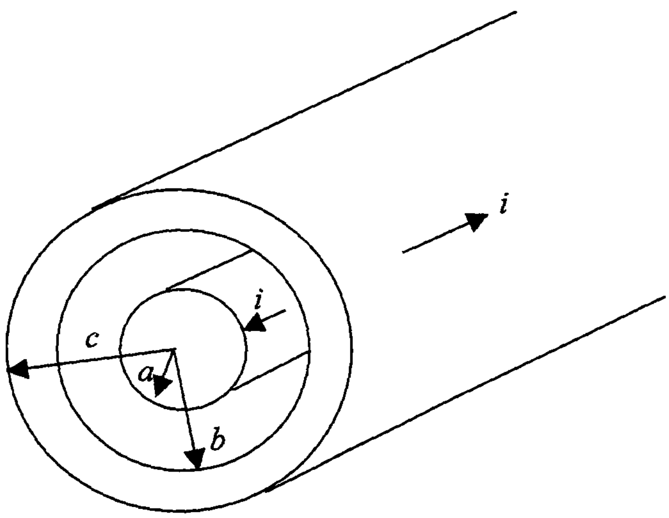
4. Suatu kabel sepaksi yang panjang terdiri daripada dua konduktor sepusat dengan jejarinya seperti ditunjukkan dalam rajah dibawah. Kabel konduktor-konduktor tersebut membawa arus i yang magnitudnya adalah sama tetapi arahnya bertentangan. Tentukan medan magnet B di r jika

   (a) $r < a$, (5/20)

   (b) $a < r < b$, (5/20)

   (c) $b < r < c$ dan (5/20)

   (d) $r > c$ (di luar kabel) (5/20)



5. Tunjukkan keupayaan vektor kemagnetan untuk dua dawai panjang, lurus dan selari yang membawa arus I yang sama tetapi bertentangan arah diberikan oleh

$$\vec{A} = \frac{\mu_o I}{2\pi} \ln\left(\frac{r_2}{r_1}\right)\hat{n},$$

dimana r_2 dan r_1 adalah jarak-jarak dari titik medan ke dawai-dawai berkenaan dan $\hat{n}$ ialah vektor unit selari dengan dawai-dawai tersebut.

(20/20)

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Third Semester Examination
2002/2003 Academic Session

April 2003

ZCT 304E/3 - Electricity and Magnetism II

Time : 3 hours

Please check that the examination paper consists of **TWELVE** printed pages before you commence this examination.

Answer all FIVE questions. Students are allowed to answer all questions in English OR bahasa Malaysia OR combinations of both.

1. (a) A circular disk of radius R has a uniform surface charge density σ. Find the electrical field at a point on the axis of the disk at a distance z from the plane of the disk. (8/20)

   (b) A right circular cylinder of radius R and height L is oriented along the z-axis. It has a nonuniform volume density of charge given by $\rho(z)=\rho_o+\beta z$ with reference to an origin at the center of the cylinder. Find the force on a point charge q placed at the center of the cylinder. (*hint: use the answer obtained from part (a))

   (12/20)

2. A charge q is distributed uniformly throughout a non-conducting spherical volume of radius R.

   (a) Show that the potential a distance r from the center, where $r < R$ is given by

   $$V=\frac{q(3R^2-r^2)}{8\pi\varepsilon_o R^3}$$ (15/20)

   (b) What is the potential at a point $r > R$?

   (5/20)

3. Two concentric, spherical, conducting shells of radii r_1 and r_2 are maintained at potentials φ_1 and φ_2 respectively. The region between the shells is filled with a dielectric medium.

   (a) Show by direct calculation that the energy stored in the dielectric is equal to $\dfrac{C(\varphi_1 - \varphi_2)^2}{2}$. (5/20)

   (b) Determine C, the capacitance of the system. (15/20)

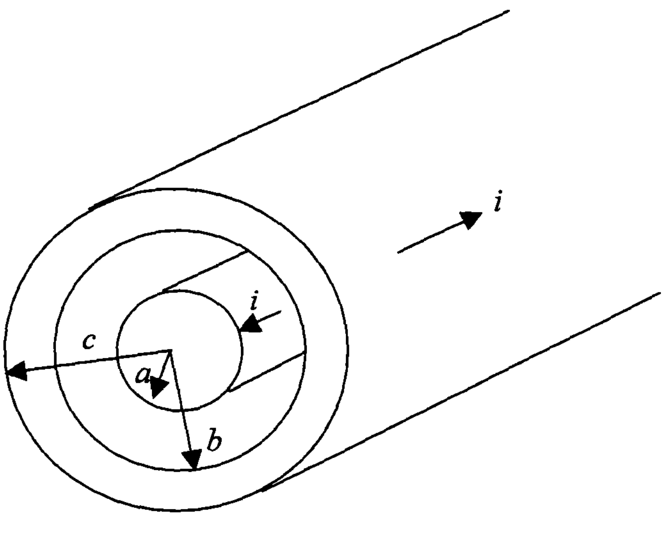
4. A long coaxial cable consists of two concentric conductors with the dimensions shown below. There are equal and opposite currents i in the conductors. Determine the magnetic field B at r if

   (a) $r < a$, (5/20)

   (b) $a < r < b$, (5/20)

   (c) $b < r < c$ and (5/20)

   (d) $r > c$ (outside the cable) (5/20)



5. Show that the magnetic vector potential for two long, straight, parallel wires carrying the same current, I in opposite directions is given by

$$\vec{A} = \frac{\mu_o I}{2\pi} \ln\left(\frac{r_2}{r_1}\right) \hat{n},$$

where r_2 and r_1 are the distances from the field point to the wires, and $\hat{n}$ is a unit vector parallel to the wires.

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Mathematical Guidance

Possibly Useful Integrals:

$$\int_{-1}^{1}\frac{(z-r\mu)d\mu}{(r^2+z^2-2zr\mu)^{3/2}}=\frac{1}{z^2}\left(\frac{z-r}{|z-r|}+\frac{z+r}{|z+r|}\right)$$

$$\int_{-1}^{1}\frac{d\mu}{(r^2+z^2-2zr\mu)^{1/2}}=\frac{1}{zr}\left(|z+r|-\lfloor z-r\rfloor\right)$$

$$\int\frac{dx}{(x^2+a^2)^{3/2}}=\frac{1}{a^2}\cdot\frac{x}{(x^2+a^2)^{1/2}}$$

$$\int xe^{ax}dx=e^{ax}\left[\frac{x}{a}-\frac{1}{a^2}\right]$$

<u>Useful Constants</u>

$$k=\frac{1}{4\pi\varepsilon_0}=8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}$$

$$e=1.60\times10^{-19}\,C$$

$$\varepsilon_0=8.85\times10^{-12}\,\frac{C^2}{N\cdot m^2}$$

$$\mu_0=4\pi\times10^{-7}\,\frac{T\cdot m}{A}$$

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Vector Calculus

Cartesian Coordinates

$$\vec{\nabla}u = \hat{x}\frac{\partial u}{\partial x} + \hat{y}\frac{\partial u}{\partial y} + \hat{z}\frac{\partial u}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \hat{x}(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}) + \hat{y}(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}) + \hat{z}(\frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x})$$

$d\tau = dxdydz$ $\quad da_x = \pm dydz \quad da_y = \pm dxdz \quad da_z = \pm dxdy$

Cylindrical Coordinates

$$\vec{\nabla}u = \hat{\rho}\frac{\partial u}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial u}{\partial \phi} + \hat{z}\frac{\partial u}{\partial z}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla}\times\vec{A} = \hat{\rho}(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}) + \hat{\phi}(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}) + \hat{z}[\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \phi}]$$

$d\tau = \rho d\rho d\phi dz$ $\quad da_\rho = \pm \rho d\phi dz \quad da_\phi = \pm d\rho dz \quad da_z = \pm \rho d\rho d\phi$

$\hat{\rho} = \cos\phi\hat{x} + \sin\phi\hat{y}$ $\quad \hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y}$

Spherical Coordinates

$$\vec{\nabla}u = \hat{r}\frac{\partial u}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial u}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial u}{\partial \phi}$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\vec{\nabla}\times\vec{A} = \frac{\hat{r}}{r\sin\theta}[\frac{\partial}{\partial \theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial \phi}] + \frac{\hat{\theta}}{r}[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)] + \frac{\hat{\phi}}{r}[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}]$$

$d\tau = r^2 \sin\theta dr d\theta d\phi$ $\quad da_r = \pm r^2 \sin\theta d\theta d\phi \quad da_\theta = \pm r\sin\theta dr d\phi \quad da_\phi = \pm r dr d\theta$

$\hat{r} = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z}$ $\quad \hat{\theta} = \cos\theta\cos\phi\hat{x} + \cos\theta\sin\phi\hat{y} - \sin\theta\hat{z}$

$\hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y}$

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Important Equations

Maxwell's Equations:

$\vec{\nabla}\cdot\vec{D}=\rho_f$ $\quad\vec{\nabla}\cdot\vec{B}=0$ $\quad\vec{\nabla}\times\vec{E}=\frac{-\partial\vec{B}}{dt}$ $\quad\vec{\nabla}\times\vec{H}=J_f+\frac{\partial\vec{D}}{dt}$

Lorentz Force: $\vec{F}=q(\vec{E}+\vec{v}\times\vec{B})$

Equation of Continuity: $\vec{\nabla}\cdot\vec{J}_f+\frac{\partial\rho_f}{dt}=0$

Coulomb's Law: $\vec{F}_q=\sum_i\frac{qq_i\vec{R}_i}{4\pi\varepsilon_0R_i^3}$ (for a collection of point charges)

$\vec{F}_q=\frac{q}{4\pi\varepsilon_0}\int_{L'}\frac{\lambda(\vec{r}\,')\vec{R}ds'}{R^3}$ (for a line charge distribution)

$\vec{F}_q=\frac{q}{4\pi\varepsilon_0}\int_{S'}\frac{\sigma(\vec{r}\,')\vec{R}da'}{R^3}$ (for a surface charge distribution)

$\vec{F}_q=\frac{q}{4\pi\varepsilon_0}\int_{V'}\frac{\rho(\vec{r}\,')\vec{R}d\tau'}{R^3}$ (for a volume charge distribution)

Electric Field: $\vec{E}=\frac{\vec{F}_q}{q}$

Electric Flux: $\Phi_e=\int\vec{E}\cdot d\vec{a}$

Gauss' Law: $\oint_S\vec{E}\cdot d\vec{a}=\frac{Q_t}{\varepsilon_0}$ (integral form)

$\vec{\nabla}\cdot\vec{E}=\frac{\rho(\vec{r})}{\varepsilon_0}$ (differential form)

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Scalar Potential: $\phi(\vec{r}) = \sum_i \frac{q_i}{4\pi\varepsilon_0 R_i}$ (for a collection of point charges)

$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{L'} \frac{\lambda(\vec{r}\,')ds'}{R}$ (for a line charge distribution)

$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{S'} \frac{\sigma(\vec{r}\,')da'}{R}$ (for a surface charge distribution)

$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{V'} \frac{\rho(\vec{r}\,')d\tau'}{R}$ (for a volume charge distribution)

Potential Energy: $U_e(\vec{r}) = q\phi(\vec{r})$ (for an isolated point charge)

$U_e = \frac{1}{2}\sum_i q_i \phi_i(\vec{r}_i)$ (for a collection of point charges)

$U_e = \frac{1}{2}\int_L \lambda(\vec{r})\phi(\vec{r})ds$ (for a line charge distribution)

$U_e = \frac{1}{2}\int_S \sigma(\vec{r})\phi(\vec{r})da$ (for a surface charge distribution)

$U_e = \frac{1}{2}\int_V \rho(\vec{r})\phi(\vec{r})d\tau$ (for a volume charge distribution)

$u_e = \frac{1}{2}\varepsilon_0 E^2$ (energy density in an electric field)

$U_e = \int u_e d\tau$ (total energy)

Multipole Moments: $Q = \sum_i q_i$ or $Q = \int_L \lambda ds$ or $Q = \int_S \sigma da$ or $Q = \int_V \rho d\tau$ (monopole)

$\vec{p} = \sum_i q_i \vec{r}_i$ or $\vec{p} = \int_L \lambda \vec{r} ds$ or $\vec{p} = \int_S \sigma \vec{r} da$ or $\vec{p} = \int_V \rho \vec{r} d\tau$ (dipole)

Boundary Conditions: $E_{t2} - E_{t1} = 0$ and $E_{n2} - E_{n1} = \frac{\sigma}{\varepsilon_0}$ (electric field)

$\phi_2 = \phi_1$ (scalar potential)

$B_{n2} - B_{n1} = 0$ and $\vec{B}_{t2} - \vec{B}_{t1} = \mu_0 \vec{K} \times \hat{n}$ (magnetic induction)

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Electricity in Matter: $\rho = \rho_f + \rho_b$ (free charge and bound charge)

$\rho_b = -\vec{\nabla} \cdot \vec{P}$ and $\sigma_b = \vec{P} \cdot \hat{n}$ (bound charge densities)

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ (definition of electric displacement)

$\vec{D} = \kappa_e \varepsilon_0 \vec{E} = \varepsilon \vec{E}$ (for an l.i.h. dielectric)

$u_e = \frac{1}{2} \vec{D} \cdot \vec{E}$ (energy density in matter)

$\oint \vec{D} \cdot d\vec{a} = Q_{f,in}$ and $\vec{\nabla} \cdot \vec{D} = \rho_f$ (Gauss' Laws for $\vec{D}$)

Electric Current: $I = \frac{dq}{dt} = \int \vec{J} \cdot d\vec{a} = \int \vec{K} \cdot d\vec{s}$

$\vec{J} = \rho \vec{v}$ $\vec{K} = \sigma \vec{v}$ (current density)

$I d\vec{s} = \vec{K} da = \vec{J} d\tau$ (current elements)

$\vec{J}_f = \sigma \vec{E}$ (Ohm's Law)

Magnetostatic Force: $\vec{F}_{C' \to C} = \frac{\mu_0}{4\pi} \oint_C \oint_{C'} \frac{I d\vec{s} \times \left(I' d\vec{s}\,' \times \hat{R} \right)}{R^2}$

Magnetic Induction: $\vec{B} = \frac{\mu_0}{4\pi} \oint_{C'} \frac{I' d\vec{s}\,' \times \hat{R}}{R^2}$ (for a filamentary current)

$\vec{B} = \frac{\mu_0}{4\pi} \int_{S'} \frac{\vec{K}' \times \hat{R} da'}{R^2}$ (for a surface current)

$\vec{B} = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{J}' \times \hat{R} d\tau'}{R^2}$ (for a volume current)

Ampere's Law: $\oint_C \vec{B} \cdot d\vec{s} = \mu_0 I_t$ (integral form)

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ (differential form)

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Vector Potential: $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\vec{A} = \frac{\mu_0}{4\pi} \oint_{C'} \frac{I' d\vec{s}'}{R}$$ (for a filamentary current)

$$\vec{A} = \frac{\mu_0}{4\pi} \int_{S'} \frac{\vec{K}' da'}{R}$$ (for a surface current)

$$\vec{A} = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{J}' d\tau'}{R}$$ (for a volume current)

Magnetic Flux: $\Phi_b = \int \vec{B} \cdot d\vec{a}$

Faraday's Law: $\varepsilon_t = \oint \vec{E}_t \cdot d\vec{s} = \frac{-d\Phi_b}{dt}$ (integral form)

$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{dt}$$ (differential form)

Magnetism in Matter: $\vec{J} = \vec{J}_f + \vec{J}_m$ (free current plus magnetisation current)

$\vec{J}_m = \vec{\nabla} \times \vec{M}$ (magnetisation volume current density)

$\vec{K}_m = \vec{M} \times \hat{n}$ (magnetisation surfacet density)

$\vec{B} = \mu_0(\vec{H} + \vec{M})$ (definition of magnetic field)

$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H}$ (for l.i.h. material)

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